Photoconductors

Dark current:
\[ J_0 = \sigma_0 E = \left( n_0 q \mu_n + p_0 q \mu_p \right) E \]

Light illumination generate electron-hole pairs, increasing the conductivity:
\[ \frac{d \delta n}{dt} = G_0 - \frac{\delta n}{\tau_n} \]
Steady state: \( \frac{d}{dt} \rightarrow 0 \)
\[ \delta n = G_0 \tau_n \]
\[ \Delta J = \delta n \cdot q \left( \mu_n + \mu_p \right) E \]

Photoconductor requires both contacts to be Ohmic.
**Photocarrier Generation Rate**

\[
G_0 = \eta \frac{P_{\text{opt}}}{\hbar \omega} \frac{1}{l \omega d} : \text{photocarrier generation rate} \left[ \frac{1}{cm^3 s} \right]
\]

\[
\eta = \eta_i (1 - R)(1 - e^{-\alpha d})
\]

- \(R\): reflectivity of photoconductor surface
- \(\alpha\): absorption coefficient
- \(d\): absorption length
- \(e^{-\alpha d}\): fraction of light remains after absorption length \(d\)

**Photoconductive Gain**

\[
\Delta I = lw \Delta J = lw \left( G_0 \tau_n q \left( \mu_n + \mu_p \right) E \right)
\]

\[
\Delta I = lw \left( \eta \frac{P_{\text{opt}}}{\hbar \omega} \frac{1}{l \omega d} \tau_n \right) q \left( \mu_n + \mu_p \right) E
\]

\[
\Delta I \approx \eta \frac{P_{\text{opt}}}{\hbar \omega} \frac{1}{d} \tau_n q (\mu_n E) = \eta P_{\text{opt}} \frac{q}{\hbar \omega} \tau_n \frac{1}{d} v_n
\]

\[
\tau_i = \frac{d}{v_n} : \text{transit time}
\]

\[
\Delta I = \left( \eta P_{\text{opt}} \frac{q}{\hbar \omega} \right) \left( \frac{\tau_n}{\tau_i} \right)
\]

**Photocarrier Generation Rate**

Light Intensity \(\propto e^{-\alpha x}\)

\[
P_{\text{opt}}(1 - R) \{1 - e^{-\alpha d}\}
\]

Area = \(w \times L\)
Analogy to Current Gain in Bipolar Transistor

Current gain in bipolar transistor:
\[ \beta = \frac{I_C}{I_B} \]

The current gain can also be expressed as
\[ \beta = \frac{\tau_{rb}}{\tau_i} \]

\( \tau_i \) : transit time

\( \tau_{rb} \) : carrier recombination lifetime in the base

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Frequency of Photoconductors

\[
\frac{dN}{dt} = \eta \frac{P_{opt}}{h \omega l w d} \frac{1}{\tau_n} - N
\]

Small signal response:

\[
N = N_0 + N_1 e^{i\omega t}
\]

\[
j \omega N_1 = \eta \frac{P}{h \omega l w d} \frac{1}{\tau_n} - N_1
\]

\[
N_1 = \frac{\eta P_1}{h \omega (l w d)} \frac{1}{j \omega + 1/\tau_n}
\]

\[
I_1 = J_l l w = (N_1 q \nu_n) l w
\]

\[
\frac{I_1}{P_1} = \left( \frac{\eta q}{h \omega} \right) \left( \frac{\tau_n}{\tau_i} \right) \frac{1}{j \omega \tau_n + 1}
\]

\[= (\text{DC Quantum Efficiency}) \times (\text{Photoconductive Gain}) \times
\]

\(\text{Normalized Frequency Response})\]
P-i-n Photodiode

- Reverse-biased p-i-n junction
- Most of the voltage drop across the i-region, the main absorption region
- High field separates photogenerated electron and hole
- Large bandgap materials are used for P and N if possible
- Fast response
- Low noise
- No gain (quantum efficiency < 100%)

I-V Curve

Dark current: \( I = I_0 \left( e^{\frac{qV}{kT}} - 1 \right) \)

Photocurrent: \( I_{ph} = \frac{\eta q}{h\omega} P_{opt} \)

Quantum efficiency: \( \eta = \eta_i(1 - R)(1 - e^{-\alpha d}) \)

Total current: \( I = I_0 \left( e^{\frac{qV}{kT}} - 1 \right) + I_{ph} \)
Absorption Coefficient

- Light intensity decays exponentially in semiconductor:
  \[ I(x) = I_0 e^{-\alpha x} \]
- Direct bandgap semiconductor has a sharp absorption edge
- Si absorbs photons with \( h\nu > E_g = 1.1 \text{ eV} \), but the absorption coefficient is small
  - Sufficient for CCD
- At higher energy (~ 3 eV), absorption coefficient of Si becomes large again, due to direct bandgap transition to higher CB

Two Types of p-i-n Photodiodes

Surface-Illuminated p-i-n
\[ \eta = \eta_i (1 - R)(1 - e^{-\alpha d}) \]
\( \eta_i \): internal quantum efficiency
\( R \): reflectivity
\( d \): absorption layer thickness

