EE 232 Lightwave Devices
Lecture 18: Photoconductors and p-i-n Photodiodes

Reading: Chuang, Chap. 14

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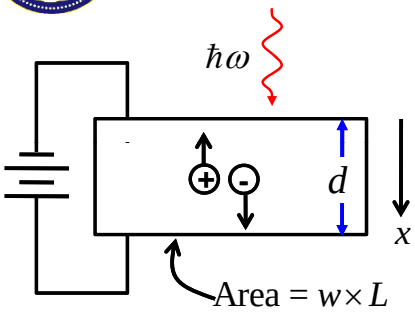
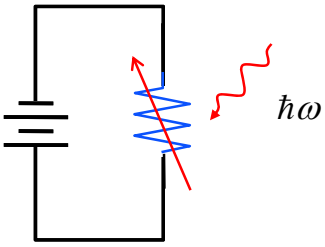
Photoconductors



Equivalent Circuit



Dark current:

$$J_0 = \sigma_0 E = \left(n_0 q \mu_n + p_0 q \mu_p\right) E$$

Light illumination generate electron-hole pairs, increasing the conductivity:

$$\frac{d\delta n}{dt} = G_0 - \frac{\delta n}{\tau_n}$$

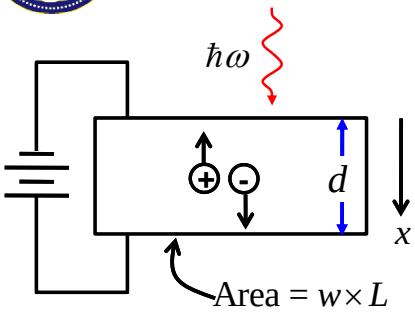
Steady state: d/dt --> 0

$$\delta n = G_0 \tau_n$$

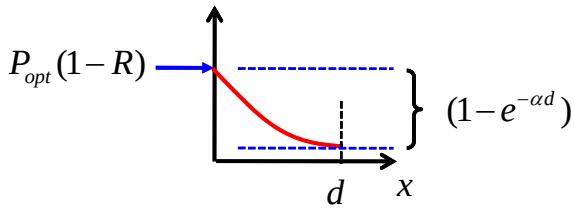
$$\Delta J = \delta n \cdot q\left(\mu_n + \mu_p\right) E$$

Photoconductor requires both contacts to be Ohmi

Photocarrier Generation Rate



Light Intensity $\propto e^{-\alpha x}$



$$G_0 = \eta \frac{P_{opt}}{\hbar\omega} \frac{1}{lwd} \quad \text{: photocarrier generation rate } [\frac{1}{cm^3 s}]$$

$$\eta = \eta_i (1-R)(1-e^{-\alpha d})$$

R: reflectivity of photoconductor surface

α: absorption coefficient

d: absorption lenght

$e^{-\alpha d}$: fraction of light remains after absorption length d

Photoconductive Gain

$$\Delta I = lw\Delta J = lw\left(G_0 \tau_n q\right)\left(\mu_n + \mu_p\right)E$$

$$\Delta I = lw\left(\eta \frac{P_{opt}}{\hbar\omega} \frac{1}{lwd} \tau_n\right) q\left(\mu_n + \mu_p\right)E$$

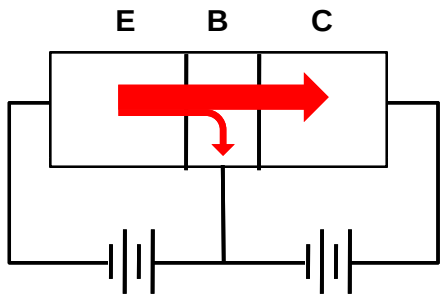
$$\Delta I \approx \eta \frac{P_{opt}}{\hbar\omega} \frac{1}{d} \tau_n q\left(\mu_n E\right) = \eta P_{opt} \frac{q}{\hbar\omega} \tau_n \frac{1}{d} v_n$$

$$\tau_t = \frac{d}{v_n} \quad \text{: transit time}$$

$$\boxed{\Delta I = \left(\eta P_{opt} \frac{q}{\hbar\omega}\right)\left(\frac{\tau_n}{\tau_t}\right)}$$

