EE 232 **Final Solution** **Fall 2009**

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$h\_bar := 1.05459\cdot 10^{-34} J\cdot s$ $m0 := 9.11\cdot 10^{-31} kg$ $\varepsilon 0 := 8.854\cdot 10^{-12} F\cdot m^{-1}$

$q := 1.6\cdot 10^{-19} C$ $eV := 1.6\cdot 10^{-19} J$ $meV := 0.001 eV$

$kB := 1.38\cdot 10^{-23}\cdot J\cdot K^{-1}$ $T := 300K$ $kB\cdot T = 0.026\cdot eV$ $\varepsilon r := 9$

1. $\alpha 0 := 10^{5} cm^{-1}$ $ve := 10^{7} cm\cdot s^{-1}$ $vh := 10^{6} cm\cdot s^{-1}$

$d := 2\mu m$ $Area := 100\mu m^{2}$

$$\alpha(hv) := \alpha 0\cdot\left(\frac{hv}{eV} - 0.8\right)^{2}$$

$\lambda 1 := 1\mu m$ $hv1 := \frac{1.24\mu m\cdot eV}{\lambda 1}$ $hv1 = 1.24\cdot eV$

$\lambda 2 := 1.5\mu m$ $hv2 := \frac{1.24\mu m\cdot eV}{\lambda 2}$ $hv2 = 0.827\cdot eV$

$\alpha 1 := \alpha(hv1) = 1.936\times 10^{4}\cdot cm^{-1}$

$\alpha 2 := \alpha(hv2) = 71.111\cdot cm^{-1}$

(a) $\boxed{\eta 1 := 1 - \exp(-\alpha 1\cdot d) = 97.918\cdot\%}$

$\boxed{\eta 2 := 1 - \exp(-\alpha 2\cdot d) = 1.412\cdot\%}$

(b) $C := \varepsilon r\cdot\varepsilon 0\cdot\frac{Area}{d} = 3.984\times 10^{-15} F$

$R := 50\Omega$

$\tau RC := R\cdot C = 1.992\times 10^{-13} s$

$\tau t := \frac{d}{vh} = 2\times 10^{-10} s$

$\tau := \tau RC + \tau t = 2.002\times 10^{-10} s$

$f3dB\_t := \frac{1}{2\cdot\pi}\cdot\frac{1}{\tau}$ $\boxed{f3dB\_t = 0.795\cdot GHz}$

(c)

$I\_ph := \frac{\frac{4\cdot kB\cdot T}{R}}{2\cdot q}$ $I\_ph = 1.035\cdot mA$

$P1 := I\_ph\cdot\frac{hv1}{q}\cdot\frac{1}{\eta 1}$ $\boxed{P1 = 1.311\cdot mW}$

$P2 := I\_ph\cdot\frac{hv2}{q}\cdot\frac{1}{\eta 2}$ $\boxed{P2 = 60.588\cdot mW}$

2. (a) $\lambda := 1.24\mu m$ $hv := \frac{1.24\cdot\mu m\cdot eV}{\lambda}$ $hv = 1\cdot eV$ $\alpha := 10^4 cm^{-1}$ $d := 2\mu m$

$\Delta\nu := 1GHz$ $P\_in := 100\mu W$ $\eta := 1 - \exp(-\alpha\cdot d) = 0.865$

$M := 100$ $k := 0.1$

$$Fn := k\cdot M + (1-k)\cdot\left(2 - \frac{1}{M}\right) = 11.791$$

First, find the photocurrent with equal SNR:

$$SNR_{APD} = SNR_{pin}$$

$$\frac{\frac{1}{2}I_p^2\langle M\rangle^2}{2eI_pF\langle M\rangle^2 dv + \frac{4k_BT\Delta v}{R}} = \frac{\frac{1}{2}I_p^2}{2eI_pdv + \frac{4k_BT\Delta v}{R}}$$

$$2eI_pF\langle M\rangle^2 dv + \frac{4k_BT\Delta v}{R} = \langle M\rangle^2\left(2eI_pdv + \frac{4k_BT\Delta v}{R}\right)$$

$$2eI_p\langle M\rangle^2(F-1) = \left(\langle M\rangle^2 - 1\right)\frac{4k_BT}{R}$$

$$I_p = \frac{\left(\langle M\rangle^2 - 1\right)\frac{4k_BT}{R}}{2e\langle M\rangle^2(F-1)}$$

