EE 232: Lightwave Devices
Lecture #17 – Steady state LED and LASER characteristics - Part 2

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Current and photon confinement

Edge-emitting stripe laser

<table>
<thead>
<tr>
<th>P-type</th>
<th>active</th>
<th>N-type</th>
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Contact | Oxide

Current confinement:

Oxide confines current in lateral direction.
No confinement in lateral direction

Photon confinement:
Index contrast confines mode in transverse direction
No confinement in lateral direction
Current and photon confinement

Edge-emitting ridge laser

- P-type
- insulator
- active
- N-type

Current confinement

- Heterostructure confines current in transverse direction.
- Ridge confines current in lateral direction.

Photon confinement

- Index contrast confines mode in transverse and lateral direction.
Current and photon confinement

Current confinement

Heterostructure and reverse-biased pn junction confines current in transverse and lateral direction.

Photon confinement

Index contrast confines mode in transverse and lateral direction.

Edge-emitting buried ridge stripe
Current and photon confinement

Surface-emitting oxide aperture laser

Current confinement

Oxide confines current in lateral direction.

Heterostructure confines current in transverse direction

Photon confinement

Index contrast confines mode in transverse and lateral direction
Current and photon confinement

Double heterostructure

Single quantum well separate confinement heterostructure

Multiple quantum well (MQW) separate confinement heterostructure

Graded index separate confinement heterostructure (GRINSCH)
Quantum well (s) can only very weakly confine a mode. Usually, quantum wells are sandwiched inside another higher bandgap material that can be engineered to improve mode guiding and electron injection into the quantum well(s). This type of structure is called a separate confinement heterostructure (SCH).
Gain in quantum well

Peak gain occurs at the bandedge

\[ g_p = g_m \left( f_c[\hbar \omega = E_{e1}^{h1}] - f_v[\hbar \omega = E_{e1}^{h1}] \right) \]

Assuming only subband is filled in conduction and valence bands we can write an approximate expression for the peak gain

Recall,

\[ n = n_c \ln \left( 1 + \exp \left[ \left( F_c - E_g - E_{e1} \right) / kT \right] \right) \]
\[ p = n_v \ln \left( 1 + \exp \left[ \left( E_{h1} - F_v \right) / kT \right] \right) \]

\[ n_c = \frac{m_e^* kT}{\pi \hbar^2 L_z} \quad n_v = \frac{m_h^* kT}{\pi \hbar^2 L_z} \]

The Fermi functions can be written in terms of the carrier density

\[ f_c (\hbar \omega = E_{e1}^{h1}) = \frac{1}{1 + \exp \left[ \left( E_g + E_{e1} - F_c \right) / kT \right]} = 1 - \exp(-n/n_c) \]
\[ f_v (\hbar \omega = E_{e1}^{h1}) = \frac{1}{1 + \exp \left[ \left( E_{h1} - F_v \right) / kT \right]} = \exp(-p/n_v) \]

Then,

\[ g_p = g_m \left[ 1 - \exp(-n/n_c) - \exp(-p/n_v) \right] \]
Gain in quantum well

\[ g_p = g_m \left[ 1 - \exp(-n/n_c) - \exp(-p/n_v) \right] \]

The plot of peak gain vs. carrier concentration (red line) is approx. linear on a semi-log plot, therefore a simpler approximate expression is often used (dashed gray curve).

\[ g_p = g_0 \ln[n/n_0] \]

As discussed previously, the current density can be written in terms of a polynomial of the carrier density (e.g. \( J \propto n^2 \) if spontaneous emission dominates). Therefore, we can write a similar approximate expression for peak gain in terms of carrier density.

\[ g_p = g_0 \ln[J/J_0] \]

Note: \( g_0 \) are not the same in both expressions
**Quantum well laser - threshold gain**

\[ g_w = g_0 \ln \left( \frac{J_w}{J_0} \right) \]

- \( g_w \): quantum well threshold gain
- \( J_w \): threshold quantum well current density

\[ n_w \Gamma_w g_w = \alpha_i + \frac{1}{2L} \ln \left( \frac{1}{R_1 R_2} \right) \]

- \( n_w \): number of quantum wells in active region
- \( \Gamma_w \): fraction of mode in quantum well

\[ \Gamma_w = \Gamma \frac{L_z}{d_{op}} \]

- \( \Gamma_w \): quantum well thickness
- \( \alpha_i \): optical confinement factor
- \( L_z \): effective width of optical mode

**Diagram:**
- Quantum wells
- P-type
- N-type
- SCH

**Equation:**

\[ \ln \left( \frac{J_w}{J_0} \right) \]
Threshold current and gain in active region with multiple quantum wells

\( n_w g_w = n_w g_0 \ln \left( \frac{n_w J_w}{n_w J_0} \right) \)

Note: \( J_{th} = \frac{n_w J_w}{\eta_i} \)

threshold current at device terminals

Then:

\[ J_{th} = \frac{n_w J_0}{\eta_i} \exp \left( \frac{g_w}{g_0} \right) \]

\[ J_{th} = \frac{n_w J_0}{\eta_i} \exp \left( \frac{1}{n_w \Gamma_w g_0} \left[ \alpha_i + \frac{1}{2L} \ln \frac{1}{R_1 R_2} \right] \right) \]

\[ I_{th} = \frac{w L n_w J_0}{\eta_i} \exp \left( \frac{1}{n_w \Gamma_w g_0} \left[ \alpha_i + \frac{1}{2L} \ln \frac{1}{R_1 R_2} \right] \right) \]