Photocurrent **Photoconductive Gain**

Analogy to Current Gain in Bipolar Transistor



Current gain in bipolar transistor:

$$\beta = \frac{I_C}{I_B}$$

The current gain can also be expressed as

$$\beta = \frac{\tau_{rb}}{\tau_t}$$

τ_t : transit time

τ_{rb} : carrier recombination lifetime in the base

Frequency of Photoconductors

$$\frac{dN}{dt} = \eta \frac{P_{opt}}{\hbar\omega} \frac{1}{lwd} - \frac{N}{\tau_n}$$

Small signal response:

$$N = N_0 + N_1 e^{j\omega t}$$

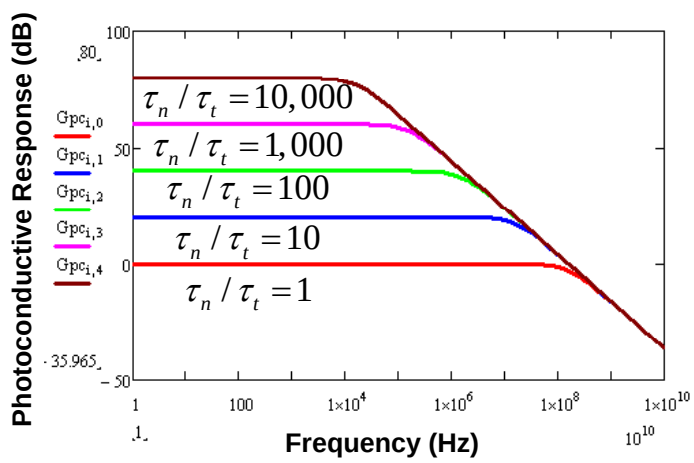
$$j\omega N_1 = \eta \frac{P_1}{\hbar\omega} \frac{1}{lwd} - \frac{N_1}{\tau_n}$$

$$N_1 = \frac{\eta P_1}{\hbar\omega(lwd)} \frac{1}{j\omega + 1/\tau_n}$$

$$I_1 = J_1 lw = (N_1 q v_n) lw$$

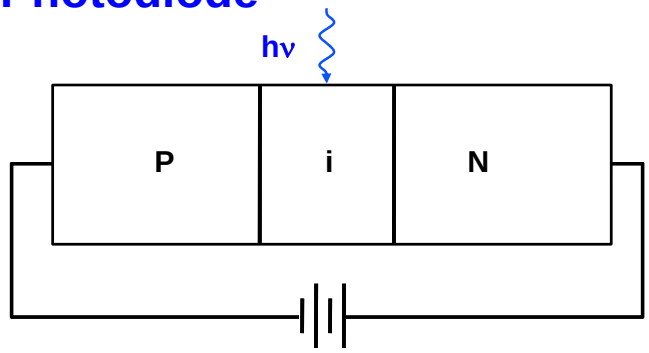
$$\frac{I_1}{P_1} = \left(\frac{\eta q}{\hbar\omega}\right)\left(\frac{\tau_n}{\tau_t}\right)\frac{1}{j\omega\tau_n + 1}$$

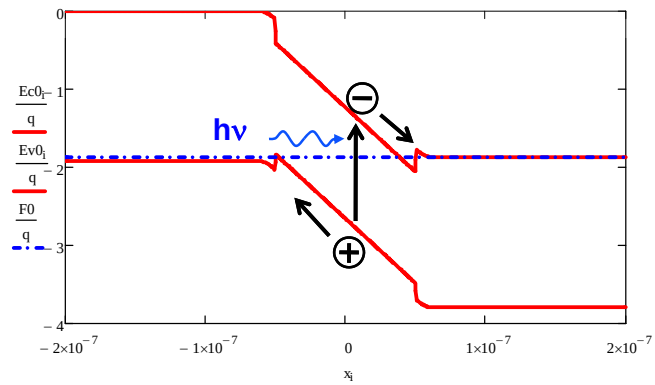
$$= (\text{DC Quantum Efficiency}) \times (\text{Photoconductive Gain}) \times$$
$$(\text{Normalized Frequency Response})$$



