Intersubband Transition in Quantum Wells

- Transition between quantized energy levels in a quantum well by absorption or emission of a photon

- Absorption
  - Infrared photodetectors
  - Thermal imager: blackbody radiation of human body ~ 10 µm
  - 3 ~ 5 and 8 ~ 10 µm wavelength bands particularly interesting

- Emission
  - Gain media for quantum cascaded lasers (QCL)
  - Long wavelength emission coincides with molecular vibration spectra

For a 10-nm QW in GaAs

- $E_{e2} - E_{e1} = 224 \text{ meV} - 56 \text{ meV} = 168 \text{ meV}$

- $\lambda = \frac{1.24}{0.168} = 7.4 \mu m$
Optical Matrix Element for Intersubband Transition

Quantum Well Wavefunction

\[ |a\rangle = \psi^\dagger_a(r) = u_e(r) e^{ik\rho} \sqrt{A} \phi_1(z) \]

\[ |b\rangle = \psi^\dagger_b(r) = u_e(r) e^{ik\rho} \sqrt{A} \phi_2(z) \]

\[ H'_{ba} = -\hat{E} \cdot \mu_{ba} \]

\[ \mu_{ba} = \langle b|e^r|a\rangle \]

\[ = \langle u_e(r)|u_e(r)\rangle \int \frac{e^{ik\rho} e^{ik\rho^*}}{A} d\rho \]

\[ \cdot \int \phi_2^*(z)e\phi_1(z)dr = 1 \cdot \delta_{k_1,k_1'} \mu_{21}^z \]

Absorption Coefficient for Intersubband Transition

\[ \alpha(\hbar\omega) = \frac{\pi \omega}{n_e e_0} \cdot \frac{2}{V} \sum_{k_i} g(E_b - E_a - \hbar\omega) \hat{\epsilon} \cdot \mu_{ba} \left| f_a - f_b \right| \]

The summation is over all electron states: \( \tilde{k}_i = k_x \hat{x} + k_y \hat{y} \)

We need to consider the finite width of the energy spread (otherwise the absorption is a delta function with infinite absorption peak)

\[ g(\Delta E) = \frac{1}{\pi} \frac{\Gamma/2}{\Delta E^2 + (\Gamma/2)^2} \] (Lorentzian Lineshape)

\[ \hat{\epsilon} \cdot \mu_{ba} = \left| \mu_{21} \right|^2 \text{ only when } \hat{\epsilon} = \hat{z} \text{ (TM polarization)} \]

\[ E_b - E_a = E_{ei}^2, \text{ independent of } \tilde{k}_i \]

\[ \alpha(\hbar\omega) = \frac{\pi \omega}{n_e e_0} g(E_{ei}^2 - \hbar\omega) \left| \mu_{21} \right|^2 \frac{2}{V} \sum_{k_i} (f_a - f_b) \]

\[ \alpha(\hbar\omega) = \frac{\pi \omega}{n_e e_0} g(E_{ei}^2 - \hbar\omega) \left| \mu_{21} \right|^2 (N_i - N_2) \]
Absorption Coefficient for Intersubband Transition

\[ \alpha(h\omega) = \frac{\pi \omega}{n \epsilon_0 c_s} g(E_{e1}^2 - h\omega) \mu_{21} \int (N_1 - N_2) \]

(1) \( E_1 < F < E_2 \)

\[ N_2 = 0 \]

\[ \alpha(h\omega) = \frac{\pi \omega}{n \epsilon_0 c_s} g(E_{e1}^2 - h\omega) \mu_{21} \int N_1 \]

is proportional to doping concentration

(2) \( E_2 < F \)

\[ N_1 = \frac{m^* k_b T}{\pi \hbar^2 L_z} \ln(1 + e^{\frac{F-E_1}{k_b T}}) = \frac{m^* k_b T}{k_b T} \]

\[ N_2 = \frac{m^*}{\pi \hbar^2 L_z} (F - E_2) \]

\[ \alpha(h\omega) = \frac{\pi \omega}{n \epsilon_0 c_s} g(E_{e1}^2 - h\omega) \mu_{21} \int \frac{m^*}{\pi \hbar^2 L_z} E_{e1}^2 \]

is a constant, independent of doping concentration

Intersubband Transition in P-Doped QW (1):
Fermi-Level Below Second Subband

Proportional to doping concentration
Intersubband Transition in N-Doped QW (2):
Fermi-Level Above Second Subband

\[ V_0 = \infty \]

\[ E_{e2}^e \]

\[ E_{e1}^e \]

\[ z = 0 \quad z = L \]

\[ E_{h1}^h \]

\[ E_{h2}^h \]

\[ \alpha(h\omega) = \frac{\pi \omega}{n_c e\varepsilon_0} g(E_{e1}^e - h\omega) \left| \mu_{21} \right|^2 \]

\[ \cdot \frac{m^*_0}{\pi \hbar^2 L_z} E_{e1}^e \]