\( w \): active region width
\( L \): active region length
Active region optimization

Optimize cavity length (L) or a fixed number of quantum wells

\[ \frac{\partial I_{th}}{\partial L} = 0 \rightarrow L_{opt} = \frac{1}{2} \frac{1}{n_w \Gamma_w g_0} \ln \left( \frac{1}{R_1 R_2} \right) \]

Optimize number of quantum wells (\(n_w\)) for a fixed cavity length

\[ \frac{\partial I_{th}}{dn_w} = 0 \rightarrow n_{opt} = \frac{1}{\Gamma_w g_0} \left( \alpha_i + \frac{1}{2L} \ln \frac{1}{R_1 R_2} \right) \]

For a general cavity,

\[ \frac{1}{\tau_p v_g} = \left( \alpha_i + \frac{1}{2L} \ln \frac{1}{R_1 R_2} \right) \]

\[ Q = \omega_0 \tau_p \]

therefore,

\[ n_{opt} = \frac{1}{\Gamma_w g_0 v_g} \frac{\omega_0}{Q} \]
Recombination rate in LED or Laser at or below threshold:

\[ R = R_{SRH} + R_{sp} + R_{Auger} \]

\[ \approx An + Bn^2 + Cn^3 \]

“ABC” approximation

The ABC approximation is widely used to estimate recombination rates in LEDs and lasers (at or below threshold). Although strictly speaking, it is valid only when Boltzmann statistics are valid so some care needs to be applied when using the approximation.

We have already proved that the spontaneous emission rate has an \( n^2 \) dependence (when Boltzmann statistics apply). See the previous lecture on spontaneous emission.

Let’s look at the Shockley-Reed-Hall and Auger rates.
The derivation of the SRH rate is found in many basic semiconductor textbooks.

\[
R_{SRH} \approx \frac{n_0 \delta n + p_0 \delta n + \delta n^2}{C_p^{-1}(n_0 + n_{ds} + \delta n) + C_n^{-1}(p_0 + p_{ds} + \delta n)}
\]

\[n = n_0 + \delta n\]
\[p = p_0 + \delta p\]
\[\delta_n = \delta_p\]
\[C_n = \sigma_n v_{n,th} N_{ds}\]
\[C_p = \sigma_p v_{p,th} N_{ds}\]

Capture rates

Capture cross-sections

\[\sigma_n\]
\[\sigma_p\]

Thermal velocity

\[v_n\]
\[v_p\]

\[N_{ds}: \text{defect density}\]

We see that in general we cannot write

\[R_{SRH} = An\]

But, we can do so if we restrict our analysis to a “low-injection” or “high-injection” regime.
Low-injection and high-injection regime

Active region materials have a background doping due to unintentional doping impurities introduced during growth. Let’s assume our active region is unintentionally doped p-type.

\[ p_0 \gg \delta_n \quad \text{Low-injection regime} \]

\[
R_{SRH} = \frac{n_0 \delta n + p_0 \delta n + \delta n^2}{C_p^{-1} (n_0 + n_{ds} + \delta n) + C_n^{-1} (p_0 + p_{ds} + \delta n)} \approx C_n \delta_n \approx A_{lown}
\]

\[ \delta_n \gg p_0 \quad \text{High-injection regime} \]

\[
R_{SRH} \approx \frac{C_n C_p}{C_n + C_p} \delta_n \approx A_{highn}
\]

We see that we can write \( R_{SRH} = An \) so long we stay in one of the two regimes. If the electron capture is the rate-limiting step (for p-type material), then the A coefficient will be identical in both regimes.
Auger recombination

Electron recombines with hole and gives up excess energy to another carrier instead of releasing a photon. Several different Auger processes are possible (as shown below). Often there is a material-dependent dominant process.
Auger recombination

The CCCH Auger rate is given by

\[ R_{\text{Auger}} = C_0 f_1 f_2 (1 - f_3) (1 - f_4) \]

\[ = C_0 \exp \left( \frac{F_c - E_1}{kT} \right) \exp \left( \frac{F_c - E_2}{kT} \right) \exp \left( \frac{E_3 - F_v}{kT} \right) \] (1)

\[ = C_0 \frac{n^2 p}{(kT)^2 N_c^2 N_v} \exp \left( \frac{E_g - E_4}{kT} \right) \]

Very likely that State 4 is empty since it is well beyond the bandedge

\[ R_{\text{Auger}} = C n^2 p = C n^3 \quad \text{for} \quad n = p \]
Auger recombination

Energy and momentum conservation needs to be simultaneously conserved. This sets a threshold value for $E_T$ which we call $E_T$. Materials with small $E_T$ will have large Auger rates since

$$R_{Auger} \propto \exp(-E_T / kT)$$

$E_T$ is related to the curvature of the bands through

$$E_T = \frac{2m_e^* + m_h^*}{m_e^* + m_h^*} E_g = aE_g$$

the value of $a$ is approximately unity for III-V semiconductors therefore,

$$R_{Auger} \propto \exp(-E_g / kT)$$

Auger recombination is higher in low bandgap materials.