P-i-n Photodiode

- Reverse-biased p-i-n junction
- Most of the voltage drop across the i-region, the main absorption region
- High field separates photogenerated electron and hole
- Large bandgap materials are used for P and N if possible
- Fast response
- Low noise
- No gain (quantum efficiency < 100%)





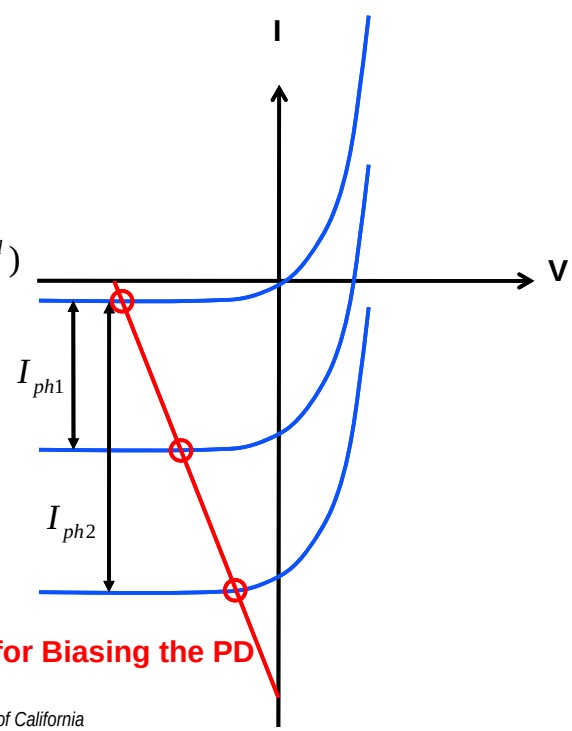
I-V Curve

Dark current: $I = I_0(e^{\frac{qV}{k_B T}} - 1)$

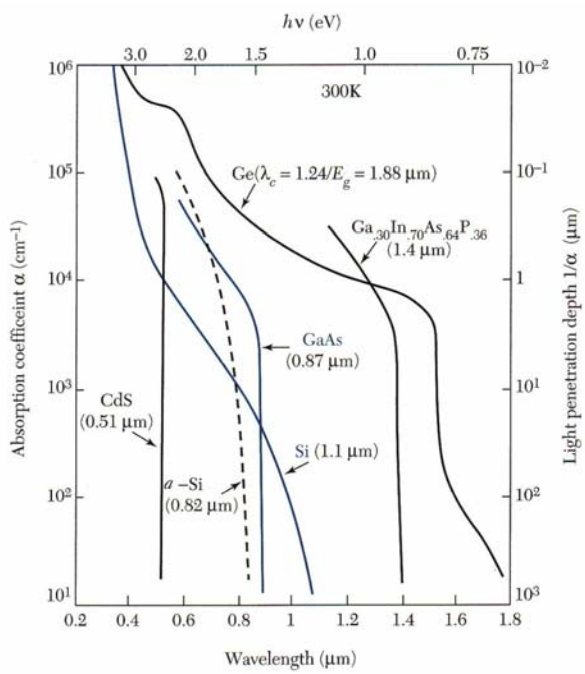
Photocurrent: $I_{ph} = \frac{\eta q}{\hbar\omega} P_{opt}$

Quantum efficiency: $\eta = \eta_i (1-R)(1-e^{-\alpha d})$

Total current: $I = I_0(e^{\frac{qV}{k_B T}} - 1) + I_{ph}$



Absorption Coefficient

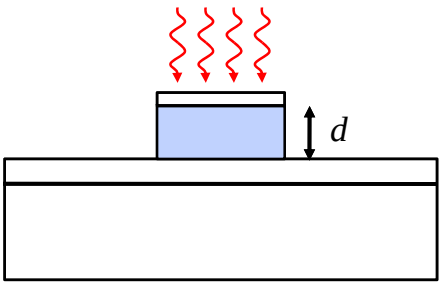


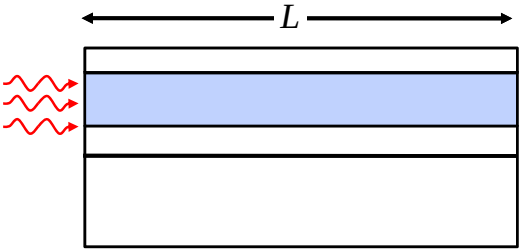
- **Light intensity decays exponentially in semiconductor:**

