EE 232 — Final Solution — Fall 2010

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$h\_bar := 1.05459\cdot10^{-34} J\cdot s$ $\quad m0 := 9.11\cdot10^{-31} kg$ $\quad \varepsilon0 := 8.854\cdot10^{-12} F\cdot m^{-1}$

$q := 1.6\cdot10^{-19} C$ $\quad eV := 1.6\cdot10^{-19} J$ $\quad meV := 0.001 eV$

$kB := 1.38\cdot10^{-23}\cdot J\cdot K^{-1}$ $\quad T := 300K$ $\quad kB\cdot T = 0.026\cdot eV$ $\quad \varepsilon r := 9$

1. (a) Higher gain receiver has best SNR at low optical power, so APD with 20dB gain has best SNR

   (b) For strong optical signal, p-i-n is best as it has the lowest noise without added noise from the gain

   (c) $R := 50\Omega$ $\quad \eta := 100\%$ $\quad hv := 1.24eV$

   $M2 := 10$ $\quad F2 := 10$ $\quad M3 := 100$ $\quad F3 := 100$

   Lower power limit: SNR_2 = SNR_3

$$SNR_{APD2} = SNR_{APD3}$$

$$\frac{\frac{1}{2} I_p^2 \langle M_2\rangle^2}{2eI_pF_2\langle M_2\rangle^2 dv + \frac{4k_BT\Delta v}{R}} = \frac{\frac{1}{2} I_p^2 \langle M_3\rangle^2}{2eI_pF_3\langle M_3\rangle^2 dv + \frac{4k_BT\Delta v}{R}}$$

$$\frac{1}{2eI_pF_2 + \frac{1}{\langle M_2\rangle^2}\frac{4k_BT}{R}} = \frac{1}{2eI_pF_3 + \frac{1}{\langle M_3\rangle^2}\frac{4k_BT}{R}}$$

$$2eI_p(F_3 - F_2) = \frac{4k_BT}{R}\left(\frac{1}{\langle M_2\rangle^2} - \frac{1}{\langle M_3\rangle^2}\right)$$

$$I_p = \frac{\left(\frac{1}{\langle M_2\rangle^2} - \frac{1}{\langle M_3\rangle^2}\right)\frac{4k_BT}{R}}{2e(F_3 - F_2)}$$

$$Ip := \frac{\left(\frac{1}{M2^2} - \frac{1}{M3^2}\right)\cdot\frac{4\cdot kB\cdot T}{R}}{2\cdot q\cdot(F3 - F2)} \qquad Ip = 0.114\cdot\mu A$$

$$Pmin := Ip\cdot\frac{hv}{q}\cdot\frac{1}{\eta} \qquad \boxed{Pmin = 1.412\times10^{-7} W}$$

Lower power limit: SNR_2 = SNR_p-i-n

$$SNR_{APD2} = SNR_{p-i-n}$$

$$\frac{\frac{1}{2} I_p^2 \langle M_2 \rangle^2}{2eI_p F_2 \langle M_2 \rangle^2 dv + \frac{4k_B T \Delta v}{R}} = \frac{\frac{1}{2} I_p^2}{2eI_p dv + \frac{4k_B T \Delta v}{R}}$$

$$\frac{1}{2eI_p F_2 + \frac{1}{\langle M_2 \rangle^2} \frac{4k_B T}{R}} = \frac{1}{2eI_p + \frac{4k_B T}{R}}$$

$$2eI_p (F_2 - 1) = \frac{4k_B T}{R} \left(1 - \frac{1}{\langle M_2 \rangle^2} \right)$$

$$I_p = \frac{\left(1 - \frac{1}{\langle M_2 \rangle^2} \right) \frac{4k_B T}{R}}{2e(F_2 - 1)}$$

$$Ip := \frac{\left(1 - \frac{1}{M2^2} \right) \cdot \frac{4 \cdot kB \cdot T}{R}}{2 \cdot q \cdot (F2 - 1)} \qquad Ip = 113.85 \cdot \mu A$$

$$Pmax := Ip \cdot \frac{hv}{q} \cdot \frac{1}{\eta} \qquad \boxed{Pmax = 1.412 \times 10^{-4} W}$$

2. $\alpha := 10^4 cm^{-1}$ $\quad ve := 10^7 cm \cdot s^{-1}$ $\quad vh := 10^6 cm \cdot s^{-1}$

$\varepsilon := 10^{-12} F \cdot cm^{-1}$ $\quad hv := 1.24eV$ $\quad R := 50\Omega$

(a)

$$f_{3dB} = \frac{1}{2\pi} \frac{1}{\tau_t + \tau_{RC}} \le \frac{1}{2\pi} \frac{1}{2\sqrt{\tau_t \tau_{RC}}} = \frac{1}{4\pi} \frac{1}{\sqrt{\frac{d}{v_h} R \frac{\varepsilon A}{d}}} = \frac{1}{4\pi} \frac{1}{\sqrt{\frac{R\varepsilon A}{v_h}}} = 100\text{GHz}$$

$$A = \frac{v_h}{R\varepsilon} \left(\frac{1}{4\pi \cdot 100\text{GHz}} \right)^2$$

