Grading Note:
(1) Make sure you see all the problems.
(2) Numerical answers within 10% will receive full credit.

1. A double heterostructure (DH) edge-emitting laser has a cavity length of 200 µm, a width of 1 µm, an active layer thickness of 0.1 µm and a confinement factor of 50%. The front and back mirror reflectivity is 50% and 100%, respectively. The intrinsic loss of the cavity is 10 cm⁻¹. The wavelength of the laser is 1.24 µm, and the effective refractive index of the laser waveguide is 3.0.
   a) What is the threshold gain of the laser?
   b) What is the photon lifetime of the cavity?
   c) What is the optical Q (quality factor) of the cavity?
   d) What is the total quantum efficiency in % and slope efficiency in Watt/Amp?

2. An astronaut brought a semiconductor sample back from Mars. It was found the semiconductor had an electron effective mass of 1 m₀, a hole effective mass of 0.1 m₀, and a bandgap energy of 1 eV. Other properties were similar to typical semiconductors on earth. The refractive index is 3, and the optical matrix element is

   \[ |\hat{e} \cdot \hat{P}_c| \leq \frac{m_0}{6} E_p \quad \text{with} \quad E_p = 24 \text{ eV} \]

   a) Draw the energy band diagram (E-vs-k) of this semiconductor. Pay attention to the curvature of the conduction and valance bands.
   b) The semiconductor is optically pumped until the quasi-Fermi level for holes is 0.1 eV below Eᵥ. What is the hole concentration in cm⁻³?
   c) What is the electron concentration? Where is the quasi-Fermi level for electrons (in reference to the conduction band edge, E_C)?
   d) Draw both quasi-Fermi levels in the diagram in a).
   e) Is there net gain at this pumping condition? How do you determine that?
   f) If the semiconductor has net gain, find the peak optical gain at zero Kelvin. If the semiconductor has net loss, find the optical absorption coefficient at photon energy of 1.1 eV.

3. A semiconductor with electron and hole effective masses of 0.1 m₀ and 1 m₀, respectively, is used for quantum well inter-subband infrared photodetectors. The bandgap energy of the well material is 1 eV. The target absorption wavelength is 10 µm. Assume both semiconductors have the same inter-subband scattering time.
   a) What is the width of the quantum well if it is doped N type?
   b) What is the width of the quantum well if it is doped P type?
c) At optimum doping concentrations (which might be different for N and P type quantum wells), which quantum well has higher absorption coefficient? What is the ratio of their absorption coefficients?

4. Two semiconductors are used for quantum well lasers: Semiconductor A has $m_e^* = m_h^* = 0.1 \, m_0$, and Semiconductor B has $m_e^* = m_h^* = 1 \, m_0$. Both semiconductors have the same bandgap energy at 1 eV, and the same optical matrix element. The quantum wells have the same width.

a) At the same pumping condition (i.e., same excess electron/hole concentration), which semiconductor quantum well has higher gain? What is the ratio of their gain coefficient?

b) To reach Bernard-Duraffourg gain condition, which semiconductor requires higher pumping (i.e., higher carrier concentration)? Qualitative and graphical argument is fine.