$$I(x) = I_0 e^{-\alpha x}$$

- **Direct bandgap semiconductor has a sharp absorption edge**
- **Si absorbs photons with hv > E_g = 1.1 eV, but the absorption coefficient is small**
  - **Sufficient for CCD**
- **At higher energy (~ 3 eV), absorption coefficient of Si becomes large again, due to direct bandgap transition to higher CB**

Two Types of p-i-n Photodiodes





Surface-Illuminated p-i-n

$\eta = \eta_i (1-R)(1-e^{-\alpha d})$

η_i : internal quantum efficiency

R : reflectivity

d : absorption layer thickness

Waveguide p-i-n

$\eta = \eta_i (1-R)(1-e^{-\Gamma \alpha L})$

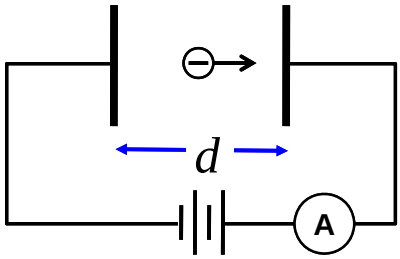
η_i : internal quantum efficiency

R : reflectivity

Γ : confinement factor

L : length of waveguide PD

Ramo's Theorem



The current caused in external circuit by a moving charge q moving at a volocity $v(t)$ in a parallel plate with a separation of d and a voltage bias of V is

$$i(t) = \frac{qv(t)}{d}$$

Proof:

Work done on the charge:

$$W = \text{Force} \times \text{Displacement}$$

$$= qEdx = q\frac{V}{d}dx$$

Work provided by power supply:

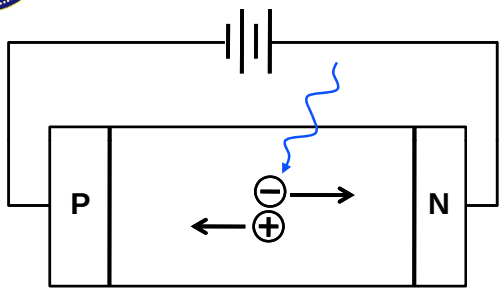
$$W = i(t)Vdt$$

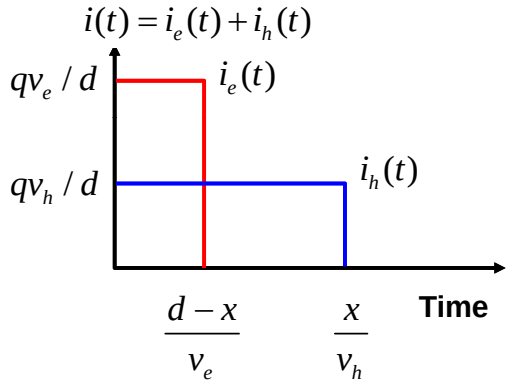
$$\Rightarrow$$

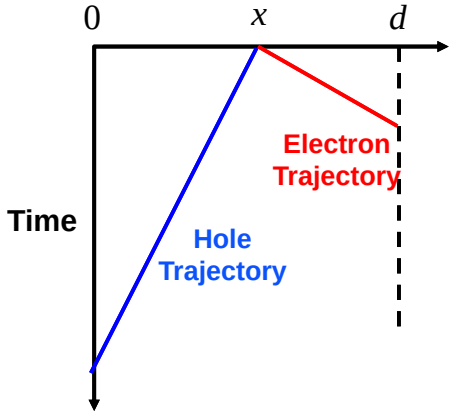
$$i(t)Vdt = q\frac{V}{d}dx$$

$$i(t) = \frac{q}{d}\frac{dx}{dt} = \frac{qv(t)}{d}$$

Response of One Photogenerated Electron-Hole Pair







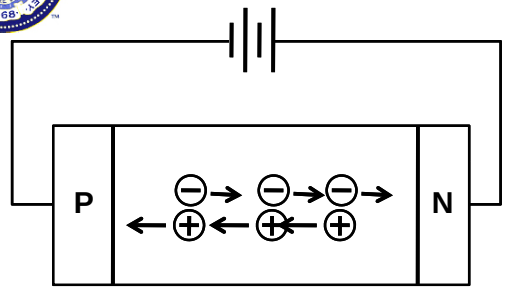
Total charge generated:

$$Q = \int_0^\infty i_e(t)dt + \int_0^\infty i_h(t)dt$$

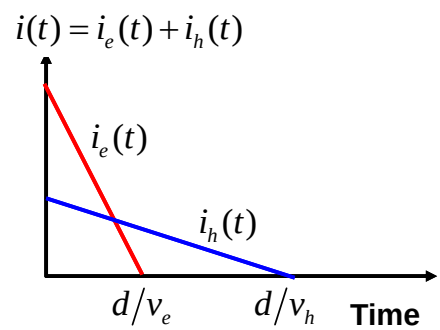
$$= \frac{qv_e}{d}\frac{d-x}{v_e} + \frac{qv_h}{d}\frac{x}{v_h} = q$$

One absorbed photon $\rightarrow$ one charge detected

Transit Time



Many electron-hole pairs generated



Electron current ends when the last electron generated near P-side reaches N-electrode: $t = d/v_e$

Hole current ends when the last hole generated near N-side reaches P-electrode: $t = d/v_h$

Hole is usually slower $\rightarrow$ A conservative estimate of the transit time: $\boxed{\tau_t = \dfrac{d}{v_h}}$

Total Response Time of p-i-n

(1) RC time:

$$\tau_{RC} = RC = R\frac{\varepsilon A}{d} \qquad (A:\ \text{area of p-i-n})$$

(2) Transit time:

$$\tau_t = \frac{d}{v_h}$$

Total response time:

$$\tau = \tau_{RC} + \tau_t$$

$$f_{3dB} \approx \frac{1}{2\pi\tau}$$

Absorption layer thickness for optimum frequency response:

$$\tau = \tau_{RC} + \tau_t = \frac{R\varepsilon A}{d} + \frac{d}{v_h}$$

$$\tau \geq 2\sqrt{\left(\frac{R\varepsilon A}{d}\right)\left(\frac{d}{v_h}\right)}$$

$$f_{3dB} \approx \frac{1}{2\pi\tau} \leq \frac{1}{4\pi}\sqrt{\frac{v_h}{R\varepsilon A}} = f_{3dB,\max}$$

Optimum bandwidth occurs when

$$\frac{R\varepsilon A}{d} = \frac{d}{v_h}$$

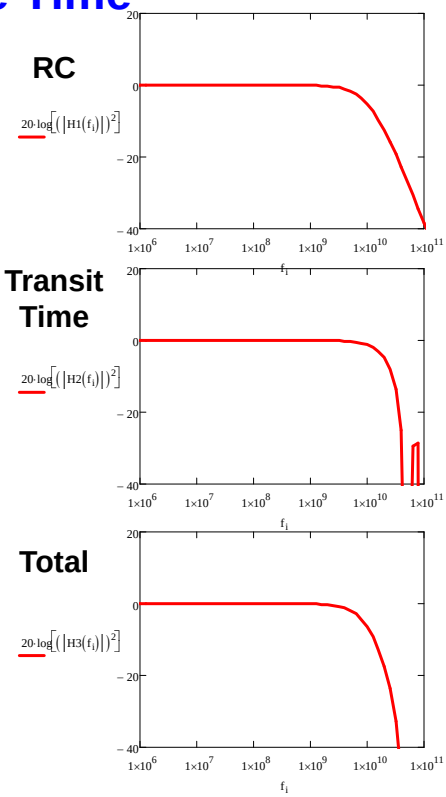
$$d_{optimum} = \sqrt{R\varepsilon A v_h}$$

