EE 232: Lightwave Devices
Lecture #16 – Steady state LED and LASER characteristics - Part 1

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Light emitting diode at steady state

At steady state:

\[ \eta_i \frac{J}{qd} = R \]

\[ \frac{dn}{dt} \approx \eta_i \frac{J}{qd} - R \]

We neglect current spreading in the lateral direction and assume quasi-Fermi level is constant throughout active region.

Rate equation for active region (ignoring generation term):

\[ \frac{\partial n}{\partial t} = -\nabla \cdot J - R \]
Carrier recombination

Leaky Bathtub analogy

At steady state, we must continuously pump the active region to maintain a certain carrier density.

\[ R = R_{SRH} + R_{sp} + R_{stim} + R_{Auger} \]

- **Shockley-Reed-Hall recombination**
- **Spontaneous emission**
- **Stimulated emission**
- **Auger recombination**

Spontaneous emission power

\[ \eta_i \frac{J}{qd} = R_{SRH} + R_{sp} + R_{Auger} \]

Let \( \eta_{sp} = \frac{R_{sp}}{R_{SRH} + R_{sp} + R_{Auger}} = \frac{R_{sp}}{\eta_i J / qd} \)

Spontaneous emission power (\( P_{sp} \))

\[ = (\text{Photon energy}) \times (\text{Emission rate}) \times (\text{Active region volume}) \]

\[ P_{sp} = \hbar \omega R_{sp} V_{act} \]
\[ = \hbar \omega V \eta_{sp} \eta_i J / qd \]
\[ = \eta_{sp} \eta_i \frac{\hbar \omega}{q} I \]

For constant internal quantum efficiency (IQE), there is a linear relationship between power and current. However, in general, IQE does change with current.
ABC approximation

\[ \eta_i \frac{J}{qd} = R_{SRH} + R_{sp} + R_{Auger} \]
\[ \approx An + Bn^2 + Cn^3 \]

\[ \eta_{IQE} = \eta_i \eta_{sp} = \eta_i \frac{R_{sp}}{R_{SRH} + R_{sp} + R_{Auger}} = \eta_i \frac{Bn^2}{An + Bn^2 + Cn^3} \]

\[ \eta_{IQE} = \eta_i \frac{Bn}{A + Bn + Cn^2} \]

**Low drive current**
\[ A > Bn + Cn^2 \]
\[ n \propto J \]
\[ \eta_{IQE} \propto J \]
\[ P_{sp} \propto J^2 \]

**Moderate drive current**
\[ Bn > A + Cn^2 \]
\[ n \propto J^{1/2} \]
\[ \eta_{IQE} \sim \text{constant} \]
\[ P_{sp} \propto J \]

**High drive current**
\[ Cn^2 > A + Bn \]
\[ n \propto J^{1/3} \]
\[ \eta_{IQE} \sim J^{-1/3} \]
\[ P_{sp} \propto J^{2/3} \]
ABC approximation

Ex. InGaAs LED

Note: At high current density, other loss mechanisms beside Auger may become important. See for example, controversy surrounding “droop” in GaN LED efficiency at high current.
Most emitted light will become trapped in the high-index semiconductor. Only light with angle smaller than critical angle can escape.
\[
\sin \theta_c = \frac{n_{\text{air}}}{n} \rightarrow \theta_c \sim \frac{1}{n}
\]

Coupling efficiency
\[
\eta_c = \frac{\int T(\theta)d\Omega}{\int d\Omega} = \frac{1}{4\pi} \int_0^{\theta_c} T(\theta)(2\pi \sin \theta)d\theta \\
\approx \frac{1}{4\pi} \int_0^{\theta_c} T(0)(2\pi \sin \theta)d\theta \\
= \frac{1}{4\pi} \int_0^{\theta_c} \frac{4n}{(n+1)^2}(2\pi \sin \theta)d\theta \\
\Rightarrow \eta_c \approx n^{-1}(n+1)^{-2}
\]

Externally collected power
\[
P_{sp} = \eta_{\text{IQE}} \eta_c \frac{\hbar \omega}{q} I = \eta_{\text{ext}} \frac{\hbar \omega}{q} I
\]

