EE 232 **Midterm-2 Solution** **Fall 2009**

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$h\_bar := 1.05459\cdot 10^{-34} J\cdot s$ $m0 := 9.11\cdot 10^{-31} kg$ $\varepsilon 0 := 8.854\cdot 10^{-12} F\cdot m^{-1}$

$q := 1.6\cdot 10^{-19} C$ $eV := 1.6\cdot 10^{-19} J$ $meV := 0.001 eV$

$kB := 1.38\cdot 10^{-23}\cdot J\cdot K^{-1}$ $T := 300K$ $kB\cdot T = 0.026\cdot eV$ $nr := 3.5$

1. $Ntr := 10^{18} cm^{-3}$ $g0 := 1000 cm^{-1}$ $g(N) := g0\cdot \ln\left(\frac{N}{Ntr}\right)$

$R1 := 30\%$ $R2 := 30\%$ $L := 500\mu m$ $w := 1\mu m$ $d := 200nm$ $Lz := 10nm$

$\tau := 1ns = 1\times 10^{-9} s$

$neff := 3.4$ $vg := \frac{c}{neff}$

$\lambda := 1.24\mu m$ $\alpha i := 10cm^{-1}$ $\eta i := 100\%$ $n\_core := 3.6$ $n\_clad := 3.2$

(a) $V(d) := \frac{2\cdot\pi}{\lambda}\cdot d\cdot\sqrt{n\_core^2 - n\_clad^2}$ $V(200nm) = 1.671$

$\Gamma\_sch := \frac{V(200nm)^2}{2 + V(200nm)^2}$ $\boxed{\Gamma\_sch = 58.277\cdot\%}$

(b)

$\Gamma := \Gamma\_sch\cdot\frac{Lz}{d}$ $\boxed{\Gamma = 2.914\cdot\%}$

(c) $\alpha m := \frac{1}{2\cdot L}\cdot\ln\left(\frac{1}{R1\cdot R2}\right)$ $\alpha m = 24.079 cm^{-1}$

$gth := \frac{1}{\Gamma}\cdot(\alpha m + \alpha i)$ $\boxed{gth = 1.17\times 10^3\cdot cm^{-1}}$

(d) $Nth := Ntr\cdot\exp\left(\frac{gth}{g0}\right)$ $Nth = 3.221\times 10^{18}\cdot cm^{-3}$

$Ith := \frac{Nth}{\tau}\cdot q\cdot Lz\cdot w\cdot L$ $\boxed{Ith = 2.576\cdot mA}$

(e) $\eta e := \frac{\alpha m}{\alpha i + \alpha m}$ $\boxed{\eta e = 70.657\cdot\%}$

$hv := \frac{1.24eV\cdot\mu m}{\lambda}$

$R := \eta e\cdot\frac{hv}{q}$ $\boxed{R = 0.707\cdot\frac{mW}{mA}}$

(f)

$Pout := (2\cdot Ith - Ith)\cdot \eta e\cdot \frac{hv}{q}$ $\boxed{Pout = 1.82\cdot mW}$

(g)

$\tau p := \frac{1}{\Gamma\cdot vg\cdot gth}$ $\tau p = 3.328 \times 10^{-12} s$

$S := \frac{Pout}{hv}\cdot\frac{\tau p}{\left(\frac{Lz\cdot w\cdot L}{\Gamma}\right)}\cdot\frac{1}{\eta e}$ $\boxed{S = 3.123 \times 10^{14}\cdot cm^{-3}}$

(h)

$a(N) := g0\cdot\frac{1}{N}$ $a(Nth) = 3.105 \times 10^{-16}\cdot cm^{2}$

$\omega r := \sqrt{\frac{vg\cdot a(Nth)\cdot S}{\tau p}}$ $f\_r := \frac{\omega r}{2\cdot\pi}$ $\boxed{f\_r = 2.551\cdot GHz}$

2 $L := 1\mu m$ $Rfront := 99\%$ $Rback := 100\%$

$\alpha m := \frac{1}{2\cdot L}\cdot \ln\left(\frac{1}{Rfront\cdot Rback}\right)$ $\alpha m = 50.252\cdot cm^{-1}$