Optimum bandwidth obtained when transit time equals RC time:

$$R \frac{\varepsilon A}{d} = \frac{d}{v_h}$$

$$d = \sqrt{R\varepsilon A v_h}$$

$$A := \frac{vh}{R \cdot \varepsilon}\left(\frac{1}{4\pi} \cdot \frac{1}{100GHz}\right)^2 \qquad \boxed{A = 1.267 \cdot \mu m^2}$$

$$d := \sqrt{R \cdot \varepsilon \cdot A \cdot vh} \qquad \boxed{d = 7.958 \times 10^{-3} \cdot \mu m}$$

(b) $\eta := 1 - \exp(-\alpha \cdot d)$ $\qquad \boxed{\eta = 0.793 \cdot \%}$

(c)

$$SNR = \frac{\frac{1}{2} I_p^2}{2eI_p dv + \frac{4k_B T \Delta v}{R}}$$

SNR := 1000

Initial value: $Ip := 1\mu A$

$$SNR = \frac{\frac{1}{2} Ip^2}{\left(2 \cdot q \cdot Ip + \frac{4kB \cdot T}{R}\right) \cdot (100GHz)}$$

Ip_1000 := Find(Ip) $\qquad$ $Ip\_1000 = 2.914 \times 10^{-4} A$

$$P := \frac{Ip\_1000}{\eta} \cdot \frac{hv}{q} \qquad \boxed{P = 45.58 \cdot mW}$$

3 (a) $hv := 1eV$ $\qquad$ $\tau\_in := 0.1ps$

$me := 0.1m0$ $\qquad$ $mh := 1m0$ $\qquad$ $Lz := 10nm$

$$\frac{\hbar^2}{2m^*}\left(\frac{\pi}{L}\right)^2 \left(2^2 - 1\right) = \frac{1.24}{12.4} eV = 0.1eV$$

$$Ln := \sqrt{\frac{h\_bar^2}{2me} \cdot \pi^2 \cdot 3 \cdot \frac{1}{0.1eV}} \qquad \boxed{Ln = 10.628 \cdot nm}$$

$$Lp := \sqrt{\frac{h\_bar^2}{2mh} \cdot \pi^2 \cdot 3 \cdot \frac{1}{0.1eV}} \qquad \boxed{Lp = 3.361 \cdot nm}$$

(b)

$$\alpha(\hbar\omega) = \frac{\pi\omega}{n_r c \varepsilon_0} g(E_{e1}^{e2} - \hbar\omega) \left|\mu_{21}\right|^2 N_1$$

$$\mu_{21} \propto L_z$$

$$\alpha\_n\_p\_ratio := \left(\frac{Ln}{Lp}\right)^2 \qquad \boxed{\alpha\_n\_p\_ratio = 10}$$

(c)

$$\alpha_{\max} = \frac{\pi\omega}{n_r c \varepsilon_0} g(E_{e1}^{e2} - \hbar\omega) \left|\mu_{21}\right|^2 \frac{m_e^*}{\pi\hbar^2 L_z} E_{e1}^{e2}$$

$$\alpha\_max\_n\_p\_ratio := \left(\frac{Ln}{Lp}\right)^2 \cdot \frac{Lp}{Ln} \cdot \frac{me}{mh}$$

$$\boxed{\alpha\_max\_n\_p\_ratio = 0.316}$$

4.

5. (a)

Bulk:

Let $\Delta = \frac{F_C - E_C}{k_B T} = \frac{F_V - E_V}{k_B T}$

$$N = 2\left(\frac{\pi m_e^* k_B T}{2\pi^2\hbar^2}\right)^{3/2} \frac{4}{3\sqrt{\pi}}\left(\frac{F_n - E_C}{k_B T}\right)^{3/2} = N_C \cdot \frac{4}{3\sqrt{\pi}}\Delta^{3/2}$$

$$P = 2\left(\frac{\pi m_h^* k_B T}{2\pi^2\hbar^2}\right)^{3/2} e^{-\frac{F_V - E_V}{k_B T}} = N_V e^{-\Delta}$$

$$N = P \;\Rightarrow\; \frac{4}{3\sqrt{\pi}}\Delta^{3/2} = \left(\frac{m_h^*}{m_e^*}\right)^{3/2} e^{-\Delta}$$

Δ depends only on the ratio of hole and electron effective masses.