External quantum efficiency
Semiconductor laser at steady state

$S \equiv$ photon density in lasing mode

$n \equiv$ carrier density in active region

$v_g \equiv$ group velocity

$\beta_{sp} \equiv$ fraction of sp. em. into lasing mode

$\tau_p \equiv$ photon lifetime

$g \equiv$ gain

$V_{cav} \equiv$ cavity volume

$V_{act} \equiv$ active region volume

$\Gamma \approx \frac{V_{act}}{V_{cav}} = \frac{(\text{Area})d}{V_{cav}} \rightarrow V_{cav} = \frac{(\text{Area})d}{\Gamma}$
Rate equations for photons and carriers

Rate equation for carriers in active region

\[
\frac{dn}{dt} = \eta_i \frac{J}{q_d} - R_{SRH} - R_{sp} - R_{stim} - R_{Auger}
\]

Shockley-Reed-Hall recombination
Spontaneous emission
Stimulated emission
Auger recombination

(steady state)

\[
J = \frac{qd}{\eta_i} \left( R_{SRH} + R_{sp} + R_{stim} + R_{Auger} \right)
\]

Rate equation for photons in cavity

\[
\frac{dS}{dt} = -\frac{S}{\tau_p} + \Gamma \beta_{sp} R_{sp} + \Gamma R_{stim}
\]

Rate of photons leaving cavity
Sp. em. rate into cavity mode
Stimulated emission

(steady state)

\[
S = \left( \Gamma \beta_{sp} R_{sp} \right) / \left( 1/\tau_p - \Gamma v_g g \right)
\]

Note, recall: \( R_{stim} = v_g g S \)
Below threshold – Spontaneous emission

From before, \( \eta_{sp} = \frac{R_{sp}}{\eta_i J / qd} \)

\[
S = \left( \Gamma \beta_{sp} R_{sp} \right) / \left( 1 / \tau_p - \Gamma v_g g \right)
\]

\[
\approx \tau_p \Gamma \beta_{sp} R_{sp}
\]

\[
= \tau_p \Gamma \beta_{sp} \eta_i \eta_{sp} \left( J / qd \right)
\]

Recall, \( \Gamma g_{th} = \frac{1}{v_g \tau_p} = \alpha_m + \alpha_i \rightarrow \tau_p = \frac{1}{v_g \left( \alpha_m + \alpha_i \right)} \)

Then, \( S = \frac{\Gamma \beta_{sp} \eta_i \eta_{sp} J}{v_g \left( \alpha_m + \alpha_i \right) \eta_i / qd} \) This is the number of spontaneously emitted photons in the cavity mode
Below threshold – Spontaneous emission

Spontaneous emission power ($P_{sp}$)

$$P_{sp} = (\hbar \omega)(v_g \alpha_m)(S)(V_{cav})$$

$$= \hbar \omega v_g \alpha_m \frac{\Gamma \beta_{sp} \eta_{sp} \eta_i}{v_g (\alpha_m + \alpha_i)} \frac{J}{qd} V_{cav}$$

Note:

$$\Gamma \approx \frac{V_{act}}{V_{cav}} = \frac{(Area)d}{V_{cav}} \rightarrow V_{cav} = \frac{(Area)d}{\Gamma}$$

$$P_{sp} = \eta_{sp} \eta_i \left( \frac{\alpha_m}{\alpha_m + \alpha_i} \right) \frac{\hbar \omega}{q} \beta_{sp} I$$

This is the spontaneous emission power into the cavity mode
At threshold

Laser threshold condition is reached when gain precisely balances cavity loss. Stimulated emission will still be small with respect to spontaneous emission.

\[ \eta_i \frac{J_{th}}{qd} = R_{SRH}(n_{th}) + R_{sp}(n_{th}) + R_{Auger}(n_{th}) \]

Above threshold

Above threshold and under steady state operation, the gain must “clamp” at the threshold gain. If this were not the case, the fields in the cavity would grow without bound (thus not achieving steady state). This also implies that the Fermi-level and current density must clamp at the threshold current density.

\[ g \approx g_{th} \quad \text{for } I > I_{th} \]
\[ n \approx n_{th} \quad \text{for } I > I_{th} \]
Gain and carrier density clamping