$\alpha i := 10cm^{-1}$

$\alpha := \alpha m + \alpha i$

$\tau p := \frac{1}{\frac{c}{3}\cdot\alpha}$ $\tau p = 1.661 \times 10^{-12} s$

$\omega := 2\cdot\pi\cdot\frac{c}{1\mu m}$ $\omega = 1.884 \times 10^{15}\frac{1}{s}$

$Q := \tau p\cdot\omega$ $\boxed{Q = 3.128 \times 10^{3}}$

3 $Eg := 1eV$ $ac := -5eV$ $av := 1eV$ $b := -2eV$

$C12\_11 := 0.5$ $\gamma 1 := 10$ $\gamma 2 := 1$ $me := 0.1m0$

$Ep := 24eV$ $nr := \sqrt{9}$ $Lz := 5nm$

$$a_C = -5eV; \quad a_V = 1eV; \quad b = -2eV; \quad \frac{C_{12}}{C_{11}} = 0.5$$

$$P_\varepsilon = -a_V(\varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}) = -2a_V\left(1 - \frac{C_{12}}{C_{11}}\right)\varepsilon = -1\varepsilon \quad \text{[eV]}$$

$$Q_\varepsilon = -\frac{b}{2}(\varepsilon_{xx} + \varepsilon_{yy} - 2\varepsilon_{zz}) = -b\left(1 + 2\frac{C_{12}}{C_{11}}\right)\varepsilon = 4\varepsilon \quad \text{[eV]}$$

$$E_{HH}(k) = -P_\varepsilon - Q_\varepsilon - \frac{\hbar^2}{2m_0}\left[(\gamma_1 + \gamma_2)k_t^2 + (\gamma_1 - 2\gamma_2)k_z^2\right]$$

$$E_{LH}(k) = -P_\varepsilon + Q_\varepsilon - \frac{\hbar^2}{2m_0}\left[(\gamma_1 - \gamma_2)k_t^2 + (\gamma_1 + 2\gamma_2)k_z^2\right]$$

$$k_z = \frac{\pi}{L_z} \text{ in QW}; \quad k_t = 0 \text{ at band edge}; \quad \gamma_1 = 10; \quad \gamma_2 = 1$$

$$E_{HH1} = -3\varepsilon - \frac{\hbar^2}{2m_0}8\left(\frac{\pi}{L_z}\right)^2$$

$$E_{LH1} = 5\varepsilon - \frac{\hbar^2}{2m_0}12\left(\frac{\pi}{L_z}\right)^2$$

$$\text{Set } E_{HH1} = E_{LH1}; \quad \boxed{\varepsilon = \frac{1}{8}\frac{\hbar^2}{2m_0}4\left(\frac{\pi}{L_z}\right)^2 = 0.75\%} > 0 \Rightarrow \boxed{\text{Compressive}}$$

$$\varepsilon := \frac{1}{8}\cdot\frac{\text{h\_bar}^2}{\text{2m0}}\cdot 4\cdot\left(\frac{\pi}{\text{Lz}}\right)^2\cdot\frac{1}{\text{eV}} \qquad \varepsilon = 0.753\,\%$$

Or calculate numerically:

$$\text{Ec\_s}(\varepsilon) := \text{Eg} + 2\cdot\text{ac}\cdot(1 - \text{C12\_11})\cdot\varepsilon$$

$$\text{Ehh\_s}(\varepsilon) := 0 + 2\cdot\text{av}\cdot(1 - \text{C12\_11})\cdot\varepsilon + \text{b}\cdot(1 + 2\cdot\text{C12\_11})\cdot\varepsilon$$

$$\text{Elh\_s}(\varepsilon) := 0 + 2\cdot\text{av}\cdot(1 - \text{C12\_11})\cdot\varepsilon - \text{b}\cdot(1 + 2\cdot\text{C12\_11})\cdot\varepsilon$$

$$\text{mhhz} := \frac{\text{m0}}{\gamma1 - 2\cdot\gamma2} = 1.139\times 10^{-31}\,\text{kg} \qquad \frac{\text{mhhz}}{\text{m0}} = 0.125$$

