HW #4
Due October 10 (Thursday) in class

1. Consider an infrared intersubband photodetector made of p-doped GaAs quantum wells. The hole effective mass is \( m_0^* = 0.5m_0 \), and the refractive index is 3.5.
   a. Find the width of the quantum well so the absorption peak is at 10 µm wavelength.
   b. Assume a lorentzian lineshape with an intraband scattering time of 0.1 ps, find the peak absorption coefficient for a doping concentration of \( 10^{18} \text{ cm}^{-3} \).
   c. What is the full-width-at-half-maximum width of the absorption spectrum?
   d. Plot the absorption spectra. Please use your favorite numeric program to calculate and plot the spectrum (no hand sketch). Be quantitative in both axes.
   e. What is the optimum doping concentration to achieve maximum absorption coefficient?

2. In this problem, you will calculate and plot the band diagram of an P-Al_{0.4}Ga_{0.6}As / i-GaAs / N-Al_{0.4}Ga_{0.6}As double heterojunction with \( N_p = 3 \times 10^{17} \text{ cm}^{-3} \) and \( N_d = 3 \times 10^{17} \text{ cm}^{-3} \). The GaAs is intrinsic. The thickness of the GaAs layer is 0.1µm. Use the material properties listed in the Table below.

<table>
<thead>
<tr>
<th>Property</th>
<th>Unit</th>
<th>GaAs</th>
<th>Al_{x}Ga_{1-x}As, 0&lt;x&lt;0.45</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bandgap Energy</td>
<td>eV</td>
<td>1.424</td>
<td>1.424 + 1.247x</td>
</tr>
<tr>
<td>Electron Effective Mass</td>
<td>m_0</td>
<td>0.067</td>
<td>0.067 + 0.083x</td>
</tr>
<tr>
<td>Hole Effective Mass</td>
<td>m_0</td>
<td>0.5</td>
<td>0.5 + 0.29x</td>
</tr>
<tr>
<td>Dielectric Constant</td>
<td>( \epsilon_0 )</td>
<td>13.1</td>
<td>13.1 – 3x</td>
</tr>
<tr>
<td>Conduction Band Discontinuity</td>
<td>%</td>
<td>-</td>
<td>( \Delta E_C \sim 67% \Delta E_g )</td>
</tr>
<tr>
<td>Valence Band Discontinuity</td>
<td>%</td>
<td>-</td>
<td>( \Delta E_V \sim 33% \Delta E_g )</td>
</tr>
</tbody>
</table>

The conduction and valence band density of states are

\[
N_c = 2 \left( \frac{\pi m_e^* k_B T}{2 \pi^2 \hbar^2} \right)^{3/2} = 2.5 \times 10^{10} \left( \frac{m_e^*}{m_0} \right) \left( \frac{T}{300} \right)^{3/2}
\]

\[
N_v = 2 \left( \frac{\pi m_h^* k_B T}{2 \pi^2 \hbar^2} \right)^{3/2} = 2.5 \times 10^{10} \left( \frac{m_h^*}{m_0} \right) \left( \frac{T}{300} \right)^{3/2}
\]

a. Calculate Fermi energy in each individual semiconductor. Find the contact potential (built-in potential), \( V_0 \).

b. Assume the depletion region on the P and the N sides are -0.05µm – \( x_P \) and 0.05µm + \( x_N \), respectively. We will solve for \( x_P \) and \( x_N \) later. Plot the charge distribution \( \rho(x) \). What is the relation between \( x_P \) and \( x_N \)? \( \textit{Hint: there is no charge in the i-region} \).

c. Calculate and plot the electric field distribution \( E(x) \). Show the analytical expression. \( \textit{Hint: the electric field in the i-region should be constant} \).
d. Calculate and plot the electron potential energy distribution, $-q\phi(x)$. Show the analytical expression. (Hint: the electron potential energy varies linearly in the i-region).

e. Equate the electron potential difference between the N- and the P-AlGaAs to the contact potential, $V_0$, solve for $x_P$ and $x_N$.

f. Now plot the entire band diagram quantitatively.