EE 232 Lightwave Devices
Lecture 19: p-i-n Photodiodes and Photoconductors

Reading: Chuang, Chap. 15 (2nd Ed)

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**P-i-n Photodiode**

- Reverse-biased p-i-n junction
- Most of the voltage drop across the i-region, the main absorption region
- High field separates photogenerated electron and hole
- Large bandgap materials are used for P and N if possible
- Fast response
- Low noise
- No gain (quantum efficiency < 100%)
I-V Curve

Dark current: \( I = I_0 (e^{\frac{qV}{kT}} - 1) \)

Photocurrent: \( I_{ph} = \frac{\eta q}{h\omega} P_{opt} \)

Quantum efficiency: \( \eta = \eta_i (1 - R) (1 - e^{-\alpha d}) \)

Total current: \( I = I_0 (e^{\frac{qV}{kT}} - 1) + I_{ph} \)

Load Line for Biasing the PD

Absorption Coefficient

- Light intensity decays exponentially in semiconductor:
  \( I(x) = I_0 e^{-\alpha x} \)

- Direct bandgap semiconductor has a sharp absorption edge

- Si absorbs photons with \( h\nu > E_g = 1.1 \text{ eV} \), but the absorption coefficient is small
  – Sufficient for CCD

- At higher energy (~ 3 eV), absorption coefficient of Si becomes large again, due to direct bandgap transition to higher CB
Two Types of p-i-n Photodiodes

Surface-Illuminated p-i-n
\[ \eta = \eta_i (1 - R)(1 - e^{-\alpha d}) \]
\( \eta_i \): internal quantum efficiency
\( R \): reflectivity
\( d \): absorption layer thickness

Waveguide p-i-n
\[ \eta = \eta_i (1 - R)(1 - e^{-\alpha \Gamma L}) \]
\( \eta_i \): internal quantum efficiency
\( R \): reflectivity
\( \Gamma \): confinement factor
\( L \): length of waveguide PD

Ramo’s Theorem

The current caused in external circuit by a moving charge \( q \) moving at a velocity \( v(t) \) in a parallel plate with a separation of \( d \) and a voltage bias of \( V \) is
\[ i(t) = \frac{q v(t)}{d} \]

Proof:
Work done on the charge:
\[ W = \text{Force} \times \text{Displacement} = q Edx = q \frac{V}{d} dx \]
Work provided by power supply:
\[ W = i(t)V dt \]
\[ \Rightarrow i(t)V dt = q \frac{V}{d} dx \]
\[ i(t) = \frac{q dx}{d dt} = \frac{q v(t)}{d} \]
Response of One Photogenerated Electron-Hole Pair

\[ i(t) = i_e(t) + i_h(t) \]

where \( qv_e / d \) and \( qv_h / d \) are the electron and hole trajectories respectively.

Total charge generated:

\[ Q = \int_0^{\infty} i_e(t) dt + \int_0^{\infty} i_h(t) dt \]

\[ Q = \frac{q v_e}{d} \int_0^{d-x} \frac{d-x}{v_e} dt + \frac{q v_h}{d} \int_0^x \frac{x}{v_h} dt \]

One absorbed photon → one charge detected

Electron current ends when the last electron generated near P-side reaches N-electrode: \( t = d/v_e \)

Hole current ends when the last hole generated near N-side reaches P-electrode: \( t = d/v_h \)

Hole is usually slower → A conservative estimate of the transit time:

\[ \tau = \frac{d}{v_h} \]
Total Response Time of p-i-n

1. RC time:
   \[ \tau_{RC} = RC = R \frac{E_A}{d} \]  \( (A: \) area of p-i-n) 
2. Transit time:
   \[ \tau_i = \frac{d}{v_h} \]

Total response time:
\[ \tau = \tau_{RC} + \tau_i \]
\[ f_{3dB} = \frac{1}{2\pi\tau} \]

Absorption layer thickness for optimum frequency response:
\[ \tau = \tau_{RC} + \tau_i = \frac{R E_A}{d} + \frac{d}{v_h} \]
\[ \tau \geq 2 \sqrt{\frac{R E_A}{d}} \left( \frac{d}{v_h} \right) \]

Optimum bandwidth occurs when
\[ \frac{R E_A}{d} = \frac{d}{v_h} \]
\[ d_{\text{optimum}} = \sqrt{R E A v_h} \]

More Rigorous Analysis of p-i-n Response Time

Small-signal analysis: assume the input light is modulated at frequency \( \omega \), the photocurrent is proportional to
\[ |i(t)| \propto \left| \frac{1}{1 + j \omega RC} \sin^2 \left( \frac{\omega \tau_i}{2} \right) \right| \]

The first term is single-pole response from RC, while the second term is the phase delay due to transit time response.
Comparison of Numeric Examples

Example:
\[ \tau_{RC} = 14.4 \text{ ps} \]
\[ \tau_t = 20 \text{ ps} \]
\[ f_{3dB} = \frac{1}{2\pi} \frac{1}{\tau_{RC} + \tau_t} = 4.6 \text{ GHz} \]

\[ |i(t)| \propto \left| \frac{1}{1 + j\omega RC} \sin^2 \left( \frac{\omega \tau_t}{2} \right) \right| = |H(\omega)| \]

Solving \[ |H(\omega)| = \frac{1}{\sqrt{2}}, \quad f_{3dB} = 9.7 \text{ GHz} \]

The discrepancy is smaller when RC dominates, and larger when transit time dominates.
(Transit time response has a sharp drop-off).