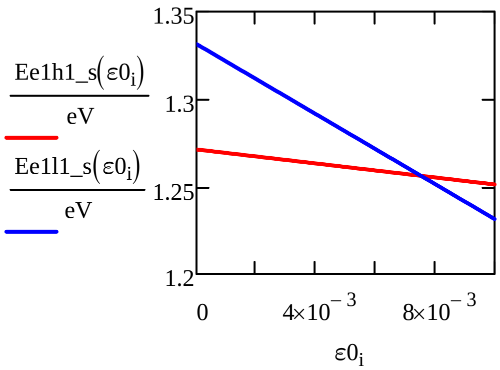
$$\text{mlhz} := \frac{\text{m0}}{\gamma1 + 2\cdot\gamma2} = 7.592\times 10^{-32}\,\text{kg} \qquad \frac{\text{mlhz}}{\text{m0}} = 0.083$$

$$\text{Ee1h1\_s}(\varepsilon) := \text{Ec\_s}(\varepsilon) + \frac{\text{h\_bar}^2}{2\cdot\text{me}}\left(\frac{\pi}{\text{Lz}}\right)^2 - \left[\text{Ehh\_s}(\varepsilon) - \frac{\text{h\_bar}^2}{2\cdot\text{mhhz}}\left(\frac{\pi}{\text{Lz}}\right)^2\right]$$

$$\text{Ee1l1\_s}(\varepsilon) := \text{Ec\_s}(\varepsilon) + \frac{\text{h\_bar}^2}{2\cdot\text{me}}\left(\frac{\pi}{\text{Lz}}\right)^2 - \left[\text{Elh\_s}(\varepsilon) - \frac{\text{h\_bar}^2}{2\cdot\text{mlhz}}\left(\frac{\pi}{\text{Lz}}\right)^2\right]$$

$i := 0 .. 100$

$\varepsilon 0_i := \frac{i}{100} \cdot 1\%$



Initial guess value: $\varepsilon 1 := 8 \cdot 10^{-3}$

Given

$$\frac{Ee1h1\_s(\varepsilon 1)}{eV} = \frac{Ee1l1\_s(\varepsilon 1)}{eV}$$

$\varepsilon\_match := Find(\varepsilon 1) = 0.753 \cdot \%$

(b)

$$g(\hbar\omega) = C_0 \left| \hat{e} \cdot \vec{P}_{cv} \right|^2 \rho_r^{2d}(\hbar\omega) f_g(\hbar\omega)$$

Without considering difference in matrix element for different polarizations, peak gain is

$$g_{peak} = C_0 \left| \hat{e} \cdot \vec{P}_{cv} \right|^2 \frac{m_r^*}{\pi \hbar^2 L_z}$$

$$\frac{g_{peak,HH}}{g_{peak,LH}} = \frac{m^*_{r,HH}}{m^*_{r,LH}} = \frac{1}{m^*_{r,LH}} / \frac{1}{m^*_{r,HH}}$$

$$\frac{1}{m^*_{r,HH}} = \frac{1}{m^*_e} + \frac{1}{m^t_{HH}} = \frac{1}{m_0}(10 + \gamma_1 + \gamma_2) = \frac{21}{m_0}$$

$$\frac{1}{m^*_{r,LH}} = \frac{1}{m^*_e} + \frac{1}{m^t_{LH}} = \frac{1}{m_0}(10 + \gamma_1 - \gamma_2) = \frac{19}{m_0}$$

$$\boxed{\frac{g_{peak,HH}}{g_{peak,LH}} = \frac{19}{21} = 0.90}$$

The elctron-light hole transition has higher peak gain.

(c) It will not be polarization insensitive as TM polarziation has higher gain than TE polarization.

(d) There are multiple ways to build polarization-insensitive semiconductor optical amplifiers that include the strained quantum well in (a) as part of the gain media. The most straightforward approach is to add some TE gain as TE gain is less than TM gain. This can be achieved by adding a compressively strained quantum well. Since waves propagating in the SOA experience “modal gain”, i.e., gain multiplied by its confinement factor, we can adjust the weight of the additional TE gain by placing it farther away from the center of the separate confinement structure, as shown below.