Independent of doping concentration

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Intersubband Transition in P-Doped QW (1):
Fermi-Level Above Second Subband

\[ V_0 = \infty \]

\[ E_{e2}^e \]

\[ E_{e1}^e \]

\[ z = 0 \quad z = L \]

\[ E_{h1}^h \]

\[ E_{h2}^h \]

\[ \alpha(h\omega) = \frac{\pi \omega}{n_c e\varepsilon_0} g(E_{h2}^h - h\omega) \left| \mu_{21} \right|^2 P \]

\[ P = \text{P-type doping concentration} \]
Intersubband Transition in P-Doped QW (2):
Fermi-Level Below Second Subband

\[ V_0 = \infty \]
\[ z = 0 \quad z = L \]

\[ E_{e1}^e \]
\[ E_{e2} \]
\[ E_{e1} \]
\[ E_{h1}^h \]
\[ E_{h2} \]
\[ k_y \]

Independent of doping concentration

\[ \alpha(\hbar \omega) = \frac{\pi \omega}{n_e e_0} g(E_{h2}^{hl} - \hbar \omega) | \mu_{z1} |^2 \]
\[ = \frac{m^*}{\pi \hbar^2 L_z} E_{h2}^{hl} \]

\[ F \quad \text{(Fermi Level)} \]

Intersubband Dipole Moment

\[ \phi_1(z) = \frac{2}{L_z} \sin \left( \frac{\pi}{L_z} z \right) \]
\[ \phi_2(z) = \frac{2}{L_z} \sin \left( \frac{2\pi}{L_z} z \right) \]

\[ \mu_{z1} = e \int_0^L \phi_1(z) \cdot z \cdot \phi_1(z) dz \]
\[ = \frac{2e}{L_z} \int_0^L \sin \left( \frac{\pi}{L_z} z \right) \cdot z \cdot \sin \left( \frac{2\pi}{L_z} z \right) dz \]
\[ = -\frac{16}{9\pi^2} e L_z \]

Compare with dipole moment of interband transition:

\[ \frac{\mu_{z1}^{\text{int}}}{\mu_{e\nu}^{\text{int}}} = \frac{16}{9\pi^2} e L_z \approx 0.2 L_z \]
\[ \approx 0.4 \text{ nm} \]
Example

\[ L_z = 10 \text{nm} \]

\[ E_{e_1} = \frac{\hbar^2}{2m_e} \left( \frac{\pi}{L_z} \right)^2 = 56 \text{ meV} \]

\[ E_{e_2} = \frac{\hbar^2}{2m_e} \left( \frac{2\pi}{L_z} \right)^2 = 224 \text{ meV} \]

\[ E_{e_1} = 168 \text{ meV} \]

\[ N = 10^{18} \text{ cm}^{-3} \]

First, determine if the second subband is occupied.

Find \( N_{1, \text{max}} \), the electron concentration in the first subband when the Fermi level is at the bottom of the second subband:

\[ N_{1, \text{max}} = \frac{m_e^*}{\pi \hbar^2 L_z} E_{e_2} = 4.7 \times 10^{18} \text{ cm}^{-3} > N = 10^{18} \text{ cm}^{-3} \]

So the second subband is not occupied. \( N_z = 0 \)

Absorption Spectra

\[ \alpha(\hbar\omega) = \frac{n\epsilon_0}{e\epsilon_0} \sqrt{\frac{16}{9\pi^2}} \frac{eL_z}{c} N \frac{1}{\Gamma} \left( \frac{16}{9\pi^2} eL_z \right)^2 \]

Peak absorption

\[ \alpha_{\text{max}} = \frac{n\epsilon_0}{e\epsilon_0} \sqrt{\frac{16}{9\pi^2}} \frac{1}{\Gamma} \left( \frac{16}{9\pi^2} eL_z \right)^2 N \approx 2 \times 10^4 \text{ cm}^{-3} \]

\[ \lambda_{\text{cm}}^{\alpha_0} \]

[Graph of absorption spectra]
Quantum Cascade Laser (QCL)

- Using intersubband transition to provide gain
- From THz to IR
- Key design
  - Upper state should be aligned with a “minigap” to prevent direct tunneling loss of upper state electrons
  - Lower state should be aligned with a “mini-band” to quickly remove the lower state population
- The electrons can be “reused” by cascading the quantum wells
- Emission wavelength tailorable by varying thickness of various layers