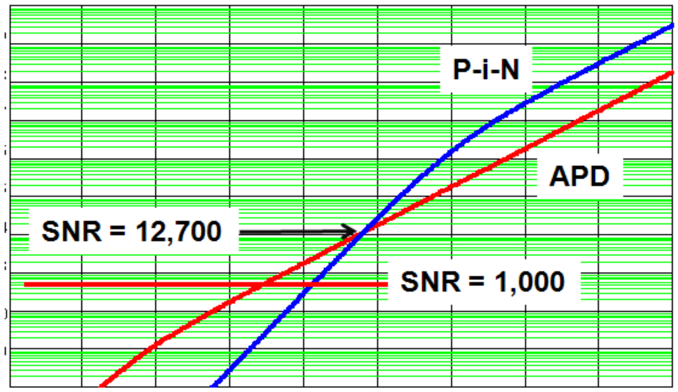
$$Ip := \frac{\left(M^2 - 1\right)\cdot 4\cdot kB\cdot T}{R\cdot 2\cdot q\cdot M^2\cdot(Fn - 1)} \qquad Ip = 95.904\cdot\mu A$$

$$\boxed{P\_in := \frac{Ip}{\eta}\cdot\frac{hv}{q} = 110.914\cdot\mu W}$$

(b) SNR at Ip = 115uW:

$$SNR := \frac{\frac{1}{2}\cdot Ip^2}{2\cdot q\cdot Ip\cdot\Delta\nu + \frac{4\cdot kB\cdot T}{R}\cdot\Delta\nu} = 1.271\times 10^4$$

Since SNR = 1000 < SNR (p-i-n = APD), APD will have better performance, i.e., it requires less photocurrent to achieve SNR of 1000, as illustrated below



(c) $\alpha e := 10^4 cm^{-1}$

$W := 1\mu m$ (initial guess value)

Given

$$M = \frac{1-k}{\exp[-(1-k)\cdot\alpha e\cdot W] - k}$$

$Wm := Find(W)$ $\quad$ $Wm = 2.463\cdot\mu m$

(d) $\tau := \frac{d}{vh} + \frac{M\cdot k\cdot Wm}{ve} = 4.463\times 10^{-10} s$

$$f3dB := \frac{1}{2\pi\cdot\tau} = 356.636\cdot MHz$$

3 (a) $hv := 1eV$

$me := 0.1m0$ $\quad$ $mh := 0.5m0$ $\quad$ $Lz := 10nm$

$$g\_max := 10^{10}\cdot\frac{1eV}{hv}\cdot\left(6\cdot 10^{-49}\right)\cdot\left(2.6\cdot 10^{46}\cdot\frac{me\cdot mh}{me+mh}\cdot\frac{1}{m0}\cdot\frac{1nm}{Lz}\right)m^{-1} = 1.3\times 10^4\cdot cm^{-1}$$

(b) $\Gamma := 1\%$

$\alpha\_max := g\_max\cdot\Gamma = 130\cdot cm^{-1}$

$\alpha i := 10cm^{-1}$

$\alpha m := \alpha\_max - \alpha i = 120\cdot cm^{-1}$

$R0 := 30\%$

$$Lmin := \frac{1}{2\alpha m}\ln\left(\frac{1}{R0^2}\right) = 100.331\cdot\mu m$$