Recall, \( S = \frac{\Gamma \beta_{sp} R_{sp}}{1/\tau_p - \Gamma v_g g} \)

Now, let’s derive a stimulated emission lifetime.

\[
R_{stim} = \frac{n}{\tau_{stim}} = v_g g S \rightarrow \tau_{stim} = \frac{1}{v_g g} \left( \frac{n}{S} \right)
\]

We see that there is a negative feedback loop that does not allow the gain to increase beyond the gain threshold. As current is increased and gain approaches the threshold gain, the photon density increases dramatically thus significantly reducing the stimulated emission lifetime. Any additional current gets immediately “used up” by stimulated emission not allowing the carrier density or gain to increase.

The laser at threshold is similar to a filled bathtub. Any additional water spills over the side. Likewise any additional injected carriers will “spill out” as stimulated emission and will not increase the carrier density.
Above threshold

From the carrier density rate equation

\[ \eta_i \frac{J}{qd} = R_{SRH}(n_{th}) + R_{sp}(n_{th}) + R_{Auger}(n_{th}) + R_{stim} \]

\[ \eta_i \frac{J}{qd} = \eta_i \frac{J_{th}}{qd} + v_g g_{th} S \]

\[ \eta_i (J - J_{th}) \frac{1}{qd} = v_g g_{th} S \]

Plug into the photon density rate equation

\[ \frac{S}{\tau_p} = \Gamma \beta_{sp} R_{sp} + \Gamma v_g g_{th} S \approx \Gamma v_g g_{th} S \]

\[ S = \Gamma \tau_p \frac{\eta_i (J - J_{th})}{qd} \]

\[ P_{stim} = v_g \alpha_m S = \eta_i \left( \frac{\alpha_m}{\alpha_m + \alpha_i} \right) \frac{\hbar \omega}{q} (I - I_{th}) \]
Above threshold (L-I curve)

Above threshold, emitted power is dominated by stimulated emission. Spontaneous emission power clamps at the threshold power.
Above threshold (L-I curve)

Plotted on log-log scale the L-I curve has an “S”-like shape.

\[ \log(\text{Current}) \]  
\[ \log(\text{Power}) \]  
\[ I_{th} \]  

Stimulated emission

Spontaneous emission
Above threshold (L-I curve)

Differential quantum efficiency

We define the external differential quantum efficiency of the laser as

$$\eta_e = \frac{q}{\hbar \omega} \frac{dP}{dI}$$

This is the probability of externally collecting a stimulated photon for an injected electron-hole pair.

$$\eta_e = \eta_i \left( \frac{\alpha_m}{\alpha_m + \alpha_i} \right) = \eta_i \frac{\ln(1/R)}{\alpha_i L + \ln(1/R)}$$

By plotting the inverse of the external differential quantum efficiency as a function of cavity length we can determine cavity loss and injection efficiency from the slope.

$$\frac{1}{\eta_e} = \frac{\alpha_i}{\eta_i \ln(1/R)} L + \frac{1}{\eta_i}$$
Summary

Light emitting diode

\[ I = V_{act} \frac{q}{\eta_i} (R_{SRH} + R_{sp} + R_{Auger}) \]

\[ P_{sp} = \eta_{IQE} \eta_c \frac{\hbar \omega}{q} I \]

Semiconductor laser

Below threshold:

\[ I = V_{act} \frac{q}{\eta_i} (R_{SRH} + R_{sp} + R_{Auger}) \]

\[ P_{sp} = \eta_{sp} \eta_i \left( \frac{\alpha_m}{\alpha_m + \alpha_i} \right) \frac{\hbar \omega}{q} \beta_{sp} I \] \hspace{1cm} \text{(Spontaneous emission power into lasing mode)}

Above threshold:

\[ I = V_{act} \frac{q}{\eta_i} \left[ (R_{SRH} + R_{sp} + R_{Auger}) \bigg|_{n = n_{th}} + R_{stim} \right] \]

\[ P_{stim} = \eta_i \left( \frac{\alpha_m}{\alpha_m + \alpha_i} \right) \frac{\hbar \omega}{q} (I - I_{th}) \] \hspace{1cm} \text{(Stimulated emission power into lasing mode)}