Grading Note:
(1) Make sure you see all the problems.
(2) Numerical answers within 10% will receive full credit.
(3) Unless specified in the problem, you can use a constant \( C_0 = \frac{\pi e^2}{n_r \varepsilon_0 m_0 \omega} = 10^{10} \frac{m^2}{kg} \) for your gain calculation, neglecting the change in optical frequency.

1. Consider a gain media consisting of a single quantum well with a well width of 10nm and an effective reduced mass of 0.2 \( m_0 \), and \( E_P = 24 \, eV \).
   a) What is the maximum gain obtainable from the quantum well for optical transitions between the first electron and first hole subbands? For simplicity, use \( C_0 = \frac{\pi e^2}{n_r \varepsilon_0 m_0 \omega} = 10^{10} \frac{m^2}{kg} \)
   b) If we use this single quantum well as the gain medium of a vertical cavity surface-emitting laser (VCSEL), what is the minimum reflectivity of the top mirror? Assume the reflectivity of the bottom mirror is 100%.

2. The measured L-I curve and the optical spectrum of a semiconductor laser are shown below. The output power reaches a maximum and then decreases due to heating of the laser. The voltage drop across the laser is about 2V after the laser turns on.

Use the data to estimate the following parameters:
   a) What is the threshold current? What is the slope efficiency [in W/A]?
   b) What is the quantum efficiency [in %]?
   c) What is the maximum wall-plug efficiency of the laser [in %]?
   d) What is the cavity length of this laser? Assume the effective refractive index of a typical semiconductor laser is 4.
3. A semiconductor with an electron effective mass of 0.1\,m_0 and a hole effective mass of 1m_0 is used as a bulk semiconductor (as in double heterostructure) and a quantum well gain media.
   a) Which gain media has lower transparency carrier concentration? Support your answer with calculation or arguments.
   b) What is the transparency carrier concentration of the quantum well gain media? The width of the quantum well is 10 nm.

4. A semiconductor has equal electron and hole effective masses: \( m_e^* = m_h^* = 0.5 \, m_0 \). Its matrix element is \( |\vec{d} \cdot \vec{P}_{c}\nu| = \frac{m_0}{6} E_p \), with \( E_p = 24 \, \text{eV} \).
   The bandgap energy is 1 eV. The effective refractive index is 4. Assume a constant \( C_0 = \frac{\pi e^2}{n_e e_0 m_0 \omega} = 10^{10} \, \frac{m^2}{kg} \).
   a) What is the spontaneous emission lifetime of this semiconductor?
   b) What is the effective “dipole length”?
   c) At T= 0 Kevin, derive the gain coefficient as a function of carrier (electron and hole) concentration.

5. A double heterostructure (DH) laser has a cavity length of 200 \( \mu m \), an active layer thickness of 0.1 \( \mu m \) and a waveguide width of 1 \( \mu m \), and a confinement factor of 50%.
   Both cleaved facets have a reflectivity of 30%. Assume a linear gain with \( g(N) = a(N - N_0) \) where \( a = 10^{-16} \, cm^2 \) and \( N_0 = 10^{18} \, cm^{-3} \). The bandgap energy is 1 eV, and the effective refractive index is 4.
   a) Find the photon density inside the laser cavity [in cm\(^{-3}\)] when the output power is 10 mW (from one facet).
   b) What is the relaxation oscillation frequency at this output power?
   c) If the confinement factor of the laser is reduced to 10%, what is the relaxation oscillation frequency of the laser at the same output power of 10 mW?

6. For a tensile strained quantum well with a heavy hole effective masses of \( m_{hh}^* = 0.25m_0 \) and \( m_{hh}^* = 0.1m_0 \), and light hole effective masses of \( m_{lh}^* = 0.08m_0 \) and \( m_{lh}^* = 0.15m_0 \). The maximum of the light hole band is higher than that of the heavy hole by 3\( \varepsilon \) eV, where \( \varepsilon \) is the amount of tensile strain [in %]. The width of the quantum well is 10 nm.
   a) Find the amount of tensile strain at which the first heavy hold band and the first light hole band has the same energy.

7. A semiconductor has an absorption coefficient of 1000 cm\(^{-1}\), an electron velocity of 10\(^7\) cm\(^{-1}\) and a hole velocity of 10\(^6\) cm\(^{-1}\). To increase light absorption, a waveguide photodetector is used. The cladding layers have higher bandgap materials and are
transparent for the photons. The detector is connected to a load resistance of $50\Omega$. The confinement factor of the absorption layer is 80%, and the waveguide is $1 \mu m$ wide. The permittivity of the absorption region is $\varepsilon_r \varepsilon_0 = 10^{-10} \text{ F/m}$.

a) Find the length of the waveguide needed to absorb 90% of light.

b) If the absorption region is $1 \mu m$ thick, what is the bandwidth of the photodiode? Is the speed limited by RC or transit time?

c) If you are allowed to change the thickness of the absorption region, and for simplicity, assume the confinement factor stays constant (at 80%), find the optimum absorber thickness with maximum bandwidth.

8. Consider the following two photoreceivers for an optical link with 10 GHz bandwidth:

Photoreceiver-A: a p-i-n photodiode with 100% quantum efficiency

Photoreceiver-B: an APD with 50% quantum efficiency, 20dB gain and 10dB noise figure.

Both receivers are connected to a load resistance of $1k\Omega$. The receiver sensitivity is defined as the minimum optical power to achieve a bit error rate of $10^{-9}$, or approximately a signal-to-noise ratio (SNR) of 12.

a) Which receiver has better sensitivity? Show your calculation to support your answer. (No credit will be given if you simply guess a receiver without showing the supporting calculation).

b) Does the answer change if both receivers are connected to a load resistance of $50\Omega$?