EE 232 Lightwave Devices
Lecture 14:
Strained Quantum Well Laser

Reading: Chuang, Sec. 10.3-10.4
(There is also a good discussion in Coldren, Appendix 11)

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Reduction of Lasing Threshold Current Density by Lowering Valence Band Effective Mass

Bernard-Duraffourg Condition:
\[ F_C - F_V \geq \omega \geq E_{e1} - E_{h1} \]

Ordinary Semiconductor

- \( m_h^* \approx 6m_e^* \)
- High transparency carrier concentration

Ideal Semiconductor

- \( m_h^* \approx m_e^* \)
- Low transparency carrier concentration

Bernard-Duraffourg Condition in Quantum Well

Bernard-Duraffourg Condition:

\[ F_C - F_V = E - E_{h1} \]

(a) \( m_h^* > m_e^* \) (as in most semiconductors)

\[ F_V > E_{h1} \]

\[ F_C \gg E_{e1} \]

\[ N_{tr} = \rho_e^2 d (F_C - E_{e1}) = \frac{m_e^*}{\pi \hbar^2 L_z} (F_C - E_{e1}) \]

Large \( N_{tr} \rightarrow \) High threshold current

(b) \( m_h^* = m_e^* \) (Ideal semiconductor)

\[ F_V = E_{h1} \]

\[ F_C = E_{e1} \]

\[ N_{tr} = \frac{m_e^*}{\pi \hbar^2 L_z} \int_{E_{e1}}^{\infty} f_C(E) dE \] is low
Transparency Carrier Concentration for “Ideal” Quantum Well

(b) Ideal Semiconductor

\[ m_h^* = m_e^* \implies F_V = E_{h1}, \quad F_C = E_{e1} \]

\[ N_{tr} = \frac{m_e^*}{\pi} \int_0^\infty \frac{1}{E-E_{e1}} dE \frac{1}{1 + e^{\frac{E-E_{e1}}{k_BT}}} \]

\[ = \frac{k_BT m_e^*}{\pi} \int_0^\infty \frac{1}{1 + e^x} dx \]

\[ = \frac{k_BT m_e^*}{\pi} \left( -\ln(1 + e^{-x}) \right)_0^\infty \]

\[ = \frac{k_BT m_e^*}{\pi} \ln 2 \]

For \( m_e^* = 0.067m_0 \)

\[ N_{tr} \approx 4.6 \times 10^{17} \text{ cm}^{-3} \]
Transparency Carrier Concentration for Normal Quantum Well

Transparency Condition: 

\[ F_C - F_V = E_{e1} - E_{h1} \]

(a) Ordinary Semiconductor 

\[ N_{tr} = \rho_e^{2d} (F_C - E_{e1}) = \frac{m_e^*}{\pi} \frac{\Delta}{2L_z} \]

To estimate \( \Delta \), note that \( N = P \)

\[ P = N_V^2 \exp\left(-\frac{-\Delta}{k_B T}\right) = \frac{k_B T m_h^*}{\pi} \frac{-\Delta}{2L_z} \exp\left(-\frac{-\Delta}{k_B T}\right) \]

\[ N = P \quad \Rightarrow \quad \exp\left(-\frac{-\Delta}{k_B T}\right) = \frac{\Delta}{k_B T} \frac{m_e^*}{m_h^*} \]

For \( m_h^* \approx 6m_e^* \) (in 1.55\,\mu m laser),

\[ \Delta = 1.43 k_B T \]

\[ N_{tr} = 1.43 \frac{k_B T m_e^*}{\pi} \frac{1}{2L_z} \]
Effective Mass Asymmetry Penalty

\[ \frac{N_{tr}^{\text{Ordinary}}}{N_{tr}^{\text{Ideal}}} = \frac{1.43}{\ln 2} = 2 \]

Threshold current density reduction is more than a factor of 2:

\[ J_{th} = J_{\text{nonrad}} + J_{rad} + J_{\text{Auger}} \]

\[ \frac{J_{th}}{qd} = AN + BN^2 + CN^3 = \frac{N}{\tau} + BN^2 + CN^3 \]

\( \tau \): Shockley-Read-Hall nonradiative recombination lifetime

\( J_{\text{Auger}} \) is greatly reduced when \( N \) is lowered

(1) \( N^3 \) is reduced by 8x

(2) C is also reduced due to band structure change by strain
Bandgap-vs-Lattice Constant of Common III-V Semiconductors