So semiconductor A and B have the same Δ.

$$\frac{N_A}{N_B} = \left(\frac{m_{e,A}^*}{m_{e,B}^*}\right)^{3/2} = \left(\frac{1}{2}\right)^{3/2}$$

$$\left(\frac{1}{2}\right)^{1.5} = 0.354$$

(b)

In quantum wel:

$$N = \frac{m_e^* k_B T}{\pi \hbar^2 L_z} \left(\frac{F_C - E_{e1}}{k_B T} \right) = N_C^{2d} \cdot \Delta$$

$$P = \frac{m_h^* k_B T}{\pi \hbar^2 L_z} e^{-\frac{F_V - E_{h1}}{k_B T}} = N_V^{2d} e^{-\Delta}$$

$$N = P \;\Rightarrow\; \Delta = \frac{m_h^*}{m_e^*} e^{-\Delta}$$

Δ depends only on the ratio of hole and electron effective masses.

So semiconductor A and B have the same Δ.

$$\frac{N_A}{N_B} = \left(\frac{m_{e,A}^*}{m_{e,B}^*} \right) = \frac{1}{2}$$

(c) Same carrier concentration:

$$N = 2\left(\frac{\pi m_e^* k_B T}{2\pi^2 \hbar^2} \right)^{3/2} \frac{4}{3\sqrt{\pi}} \left(\frac{F_n - E_C}{k_B T} \right)^{3/2}$$

$F_n - E_C$ is inversely proportional to m_e^*

$$g_{\max} = C_0 \left| \hat{e} \cdot \vec{P}_{cv} \right|^2 \rho_r (\Delta F - E_g)$$

$$F_n - E_C = \left(\Delta F - E_g \right) \frac{m_h^*}{m_e^* + m_h^*}$$

$$\Delta F - E_g = \left(F_n - E_C \right) \frac{m_e^* + m_h^*}{m_h^*}$$

$$\frac{g_{\max,A}}{g_{\max,B}} = \left(\frac{m_{e,A}^*}{m_{e,B}^*} \right)^{3/2} \left(\frac{\left(\Delta F - E_g \right)_A}{\left(\Delta F - E_g \right)_B} \right)^{1/2} = \left(\frac{m_{e,A}^*}{m_{e,B}^*} \right)^{3/2} \left(\frac{m_{e,B}^*}{m_{e,A}^*} \right)^{1/2} = \frac{m_{e,A}^*}{m_{e,B}^*} = \frac{1}{2}$$

(d)

$$g_{\max} = C_0 \left| \hat{e} \cdot \vec{P}_{cv} \right|^2 \frac{m_r^*}{\pi \hbar^2 L_z}$$

$$\frac{g_{\max,A}}{g_{\max,B}} = \frac{m_{e,A}^*}{m_{e,B}^*} = \frac{1}{2}$$

6 (a) $\lambda := 1\mu m$ $n := 3.33$ $me := 0.5m0$ $mh := 0.5m0$ $Ep := 24eV$

$\omega := \frac{1.24eV}{h\_bar}$ $mr := \left(me^{-1} + mh^{-1}\right)^{-1}$ $Lz := 10nm$

$C0 := \frac{\pi q^2}{n \cdot c \cdot \varepsilon 0 \cdot m0^2 \cdot \omega}$ $C0 = 5.828 \times 10^9 \frac{m^2}{kg}$

$g\_max := C0 \cdot \frac{m0}{6} \cdot Ep \cdot \frac{mr}{\pi \cdot h\_bar^2 \cdot Lz}$ $g\_max = 2.215 \times 10^6 \frac{1}{m}$

(b)

$$g_{\max} = \frac{1}{2L_z} \ln\left(\frac{1}{R \cdot 100\%}\right)$$

$$R_{\min} = \exp(-2 \cdot L_z g_{\max})$$

$R\_min := \exp(-2Lz \cdot g\_max)$ $R\_min = 0.957$

(c) $\tau p := \frac{1}{g\_max \cdot \frac{c}{n}}$ $\tau p = 5.015 \times 10^{-15} s$

$Q := \omega \cdot \tau p$ $Q = 9.435$

7.

$$f_R = \frac{1}{2\pi}\sqrt{\frac{v_g a S}{\tau_p}}$$

$$P_{out} = \frac{\hbar\omega}{\tau_p}\frac{\alpha_m}{\alpha_i + \alpha_m}\frac{VS}{\Gamma} \approx \frac{\hbar\omega}{\tau_p}\frac{VS}{\Gamma}$$

$$\frac{1}{\tau_p} = \frac{c}{n}\frac{1}{2\Gamma L}\ln\left(\frac{1}{R_1 R_2}\right)$$

Note that to keep output power constant at 1 mW

$$\frac{S}{\tau_p} = \frac{P_{out}}{\hbar\omega}\frac{\Gamma}{V} \propto \frac{\Gamma}{V}$$

(a) L reduced by 4x, V also reduced by 4x, S/τp increased by 4x, f_r =2x 10GHz = 20 GHz

(b) R increases to 90%, S/τp stay the same. So f_r = 10 GHz

(c) Γ goes up by 4x, S/τp increases by 4x, f_r goes up by 2x --> 20 GHz