EE 232 **Midterm-1 Solution** **Fall 2010**

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h_bar := $1.05459\cdot10^{-34}$ J·s q := $1.6\cdot10^{-19}$ C m0 := $9.11\cdot10^{-31}$ kg eV := $1.6\cdot10^{-19}$ J

nr := 3.5 ε0 := $8.854\cdot10^{-12}$ F·m^{-1} meV := 0.001eV

kB := $1.38\cdot10^{-23}$·J·K^{-1} T := 298K kB·T = 0.026·eV

1. R := e^{-1} Γ := 10% αi := 0cm^{-1} g_max := 10^4 cm^{-1}

R = 36.788·%

$$L\_min := \frac{1}{2\cdot(g\_max\cdot\Gamma - \alpha i)}\cdot\ln\left(\frac{1}{R^2}\right)$$

L_min = 10·μm

2. Assume the electron energy is at F_n

Find the corresponding energy for hole (E_h) that has the same momentum, k

$$E_e = F_n = E_C + \frac{\hbar^2 k^2}{2m_e^*}$$

$$\hbar^2 k^2 = 2m_e^*\cdot(F_n - E_C)$$

$$E_h = E_V - \frac{\hbar^2 k^2}{2m_h^*}$$

$$E_V - E_h = \frac{\hbar^2 k^2}{2m_h^*} = \frac{m_e^*}{m_h^*}(F_n - E_C)$$

$$n = N_C\cdot\frac{4}{3\sqrt{\pi}}\left(\frac{F_n - E_C}{k_B T}\right)^{3/2}$$

$$p = N_V\cdot\frac{4}{3\sqrt{\pi}}\left(\frac{E_V - F_p}{k_B T}\right)^{3/2}$$

Since $n = p$

$$\frac{F_n - E_C}{E_V - F_p} = \left(\frac{N_V}{N_C}\right)^{2/3} = \frac{m_h^*}{m_e^*}$$

Therefore

$$E_V - E_h = \frac{m_e^*}{m_h^*}(F_C - E_C) = \frac{m_e^*}{m_h^*}\frac{m_h^*}{m_e^*}\left(E_V - F_p\right) = E_V - F_p$$

$$\Rightarrow E_h = F_p$$

3. $g = \text{constant} \cdot \sqrt{\hbar\omega - E_g}$

$$2\times10^3\ \text{cm}^{-1} = \text{constant}\cdot\sqrt{1.04\text{eV} - E_g}$$

$$4\times10^3\ \text{cm}^{-1} = \text{constant}\cdot\sqrt{1.16\text{eV} - E_g}$$

$$\Rightarrow 2^2 = \frac{1.16\text{eV} - E_g}{1.04\text{eV} - E_g} = 4$$

$$\Rightarrow E_g = 1\ \text{eV}$$

$$\text{constant} = 10^4\ \text{cm}^{-1} /\ \text{eV}^{1/2}$$

$$\text{Eg} := 1\text{eV} \qquad \text{Const} := 10^4\cdot\frac{\text{cm}^{-1}}{\sqrt{\text{eV}}}$$

$$\text{g3} := \text{Const}\cdot\left(\sqrt{1.25\text{eV} - \text{Eg}}\right) = 5\times10^3\cdot\text{cm}^{-1}$$

4.

$$n \approx N_C \cdot \frac{4}{3\sqrt{\pi}}\left(\frac{F_n - E_C}{k_B T}\right)^{3/2}$$

$$F_n - E_C = k_B T\left(\frac{3\sqrt{\pi}}{4}\right)^{2/3}\frac{1}{N_C^{2/3}}n^{2/3}$$

$$N_C = 2\left(\frac{\pi m_e^* k_B T}{2\pi^2\hbar^2}\right)^{3/2}$$

$$F_n - E_C = k_B T\left(\frac{3\sqrt{\pi}}{4}\right)^{2/3}\frac{2\pi^2\hbar^2}{2^{2/3}\pi m_e^* k_B T}n^{2/3}$$

$$F_n - E_C = \left(\frac{3\sqrt{\pi}}{4}\right)^{2/3}\frac{2\pi^2\hbar^2}{2^{2/3}\pi m_e^*}n^{2/3} = \text{const}\cdot\frac{n^{2/3}}{m_e^*}$$

Similarly

$$E_V - F_p = \text{const}\cdot\frac{p^{2/3}}{m_h^*}$$

Note that $n = p$

Adding the last 2 equations:

$$\Delta F - E_g = \text{const}\cdot\frac{n^{2/3}}{m_r^*}$$

$$g(\hbar\omega) = C_0\left|\hat{e}\cdot\vec{P}_{cv}\right|^2 \rho_r(\hbar\omega - E_g)\, f_g(\hbar\omega - E_g)$$

At T = 0 Kevin

$$g_{peak} = C_0 \left|\hat{e}\cdot\vec{P}_{cv}\right|^2 \rho_r(\Delta F - E_g) = \text{const}_1 \cdot \left(m_r^*\right)^{3/2} \cdot \sqrt{\Delta F - E_g}$$

$$g_{peak} = \text{const} \cdot \text{const}_1 \cdot \left(m_r^*\right)^{3/2} \cdot \sqrt{\frac{n^{2/3}}{m_r^*}} = \text{const}_2 \cdot \left(m_r^*\right) n^{1/3}$$

(a) gain peak ratio:

$$\frac{g_{peak,A}}{g_{peak,B}} = \left(\frac{m_{rA}^*}{m_{rB}^*}\right) = \frac{1}{10} = 0.1$$

(b) Optical gain bandwidth

$$BW = \Delta F - E_g$$

$$\frac{BW_A}{BW_B} = \frac{\dfrac{n^{2/3}}{m_{rA}^*}}{\dfrac{n^{2/3}}{m_{rB}^*}} = \frac{m_{rB}^*}{m_{rA}^*} = 10$$