EE 232 **Midterm-1 Solution** **Fall 20**

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$h\_bar := 1.05459\cdot 10^{-34} J\cdot s$ $q := 1.6\cdot 10^{-19} C$ $m0 := 9.11\cdot 10^{-31} kg$ $eV := 1.6\cdot 10^{-19} J$

$nr := 3.1$ $\varepsilon 0 := 8.854\cdot 10^{-12} F\cdot m^{-1}$ $meV := 0.001 eV$

$Eg := 1eV$ $Ev := 0$ $Ec := Ev + Eg$ $\omega := \dfrac{Eg}{h\_bar}$

$kB := 1.38\cdot 10^{-23} \cdot J\cdot K^{-1}$ $T := 300K$ $kB\cdot T = 0.026\cdot eV$

1(a) T = 0 K

$m\_e := 0.5\cdot m0$ $m\_h := 0.5\cdot m0$ $m\_r := \dfrac{m\_e\cdot m\_h}{m\_e + m\_h}$ $\dfrac{m\_r}{m0} = 0.25$

$Lz := 10nm$

$C0 := \dfrac{\pi\cdot q^2}{m0^2\cdot \omega\cdot \varepsilon 0\cdot c\cdot nr}$ $C0 = 7.762 \times 10^9 \dfrac{m^2}{kg}$ $Ep := 24eV$ $\dfrac{m0}{6}\cdot Ep = 5.83 \times 10^{-49}\cdot kg\cdot J$

$\rho r(hv) := if\left(hv \ge Eg, \dfrac{m\_r}{\pi\, h\_bar^2\cdot Lz}, 0\right)$

$\alpha 0(hv) := C0\cdot \dfrac{m0}{6}\cdot Ep\cdot \rho r(hv)$

At T= 0 K, the gain is flat from Eg to ΔF for quantum well gain media.

$\boxed{\alpha 0(1.052eV) = 2.95 \times 10^4\cdot cm^{-1}}$

(b) Gain spectral width = Δhv = ΔF - Eg = 52 meV

(c) Electron quasi-Fermi level is 1/2 of ΔF - Eg = 26 meV

$N := \dfrac{m\_e}{\pi\, h\_bar^2\cdot Lz}\cdot 26meV$ $\boxed{N = 5.423 \times 10^{18}\cdot cm^{-3}}$

$P := N$ $\boxed{P = 5.423 \times 10^{18}\cdot cm^{-3}}$

(d) The slope of Fermi inversion function at ΔF is 1/4kT
Gain peak at 300K now appear at band edge, Eg

$T := 300K$ $\Delta F := Eg + 52meV$

$f\_g(hv) := \dfrac{-1}{4kB\cdot T}\cdot (hv - \Delta F)$

$$g(hv) := C0 \cdot \frac{m0}{6} \cdot Ep \cdot \rho r(hv) \cdot f\_g(hv)$$

$$g\_peak := g(Eg) \qquad \boxed{g\_peak = 1.482 \times 10^{4} \cdot cm^{-1}}$$

(e) Peak wavelength occurs at the effective bandgap, hv = Eg = 1 eV = **1.24 um**

(f) Like Fermi inversion factor, the Fermi-Dirac distribution can be approximted by a line with slope of 1/4kT

$$Fc := Ec + \frac{1}{2}(\Delta F - Eg) \qquad Fc = 1.026 \cdot eV$$

Taylor expansion of Fermi-Dirac distribution around Fc:

$$F\_e(E) := \frac{1}{2} - \frac{1}{4kB \cdot T} \cdot (E - Fc)$$

$$F\_e(Eg) = 0.751$$

$$N := \frac{m\_e}{\pi\, h\_bar^{2} \cdot Lz} \cdot \frac{1}{2} \cdot F\_e(Eg) \cdot (Fc - Eg) \qquad \boxed{N = 2.037 \times 10^{18} \cdot cm^{-3}}$$

Hole concentration is the same as electron concentration.

2.(a)

$$g(\hbar\omega) = C_0 \left|\hat{e} \cdot \vec{P}_{cv}\right|^2 \rho_r(\hbar\omega) f_g(\hbar\omega)$$

At T = 0 K, gain peak occurs at $\hbar\omega = \Delta F$

$$g_{peak} = C_0 \left|\hat{e} \cdot \vec{P}_{cv}\right|^2 \frac{1}{2\pi^2} \left(\frac{2m_r^*}{\hbar^2}\right)^{3/2} \sqrt{\Delta F - E_g}$$

$$\frac{g_{peak,A}}{g_{peak,B}} = \left(\frac{m_{r,A}^*}{m_{r,B}^*}\right)^{3/2}$$

$$m\_rA := \frac{0.1 \cdot 0.1}{0.1 + 0.1} \cdot m0 \qquad m\_rB := \frac{0.1 \cdot 1}{0.1 + 1} \cdot m0$$

$$Peak\_gain\_ratio := \left(\frac{m\_rA}{m\_rB}\right)^{1.5} \qquad \boxed{Peak\_gain\_ratio = 0.408}$$

(b) At room temperature, the Fermi inversion factor at gain peak is < 1. However, because the gain peak photon energy is the same for Semiconductor A and B, the Fermi inversion factors cancel out. So the peak gain ratio remains the same.

