1. Refer to the diagram on the right. Under biased condition, both conduction and valence bands are populated. The electron distribution in conduction band is described by Fermi-Dirac distribution, \( f_C(E_2) \), with quasi-Fermi energy \( F_C \). The electron distribution in valence band is described by Fermi-Dirac distribution, \( f_V(E_1) \), with quasi-Fermi energy \( F_V \). Here, \( E_1 \) and \( E_2 \) are related by an optical transition (i.e., they have the same \( k \)). The optical matrix element is
\[
\hat{\psi} \cdot \vec{P}_{cv} = \frac{m_e}{6} E_p \quad \text{with} \quad E_p = 25.7 \text{ eV}
\]
and the refractive index of the semiconductor is 3.5. In calculating \( f_C \) and \( f_V \), you can assume charge neutrality, i.e., electron concentration is equal to hole concentration, and assume the doping concentration is negligible compared with injected electrons/holes.

   a. Use the energy reference below (i.e, \( E_V = 0 \) and \( E_C = E_g \), the bandgap energy), find \( E_1 \) and \( E_2 \) as functions of the photon energy, \( \hbar \omega \).

   b. Derive \( f_C(E_2(\hbar \omega)) \) as a function of \( \hbar \omega \).

   c. Derive \( f_V(E_1(\hbar \omega)) \) as a function of \( \hbar \omega \).

   d. Assuming \( E_g = 1 \text{ eV}, F_C - F_V = 1.2 \text{ eV}, m_e^* = 0.1m_0, m_h^* = 0.4m_0 \). Calculate and plot the emission probability \( f_e(\hbar \omega) = f_C(E_2(\hbar \omega)) \cdot [1 - f_V(E_1(\hbar \omega))] \) for photon energies from 0.8 to 1.5 eV. Plot for two temperatures: \( T = 0 \) and \( T = 300 \text{ K} \).

   e. Repeat part d) for the Fermi inversion factor: \( f_g(\hbar \omega) = f_C(E_2(\hbar \omega)) - f_V(E_1(\hbar \omega)) \)

   f. Plot the gain spectra for \( T = 0 \) and \( T = 300 \text{ K} \) for the condition given in d).

   g. Plot the spontaneous emission spectra for \( T = 0 \) and \( T = 300 \text{ K} \) for the condition given in d).