More Rigorous Analysis of p-i-n Response Time

Small-signal analysis: assume the input light is modulated at frequency ω, the photocurrent is proportional to

$$|i(t)| \propto \left|\frac{1}{1+j\omega RC}\right| \left|\frac{\sin^2\left(\frac{\omega\tau_t}{2}\right)}{\left(\frac{\omega\tau_t}{2}\right)^2}\right|$$

The first term is single-pole response from RC, while the second term is the phase delay due to transit time response.



Comparison of Numeric Examples

Example:

$\tau_{RC} = 14.4\,ps$

$\tau_t = 20\,ps$

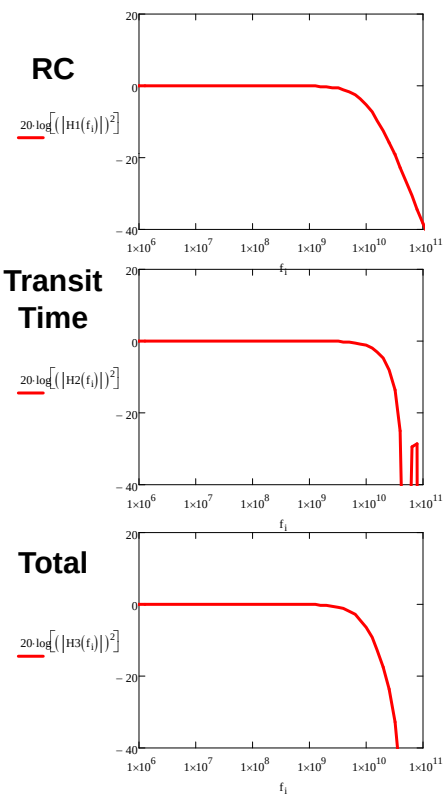
$$f_{3dB} = \frac{1}{2\pi}\frac{1}{\tau_{RC}+\tau_t} = 4.6 \text{ GHz}$$

$$|i(t)| \propto \left|\frac{1}{1+j\omega RC}\right| \left|\frac{\sin^2\left(\frac{\omega\tau_t}{2}\right)}{\left(\frac{\omega\tau_t}{2}\right)^2}\right| = |H(\omega)|$$

Solving $|H(\omega)| = \frac{1}{\sqrt{2}}$, $f_{3dB} = 9.7$ GHz

The discrepancy is smaller when RC dominates, and larger when transit time dominates.

(Transit time response has a sharp drop-off).



Bandwidth-Efficiency Product

(1) For surface-illuminated p-i-n (assume AR coating: R=0%), in the extreme of thin absorbing layer and transit-time-dominated response:

$$\eta = \eta_i(1-e^{-\alpha d}) \approx \eta_i(1-(1-\alpha d)) = \eta_i \alpha d$$

$$f_{3dB} \approx \frac{1}{2\pi}\frac{v_h}{d}$$

Bandwidth-efficiency product: $f_{3dB} \times \eta \approx \left(\frac{1}{2\pi}\frac{v_h}{d}\right)\left(\eta_i \alpha d\right) = \frac{\eta_i \alpha v_h}{2\pi}$

(2) On the other hand, the efficiency of waveguide p-i-n is

$$\eta = \eta_i(1-e^{-\Gamma \alpha L}) \approx \eta_i \Gamma \alpha L$$

RC-limited bandwidth: $f_{3dB} \approx \frac{1}{2\pi}\frac{d}{R\varepsilon L w}$

Bandwidth-efficiency product: $f_{3dB} \times \eta \approx \left(\frac{1}{2\pi}\frac{d}{R\varepsilon L w}\right)\left(\eta_i \Gamma \alpha L\right) = \frac{\eta_i \Gamma \alpha d}{2\pi R\varepsilon w}$

$\Rightarrow$ In general, there is a bandwidth-efficiency trade-off