Waveguide p-i-n
\[ \eta = \eta_i (1 - R)(1 - e^{-\Gamma \alpha L}) \]
\( \eta_i \): internal quantum efficiency
\( R \): reflectivity
\( \Gamma \): confinement factor
\( L \): length of waveguide PD
**Ramo’s Theorem**

Proof:

Work done on the charge:

\[ W = \text{Force} \times \text{Displacement} \]

\[ = qEdx = q \frac{V}{d} dx \]

Work provided by power supply:

\[ W = i(t)Vdt \]

\[ \Rightarrow \]

\[ i(t)Vdt = q \frac{V}{d} dx \]

\[ i(t) = \frac{q}{d} \frac{dx}{dt} = \frac{qv(t)}{d} \]

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**Response of One Photogenerated Electron-Hole Pair**

Total charge generated:

\[ Q = \int_{0}^{\infty} i_e(t)dt + \int_{0}^{\infty} i_h(t)dt \]

\[ = \frac{qv_e}{d} \frac{d-x}{v_e} + \frac{qv_h}{d} \frac{x}{v_h} = q \]

One absorbed photon → one charge detected
Transit Time

Electron current ends when the last electron generated near P-side reaches N-electrode: \( t = \frac{d}{v_e} \)

Hole current ends when the last hole generated near N-side reaches P-electrode: \( t = \frac{d}{v_h} \)

Hole is usually slower → A conservative estimate of the transit time: \( \tau_t = \frac{d}{v_h} \)

Total Response Time of p-i-n

(1) RC time:
\[ \tau_{RC} = RC = R \frac{\epsilon A}{d} \quad (A: \text{area of p-i-n}) \]

(2) Transit time:
\[ \tau_t = \frac{d}{v_h} \]

Total response time:
\[ \tau = \tau_{RC} + \tau_t \]
\[ f_{3dB} \approx \frac{1}{2\pi\tau} \]

Absorption layer thickness for optimum frequency response:
\[ \tau = \tau_{RC} + \tau_t = \frac{R \epsilon A}{d} + \frac{d}{v_h} \]
\[ \tau \geq 2 \sqrt{\left( \frac{R \epsilon A}{d} \right) \left( \frac{d}{v_h} \right)} \]
\[ f_{3dB} \approx \frac{1}{2\pi\tau} \leq \frac{1}{4\pi} \sqrt{\frac{v_h}{R \epsilon A}} = f_{3dB,\text{max}} \]

Optimum bandwidth occurs when
\[ \frac{R \epsilon A}{d} = \frac{d}{v_h} \]
\[ d_{\text{optimum}} = \sqrt{R \epsilon A v_h} \]
More Rigorous Analysis of p-i-n Response Time

Small-signal analysis: assume the input light is modulated at frequency $\omega$, the photocurrent is proportional to

$$|i(t)| \propto \left| \frac{1}{1 + j\omega RC} \right| \left| \frac{\sin^2 \left( \frac{\omega \tau_i}{2} \right)}{\left( \frac{\omega \tau_i}{2} \right)^2} \right| = |H(\omega)|$$

The first term is single-pole response from RC, while the second term is the phase delay due to transit time response.

Comparison of Numeric Examples

Example:

$\tau_{RC} = 14.4 \text{ ps}$

$\tau_i = 20 \text{ ps}$

$$f_{3\, dB} = \frac{1}{2\pi} \tau_{RC} + \tau_i = 4.6 \text{ GHz}$$

$$|i(t)| \propto \left| \frac{1}{1 + j\omega RC} \right| \left| \frac{\sin^2 \left( \frac{\omega \tau_i}{2} \right)}{\left( \frac{\omega \tau_i}{2} \right)^2} \right| = |H(\omega)|$$

Solving $|H(\omega)| = \frac{1}{\sqrt{2}}$, $f_{3\, dB} = 9.7 \text{ GHz}$

The discrepancy is smaller when RC dominates, and larger when transit time dominates. (Transit time response has a sharp drop-off.)
Bandwidth-Efficiency Product

(1) For surface-illuminated p-i-n (assume AR coating: R=0%), in the extreme of thin absorbing layer and transit-time-dominated response:
\[ \eta = \eta_i (1 - e^{-\alpha d}) \approx \eta_i (1 - (1 - \alpha d)) = \eta_i \alpha d \]
\[ f_{3dB} \approx \frac{1}{2\pi} \frac{v_h}{d} \]
Bandwidth-efficiency product: \[ f_{3dB} \times \eta \approx \left( \frac{1}{2\pi} \frac{v_h}{d} \right) (\eta_i \alpha d) = \frac{\eta_i \alpha v_h}{2\pi} \]

(2) On the other hand, the efficiency of waveguide p-i-n is
\[ \eta = \eta_i (1 - e^{-\Gamma\alpha L}) \approx \eta_i \Gamma \alpha L \]
RC-limited bandwidth: \[ f_{3dB} \approx \frac{1}{2\pi} \frac{d}{R\epsilon Lw} \]
Bandwidth-efficiency product: \[ f_{3dB} \times \eta \approx \left( \frac{1}{2\pi} \frac{d}{R\epsilon Lw} \right) (\eta_i \Gamma \alpha L) = \frac{\eta_i \Gamma \alpha d}{2\pi R\epsilon w} \]
⇒ In general, there is a bandwidth-efficiency trade-off