EE 232 **Midterm-2 Solution** **Fall 2010**

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$h\_bar := 1.05459 \cdot 10^{-34} J \cdot s$ $m0 := 9.11 \cdot 10^{-31} kg$ $\varepsilon 0 := 8.854 \cdot 10^{-12} F \cdot m^{-1}$

$q := 1.6 \cdot 10^{-19} C$ $eV := 1.6 \cdot 10^{-19} J$ $meV := 0.001 eV$

$kB := 1.38 \cdot 10^{-23} \cdot J \cdot K^{-1}$ $T := 300K$ $kB \cdot T = 0.026 \cdot eV$ $nr := 3.5$

1 $\lambda := 1.55 \mu m$ $n\_core := 5$ $n\_clad := 3$

(a) $$V(d) := \frac{2 \cdot \pi}{\lambda} \cdot d \cdot \sqrt{n\_core^2 - n\_clad^2}$$

$$d\_max := \frac{\pi}{\left(\frac{2 \cdot \pi}{\lambda}\right) \cdot \sqrt{n\_core^2 - n\_clad^2}}$$ $$\boxed{d\_max = 193.75 \cdot nm}$$

(b) $$\Gamma := \frac{V(100nm)^2}{2 + V(100nm)^2}$$ $$\boxed{\Gamma = 56.796 \cdot \%}$$

2

	Transition	Allowed/Not allowed	Primary Polarization
Interband	E1-HH1	Allowed	TE
	E1-LH1	Allowed	TM
	E2-HH2	Allowed	TE
	E2-LH2	Allowed	TM
	E1-HH2	Not allowed	
	E2-HH1	Not allowed	
	E1-LH2	Not allowed	
	E2-LH1	Not allowed	
Inter-subband	E1-E2	Allowed	TM
	HH1-HH2	Allowed	TM
	LH1-LH2	Allowed	TM

- Note: Intra-valance subband transitions between heavy and light hole bands are allowed for both polarizations due to mixing of heavy and light hole bands.

3 (a)

$$\Delta = \frac{F_C - E_{e1}}{k_B T} = \frac{F_V - E_{h1}}{k_B T}$$

Electron concentration: $\because F_C > E_{e1}$

$$N = \frac{m_e^* k_B T}{\pi \hbar^2 L_z}\left(\frac{F_C - E_{e1}}{k_B T}\right) = N_C^{2d} \cdot \Delta$$

Hole concentration: $\because F_V > E_{h1}$

$$P = \frac{m_h^* k_B T}{\pi \hbar^2 L_z} e^{-\frac{F_V - E_{h1}}{k_B T}} = N_V^{2d} e^{-\Delta}$$

$$N = P \ \Rightarrow \ \Delta = \frac{m_h^*}{m_e^*} e^{-\Delta} = 2e^{-\Delta} \approx 2(1-\Delta)$$

$$\Delta := \frac{2}{3}$$

Lz := 10nm

$$\text{Nc\_2d} := \frac{0.1\text{m0}\cdot\text{kB}\cdot\text{T}}{\pi\cdot\text{h\_bar}^2\cdot\text{Lz}} = 1.079 \times 10^{18}\cdot\text{cm}^{-3}$$

$$\text{N\_tr} := \text{Nc\_2d}\cdot\Delta = 7.196 \times 10^{17}\cdot\text{cm}^{-3}$$

4 Eg := 1eV   ac := −5eV   av := 1eV   b := −2eV

C12_11 := 0.5   γ1 := 3   γ2 := 1   me := 0.1m0   Lz := 10nm

Focus on the valence band edge shift due to shear potential, and the quantization of HH and LH levels:

$$Q_\varepsilon(\varepsilon) = -b\left(1 + 2\frac{C_{12}}{C_{11}}\right)\varepsilon = -2b\varepsilon$$

$$E_{HH}(\varepsilon) = -Q_\varepsilon - \frac{\hbar^2}{2m_0}\left[(\gamma_1 - 2\gamma_2)\left(\frac{\pi}{L_z}\right)^2\right] = 2b\varepsilon - \frac{\hbar^2}{2m_0}\left(\frac{\pi}{L_z}\right)^2$$

$$E_{LH}(\varepsilon) = Q_\varepsilon - \frac{\hbar^2}{2m_0}\left[(\gamma_1 + 2\gamma_2)\left(\frac{\pi}{L_z}\right)^2\right] = -2b\varepsilon - \frac{5\hbar^2}{2m_0}\left(\frac{\pi}{L_z}\right)^2$$

Set $E_{HH}(\varepsilon) = E_{LH}(\varepsilon)$, solve ε

$$\varepsilon = -\frac{1}{4b}\frac{4\hbar^2}{2m_0}\left(\frac{\pi}{L_z}\right)^2$$

$$\varepsilon := \frac{-1}{4\cdot\text{b}}\cdot\frac{4\text{h\_bar}^2}{2\text{m0}}\left(\frac{\pi}{\text{Lz}}\right)^2 = 1.883 \times 10^{-3}$$

5.

$$g_{th} = \frac{\alpha}{n\Gamma_0} = g_0 \ln\left(\frac{N_{th}}{N_{tr}}\right)$$

(a) $N_{th} = N_{tr} e^{\frac{\alpha}{n\Gamma_0 g_0}}$

(b) $J_{th} = \frac{qN_{th}nL_z}{\tau} = \frac{qN_{tr}nL_z}{\tau} e^{\frac{\alpha}{n\Gamma_0 g_0}} = \frac{qN_{tr}L_z}{\tau} \cdot n \cdot e^{\frac{\alpha}{\Gamma_0 g_0}\frac{1}{n}}$

(c) Optimum number of quantum wells:

$$\frac{dJ_{th}}{dn} = 0 = e^{\frac{\alpha}{\Gamma_0 g_0}\frac{1}{n}} + n \cdot \frac{\alpha}{\Gamma_0 g_0}\left(\frac{-1}{n^2}\right) e^{\frac{\alpha}{\Gamma_0 g_0}\frac{1}{n}}$$

$$n = \frac{\alpha}{\Gamma_0 g_0}$$