Bandwidth-Efficiency Product

(1) For surface-illuminated p-i-n (assume AR coating: \( R=0\% \)), in the extreme of thin absorbing layer and transit-time-dominated response:

\[ \eta = \eta_s (1 - e^{-\alpha d}) = \eta_s (1 - (1 - \alpha d)) = \eta_s \alpha d \]

\[ f_{3dB} = \frac{1}{2\pi} \frac{v_h}{d} \]

Bandwidth-efficiency product: \[ f_{3dB} \times \eta = \left( \frac{1}{2\pi} \frac{V_h}{d} \right) \eta_s \alpha d = \frac{\eta_s \alpha V_h}{2\pi} \]

(2) On the other hand, the efficiency of waveguide p-i-n is

\[ \eta_s (1 - e^{-\Gamma L}) = \eta_s \Gamma \alpha L \]

RC-limited bandwidth: \[ f_{3dB} = \frac{1}{2\pi} \frac{d}{R_e L_w} \]

Bandwidth-efficiency product: \[ f_{3dB} \times \eta = \left( \frac{1}{2\pi} \frac{d}{R_e L_w} \right) \eta_s \Gamma \alpha L = \frac{\eta_s \Gamma \alpha d}{2\pi R_e W} \]

\[ \Rightarrow \quad \text{In general, there is a bandwidth-efficiency trade-off} \]
**Photoconductors**

Dark current:

\[ J_0 = \sigma_0 E = \left( n_0 q \mu_n + p_0 q \mu_p \right) E \]

Light illumination generate electron-hole pairs, increasing the conductivity:

\[ \frac{d\delta n}{dt} = G_0 - \frac{\delta n}{\tau_n} \]

Steady state: \( \frac{d}{dt} \rightarrow 0 \)

\[ \delta n = G_0 \tau_n \]

\[ \Delta J = \delta n \cdot q \left( \mu_n + \mu_p \right) E \]

Photoconductor requires both contacts to be Ohmic and the semiconductor doping type to be the same.

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**Photocarrier Generation Rate**

Light Intensity \( \propto e^{-\alpha x} \)

\[ P_{\text{gpr}}(1-R) \]

\( (1-e^{-\alpha d}) \)

\[ G_0 = \eta \frac{P_{\text{gpr}}}{\hbar \omega lwd} \] : photocarrier generation rate \( \frac{1}{\text{cm}^3 \text{s}} \)

\[ \eta = \eta \left( 1-R \right) \left( 1-e^{-\alpha d} \right) \]

\( R \) : reflectivity of photoconductor surface

\( \alpha \) : absorption coefficient

\( d \) : absorption length

\( e^{-\alpha d} \) : fraction of light remains after absorption length \( d \)
Photoconductive Gain

\[ \Delta I = \frac{\Delta \Phi}{\omega} = \frac{\Phi}{\omega} \left( G_n \tau_n \right) \left( \mu_n + \mu_p \right) E \]

\[ \Delta I = \frac{\Delta \Phi}{\omega} \left( \eta \frac{P_{\text{opt}}}{h \omega} \frac{1}{\tau_n} \right) q \left( \mu_n + \mu_p \right) E \]

\[ \Delta I = \eta \frac{P_{\text{opt}}}{h \omega} \frac{1}{\tau_n} q \left( \mu_n E \right) = \eta \frac{P_{\text{opt}}}{h \omega} \frac{1}{\tau_n} \frac{1}{d} V_n \]

\[ \tau_i = \frac{d}{v_n} \quad \text{transit time} \]

\[ \Delta I = \left( \eta \frac{P_{\text{opt}}}{h \omega} \frac{q}{\tau_n} \right) \left( \frac{\tau_n}{\tau_i} \right) \]

Photocurrent \quad \text{Photoconductive Gain}

Analogy to Current Gain in Bipolar Transistor

Current gain in bipolar transistor:

\[ \beta = \frac{I_C}{I_B} \]

The current gain can also be expressed as

\[ \beta = \frac{\tau_{\text{re}}}{\tau_i} \]

\[ \tau_i \quad \text{transit time} \]

\[ \tau_{\text{re}} \quad \text{carrier recombination lifetime in the base} \]
Frequency of Photoconductors

\[
\frac{dN}{dt} = \eta P_{\text{opt}} \frac{1}{\hbar \omega \text{w} \text{d}} - \frac{N}{\tau_n} \\
\text{Small signal response:} \\
N = N_0 + N e^{\nu t} \\
\eta N_1 = \frac{N_1}{\hbar \omega \text{w} \text{d}} - \frac{N_1}{\tau_n} \\
N_1 = \frac{\eta P_1}{\hbar \omega \text{w} \text{d}} \frac{1}{\nu \omega + 1/\tau_n} \\
I_1 = J_\nu w = (N_0 q \nu) \nu w \\
\frac{I_1}{P_1} = \left( \frac{\eta \nu}{\hbar \omega} \right) \left( \frac{\tau_n}{\nu \omega} \right) \frac{1}{\nu \omega + 1/\tau_n} = (\text{DC Quantum Efficiency}) \times (\text{Photoconductive Gain}) \times (\text{Normalized Frequency Response})
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