![Graph showing bandgap energy and lattice constant for various III-V semiconductors](image)

- **Compressive Strain**
- **Tensile Strain**
- **Lattice Matched**
  - $\text{In}_{0.53}\text{Ga}_{0.47}\text{As}$

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Fig. 7.6. Bandgap energy and lattice constant of various III-V semiconductors at room temperature (adopted from Tien, 1988).
Qualitative Band Energy Shifts Under Strain

- **Biaxial Strain**
- **Hydrostatic Strain**
- **Tensile Strain**
- **Compressive Strain**

\[ \delta E_C \]

- Compressive Strain
- Tensile Strain
- Hydrostatic Strain

\[ HH, LH \]

\[ SO \]
Strain and Stress

\[ \varepsilon = \varepsilon_{xx} = \varepsilon_{yy} = \frac{a_0 - a(x)}{a_0} \]

- \( a_0 \) : lattice constant of InP
- \( \varepsilon < 0 \) : compressive strain
- \( \varepsilon > 0 \) : tensile strain

\[ \varepsilon_{\perp} = \varepsilon_{zz} = -2 \frac{C_{12}}{C_{11}} \varepsilon \]

- \( C_{ij} \) : Compliance Tensor
- \( C_{12} \approx 0.5C_{11} \)

\[
\begin{bmatrix}
\sigma_{xx} \\
\sigma_{yy} \\
\sigma_{zz}
\end{bmatrix} =
\begin{bmatrix}
C_{11} & C_{12} & C_{12} \\
C_{12} & C_{11} & C_{12} \\
C_{12} & C_{12} & C_{11}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_{xx} \\
\varepsilon_{yy} \\
\varepsilon_{zz}
\end{bmatrix}
\]

Biaxial stress:
- \( \sigma_{xx} = \sigma_{yy} = \sigma \)
- \( \sigma_{zz} = 0 \)

\[ \Rightarrow C_{12} \varepsilon_{xx} + C_{12} \varepsilon_{yy} + C_{11} \varepsilon_{zz} = 0 \]

\[ \varepsilon_{zz} = -2 \frac{C_{12}}{C_{11}} \varepsilon \]
Band Edge Shift

\[ E_C = E_g(x) + \delta E_C \]
\[ E_{HH} = -P_\varepsilon - Q_\varepsilon \]
\[ E_{LH} = -P_\varepsilon + Q_\varepsilon \]

\[ \delta E_C = a_C (\varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}) = 2a_C \left(1 - \frac{C_{12}}{C_{11}}\right) \varepsilon \]

\[ P_\varepsilon = -a_V (\varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}) = -2a_V \left(1 - \frac{C_{12}}{C_{11}}\right) \varepsilon \]

\[ Q_\varepsilon = -b \left(\frac{\varepsilon_{xx} + \varepsilon_{yy}}{2} - \varepsilon_{zz}\right) = -b \left(1 + 2 \frac{C_{12}}{C_{11}}\right) \varepsilon \]

\[ a = a_C - a_V \quad \text{: hydrostatic potential} \]
\[ b \quad \text{: shear potential} \]
# Strain Parameters in III-V

(Coldren, p.535)

<table>
<thead>
<tr>
<th>Material</th>
<th>$a$(Å)</th>
<th>$a$</th>
<th>$b$</th>
<th>$d$</th>
<th>$C_{11}$</th>
<th>$C_{12}$</th>
<th>$C_{44}$</th>
<th>$dE/dP$</th>
<th>$\Delta$(eV)</th>
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<tbody>
<tr>
<td>GaAs</td>
<td>5.6533</td>
<td>−8.68</td>
<td>−1.7</td>
<td>−4.55</td>
<td>11.88</td>
<td>5.38</td>
<td>5.94</td>
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<td>0.34</td>
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<td>−1.8</td>
<td>−3.6</td>
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<td>4.526</td>
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<td>5.89</td>
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<td>−4.6</td>
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<td>6.15</td>
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</tbody>
</table>

* Indirect gap.
Band-Edge Profile and Subband Dispersion

(b) An unstrained quantum well

E_c(0)

E_{HH}(0) = E_{LH}(0)

(c) A quantum well under a tensile strain

E_c(0)

E_{LH}(0)

E_{HH}(0)

(a) A quantum well under a compressive strain

E_c(0)

k_x

HH1

LH1

HH2

HH1

LH1

HH2