(c) the entire n = 1 subband should be completely filled, the threshold electron concentration must be larger than

$$N1 := \frac{me}{\pi \cdot h\_bar^2 \cdot Lz} \cdot 3 \cdot \frac{h\_bar^2}{2 \cdot me} \cdot \left(\frac{\pi}{Lz}\right)^2 = 4.712 \times 10^{18} \cdot cm^{-3}$$

4 (a)

$$n_{core} = 5$$

$$n_{clad} = 4$$

$$V = \frac{2\pi}{\lambda} d \sqrt{n_{core}^2 - n_{clad}^2} = \frac{6\pi}{\lambda} d$$

$$\Gamma = \frac{V^2}{2 + V^2} = \frac{\left(\frac{6\pi}{\lambda} d\right)^2}{2 + \left(\frac{6\pi}{\lambda} d\right)^2}$$

(b)

$$I_{th} = \frac{qN_{th}}{\tau} V_{active} = \frac{qN_{th}}{\tau} wLd$$

$$N_{th} = N_{tr} + \frac{\alpha_i + \alpha_m}{a\Gamma} = N_{tr} + \frac{\alpha}{a\Gamma} = N_{tr} + \frac{\alpha}{a}\left(\frac{2 + \left(\frac{6\pi}{\lambda} d\right)^2}{\left(\frac{6\pi}{\lambda} d\right)^2}\right)$$

$$N_{th} = \left(N_{tr} + \frac{\alpha}{a}\right) + \frac{2\alpha}{a} \frac{1}{\left(\frac{6\pi}{\lambda} d\right)^2}$$

$$I_{th} = \frac{q}{\tau} wL \cdot d \left(\left(N_{tr} + \frac{\alpha}{a}\right) + \frac{2\alpha}{a} \frac{1}{\left(\frac{6\pi}{\lambda} d\right)^2}\right) = \frac{q}{\tau} wL \left(\left(N_{tr} + \frac{\alpha}{a}\right) d + \frac{2\alpha}{a} \frac{\lambda^2}{(6\pi)^2 d}\right)$$

(c) The first term in the expression of I_th is proportional to d, while the second term is inversely proportional to 1/d. So there is an optimum d.

(d) and (e)

$$I_{th} \geq \frac{qwL}{\tau} 2\sqrt{\left(N_{tr} + \frac{\alpha}{a}\right)\left(\frac{2\alpha}{a}\frac{\lambda^2}{(6\pi)^2}\right)}$$

Minimum threshold is reached when the two terms equal to each other:

$$d = \sqrt{\frac{\frac{2\alpha}{a}\frac{\lambda^2}{(6\pi)^2}}{\left(N_{tr} + \frac{\alpha}{a}\right)}}$$

5. (a) QCL is TM polarized, QWL is also TM polarized (when the lowest transition energy is LH)

(b) Peak QCL gain:

$$g_{QCL,\max} = \frac{\pi\omega}{n_r c\varepsilon_0} \frac{1}{\pi\left(\frac{\hbar}{\tau_{in}}\right)} \left(\frac{16}{9\pi^2} eL_z\right)^2 \frac{m_e^*}{\pi\hbar^2 L_z} E_{e1}^{e2}$$

Peak QWL gain:

$$g_{QWL,\max} = \left(\frac{\pi e^2}{n_r c\varepsilon_0 m_0{}^2 \omega}\right)\left(2 \cdot \frac{m_0}{6} E_p\right)\left(\frac{m_r^*}{\pi\hbar^2 L_z}\right)$$

(Note the factor of 2 in front of matrix element represents the polarization dependence of tensile QW)

$$\frac{g_{QCL,\max}}{g_{QWL,\max}} = \frac{\frac{\pi\omega}{n_r c\varepsilon_0} \frac{1}{\pi\left(\frac{\hbar}{\tau_{in}}\right)} \left(\frac{16}{9\pi^2} eL_z\right)^2 \frac{m_e^*}{\pi\hbar^2 L_z} E_{e1}^{e2}}{\left(\frac{\pi e^2}{n_r c\varepsilon_0 m_0{}^2 \omega}\right)\left(2 \cdot \frac{m_0}{6} E_p\right)\left(\frac{m_r^*}{\pi\hbar^2 L_z}\right)}$$

$$\frac{g_{QCL,\max}}{g_{QWL,\max}} = \frac{128}{\pi^3} \frac{m_0}{m_r^*} \frac{\hbar\omega}{E_p} \omega\tau_{in}$$

