1. (a) Show that, in general, the Hamiltonian operator is Hermitian.

(b) Show that if $|p>$ and $|q>$ are the eigenfunctions of a Hermitian operator corresponding to different eigenvalues then $|p>$ and $|q>$ must be orthogonal.

2. (a) Show that if $n(g)$ is any function in the reciprocal space, then $n(g+G)=n(g)$ where $G$ is the reciprocal lattice vector.

(b) Show that the distance between two adjacent parallel planes (hkl) is $2\pi/|G|$

3. (E.5.4) Write down the eigen values of the following matrix using the principles of bandstructure

\[
\begin{pmatrix}
   a & b & c & 0 & c & b \\
   b & a & b & 0 & c \\
   c & b & a & b & c \\
   0 & c & b & a & b \\
   c & 0 & c & b & a \\
   b & c & 0 & c & b & a \\
\end{pmatrix}
\]

4. Using a two atom unit cell and one basis per cell (c.f. Fig. 5.1.5), write down the Hamiltonian so that the upper band is a conduction band (minima at $k=0$) and the lower band is a valence band (maxima at $k=0$). Use $E_0$, $E_{ss}$ and $E_{ss'}$ of your choice to obtain the desired result. Also write down the dispersion relation.

5. Using the parameters given for GaAs in pg. 120 (given below), write a program to plot the E-K diagrams (i)in the (110) (ii)(101) and (0 11) directions.

\[
\begin{align*}
E_{ss} &= -8.3431 \text{ eV} & E_{pa} &= 1.0414 \text{ eV} & E_{sa} &= 8.5914 \text{ eV} \\
E_{sc} &= -2.6569 \text{ eV} & E_{pc} &= 3.6686 \text{ eV} & E'_{sa} &= 6.7386 \text{ eV} \\
4E_{ss} &= -6.4513 \text{ eV} & 4E_{pasc} &= -5.7839 \text{ eV} & 4E_{pasc} &= -4.8077 \text{ eV} \\
4E_{sa} &= 4.48 \text{ eV} & 4E_{s'pasc} &= 4.8422 \text{ eV} & \\
4E_{sy} &= 1.9546 \text{ eV} & 4E_{s'y} &= 5.0779 \text{ eV}
\end{align*}
\]