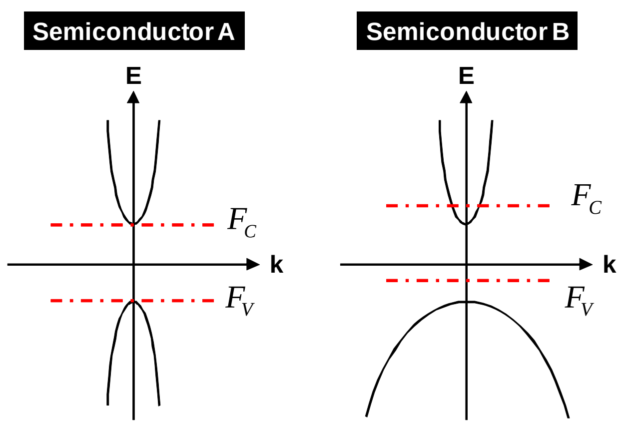
(c)

$$n = 2\left(\frac{\pi m_e^* k_B T}{2\pi^2\hbar^2}\right)^{3/2} \frac{4\left(\frac{F_n - E_C}{k_B T}\right)^{3/2}}{3\sqrt{\pi}}$$

$$n = \frac{8}{3\sqrt{\pi}}\left(\frac{\pi m_e^*\left(F_n - E_C\right)}{2\pi^2\hbar^2}\right)^{3/2}$$

$$\frac{n_A}{n_B} = \left(\frac{m_{e,A}^*\left(F_n - E_C\right)_A}{m_{e,B}^*\left(F_n - E_C\right)_B}\right)^{3/2} = \left(\frac{\left(F_n - E_C\right)_A}{\left(F_n - E_C\right)_B}\right)^{3/2} = \left(\frac{\frac{\Delta F}{2}}{\left(F_n - E_C\right)_B}\right)^{3/2} < 1$$

(d) Semiconductor A will have lower transparency carrier concentration. Since it has symmetric effective masses, Fc is exactly at Ec for Semiconductor A. In Semiconductor B, Fc will lie above Ec because of larger hole effective mass. As a result, it has higher electron concentration.



$$m_e^* = m_h^* = 0.1m_0$$

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$$m_h^* = 1.0m_0$$

3. (a)

$hv := \frac{1.24 eV \cdot \mu m}{5 \mu m}$ $\quad hv = 0.248 \cdot eV$ $\quad m\_e := 0.1 m0$ $\quad m\_h := 0.5 m0$

$E1\_n(Lz) := \frac{h\_bar^2}{2 \cdot m\_e} \cdot \left(\frac{\pi}{Lz}\right)^2$ $\quad E1\_p(Lz) := \frac{h\_bar^2}{2 \cdot m\_h} \cdot \left(\frac{\pi}{Lz}\right)^2$

$Lz := 10 nm$ (initial guess value)

Given

$E1\_n(Lz) = \frac{hv}{3}$ $\quad Lz\_n := Find(Lz)$ $\quad \boxed{Lz\_n = 6.749 \cdot nm}$

Given

$E1\_p(Lz) = \frac{hv}{3}$ $\quad Lz\_p := Find(Lz)$ $\quad \boxed{Lz\_p = 3.018 \cdot nm}$

(b)

$$\alpha(\hbar\omega) = \frac{\pi\omega}{n_r c \varepsilon_0} g(E_{e1}^{e2}) |\mu_{21}|^2 \frac{m_e^*}{\pi \hbar^2 L_z} E_{e1}^{e2}$$

$$= \frac{\pi\omega}{n_r c \varepsilon_0} g(E_{e1}^{e2}) \left(-\frac{16}{9\pi^2} e L_z\right)^2 \frac{m_e^*}{\pi \hbar^2 L_z} E_{e1}^{e2}$$

$$\frac{\alpha_{\max,N}}{\alpha_{\max,P}} = \frac{L_{z,N} m_e^*}{L_{z,P} m_h^*}$$

$\alpha\_ratio := \frac{Lz\_n \cdot m\_e}{Lz\_p \cdot m\_h}$ $\quad \boxed{\alpha\_ratio = 0.447}$

(c)

$$N_{\max} = \frac{m_e^*}{\pi \hbar^2 L_{z,N}} \Delta E$$

$$P_{\max} = \frac{m_h^*}{\pi \hbar^2 L_{z,P}} \Delta E$$

$$\frac{N_{\max}}{P_{\max}} = \frac{\frac{m_e^*}{L_{z,N}}}{\frac{m_h^*}{L_{z,P}}}$$

$N\_P\_ratio := \frac{m\_e \cdot Lz\_p}{m\_h \cdot Lz\_n}$ $\quad \boxed{N\_P\_ratio = 0.089}$