(c) $m\_e := 0.1m0$ $\quad m\_lhz := 0.1m0$ $\quad m\_lht := 0.5m0$ $\quad \tau\_in := 0.1ps$

$$m\_r := \frac{m\_e \cdot m\_lht}{m\_e + m\_lht} \qquad \frac{m0}{m\_r} = 12$$

$hv := 1eV$ $\quad Ep := 6eV$

$$\boxed{g\_ratio := \frac{128}{\pi^3} \cdot \frac{m0}{m\_r} \cdot \frac{hv}{Ep} \cdot \frac{hv}{h\_bar} \cdot \tau\_in = 1.253 \times 10^3}$$

(d) Note that the effective masses are the same for electron and longitudinal light hole.

$$E_{e1} = \frac{\hbar^2}{2m_e^2}\frac{\pi^2}{L_z^2}$$

$$E_{e1}^{e2} = 3E_{e1} = 1 \text{ eV}$$

$$E_{lh1}^{e1} = E_{e1} + E_{lh1} + E_g = 2E_{e1} + E_g$$

$$E_{e1}^{e2} = E_{lh1}^{e1} \Rightarrow \boxed{E_g = E_{e1} = \frac{1}{3}E_{e1}^{e2} = \frac{1}{3}\text{eV}}$$

(e) For LH-electron transition:

$$\alpha_{TE} \propto \left(\frac{5}{4} - \frac{3}{4}\cos^2\theta\right)M_b^2$$

$$\alpha_{TM} \propto \left(\frac{1}{2} + \frac{3}{2}\cos^2\theta\right)M_b^2$$

$$R = \frac{\alpha_{TE}}{\alpha_{TM}} = \frac{\left(\frac{5}{4} - \frac{3}{4}\cos^2\theta\right)}{\left(\frac{1}{2} + \frac{3}{2}\cos^2\theta\right)}$$

At bandedge, θ = 0

$$\boxed{R := \frac{\frac{2}{4}}{2} = 0.25}$$

(f) At photon energy of 0.1eV above bandedge:

Angular factor: $\cos^2\theta = \frac{k_z^2}{k^2} = \frac{k_z^2}{k_t^2 + k_z^2}$

In quantum wells, k_z is quantized: $E_{en} = \frac{\hbar^2 k_z^2}{2m_e^*}$

$$\cos^2\theta = \frac{\frac{\hbar^2 k_z^2}{2m_e^*}}{\frac{\hbar^2 k_t^2}{2m_e^*} + \frac{\hbar^2 k_z^2}{2m_e^*}}$$

$$\frac{\hbar^2 k_t^2}{2m_r^*} = \hbar\omega - E_{h1}^{e1} = 0.1\ \text{eV}$$

$$\frac{\hbar^2 k_t^2}{2m_e^*} = \frac{\hbar^2 k_t^2}{2m_r^*}\frac{m_r^*}{m_e^*}$$

$$\text{E\_e1} := \frac{1}{3}\text{eV}$$

$$\theta := \text{acos}\left(\sqrt{\frac{\text{E\_e1}}{0.1\text{eV}\cdot\frac{\text{m\_r}}{\text{m\_e}} + \text{E\_e1}}}\right) \qquad \theta = 0.464 \qquad \cos(\theta)^2 = 0.8$$

$$R := \frac{5 - 3\cdot\cos(\theta)^2}{2 + 6\cdot\cos(\theta)^2} \qquad \boxed{R = 0.382}$$

***** Note: there are too many condisitons given in the problem. Lz should not be specified. If you use the Lz given, you will get some different answers (which you still receive credits) ******