EE 233. LIGHTWAVE SYSTEMS
Chapter 6. Amplifiers

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• BASICS

• SEMICONDUCTOR OPTICAL AMPLIFIER (SOA)

• RAMAN AMPLIFIER

• ERBIUM-DOPED FIBER AMPLIFIER (EDFA)
BASICS
\[ \frac{dP}{dy} = qP, \quad E(y) \propto E_{in} e^{-\frac{y}{a}} \]

\[ g(w) = \frac{g_0}{1 + (w - w_0)^2 T_2^2 + P/P_s} \quad (6.1.1) \]

- \( g_0 \) = peak gain coefficient
- \( T_2 \) = dipole relaxation time (~1 ps)
- \( P_s \) = saturation power

**Unsaturated regime -** \( P/P_s \ll 1 \)

\[ g(w) = \frac{g_0}{1 + (w - w_0)^2 T_2^2} \]

Lorentzian shape

\[ \text{Gain bandwidth (FWHM)} = \]
\[ \Delta \omega_g = 2/T_2 \]
\[ \Delta \nu_g = \Delta \omega_g / 2\pi = 1/\pi T_2 \]
\[ G = \frac{P_{out}}{P_{in}}, \quad P(0) = P_{in}, \quad P(3) = P_{out} \]
\[ P(3) = P_{in} e^{\frac{q}{3}} \quad (6.1.6) \]
\[ G(\omega) = e^{\frac{q}{4}} \]
\[ \Delta \nu_A = \Delta \nu_0 \left[ \frac{\ln 2}{\ln (\frac{q_{out}}{q_{in}})} \right]^{\frac{1}{2}} \quad (6.1.8) \]

Fig 6.1.
Figure 6.1: Lorentzian gain profile $g(\omega)$ and the corresponding amplifier-gain spectrum $G(\omega)$ for a two-level gain medium.
GAIN SATURATION

\[ \omega = \omega_0 \]

\[ \frac{dP}{dz} = \frac{\alpha_0 P}{1 + P/P_s} \quad (6.1.4) \]

\[ G = G_0 \exp \left(- \frac{G-1}{G} \frac{P_{out}}{P_s} \right) \quad (6.1.10) \]

\[ \Rightarrow G_0 \exp \left(- \frac{P_{out}}{P_s} \right) \]

[Fig. 6.2]

\[ g = \frac{G_0}{2} \]

\[ P_{out}^* = \frac{G_0 \ln^2}{G_0 - 2} \frac{P_s}{P_s} \quad (6.1.11) \]
Figure 6.2: Saturated amplifier gain $G$ as a function of the output power (normalized to the saturation power) for several values of the unsaturated amplifier gain $G_0$. 
AMPLIFIER NOISE

\[ F_n = \frac{(S/N)_\text{in}}{(S/N)_\text{out}} \quad \text{noise figure} \]

\[ (SNR)_\text{in} = \frac{\langle I \rangle^2}{\sigma_s^2} = \frac{(RP_{\text{in}})^2}{2q(RP_{\text{in}})\Delta f} = \frac{P_{\text{in}}}{2h
u\Delta f} \]

\[ P_{\text{out}} = 9P_{\text{in}} \]
\[ \langle I \rangle = \text{avg. photocurrent} = RP_{\text{in}} \]
\[ R = q/h\nu, \quad \eta = 1 \]

\[ \sigma_s^2 = 2q(RP_{\text{in}})\Delta f \quad \text{shot noise levels} \]
\[ I_\text{d} = 0 \]
\[ \Delta f = \text{rcvr bandwidth} \]
AMPLIFIER
SPONTANEOUS EMISSION NOISE

white noise

\[ S_p (\nu) = (G-1) \text{np} \ h \nu, \text{fluctuations due to spontaneous emiss.} \]
\[ \rightarrow 0, \ G \rightarrow 0 \]

np = spontaneous-emission factor

\[ = \frac{N_2}{(N_2-N_1)}, \text{number populations} \]
\[ N_2/N_1 \sim e^{-\frac{h\nu}{kT}}, \text{thermal equil.} \]
\[ N_2 = 0, N_2 = N, \text{np} = 1, \text{total inversion} \]
[rough approx - ]
SPONTANEOUS EMISSION OCCURS AS NOISE.

\[
\begin{align*}
\left( \text{SPONT. EMISSION CURRENT} + \text{AMP. SIG} \right)^2 &= (\text{SPONT} - \text{SPONT}^0)^2 + (\text{SIG} - \text{SPONT})^2 + (\text{SIG})^2 \\
\text{Photo current, } I &= R \left| \sqrt{\text{E}_in + \text{E}_sp} \right|^2 \\
\text{SIG} - \text{SPONT}, \quad &\Delta I = 2R \left( \text{AP} \right)^{1/2} \left| \text{E}_sp \right| \cos \theta \\
\cos \theta^2 &= \frac{1}{2} \\
\sigma_{\text{SPONT}}^2 &\sim 4R \text{PIN} (R_{sp}) \Delta f \quad (6.117) \\
(S/N)_{\text{OUT}} &= \frac{\langle I \rangle}{\Delta I} = \frac{(RG \text{PIN})^2}{\Delta I} \\
&\approx \frac{\text{PIN}}{4R_{sp} \Delta f} \\
&\quad (\text{not } 11, 12, 15, 18) \\
\text{For } &\frac{1}{R_{sp}} \approx \frac{2n_{sp} (G - 1)}{G} \approx 2n_{sp} \quad G >> 1 \\
\text{Min } &\frac{1}{R_{sp}} \rightarrow 2 \rightarrow 3 \text{dB}
\end{align*}
\]
Amplifier Noise

\[ SNR_i = \frac{\langle I \rangle^2}{\sigma_{\text{shot}}^2} = \frac{(R \cdot P)^2}{2 \cdot R \cdot e \cdot P \cdot B_e} \]

\[ SNR_o = \frac{\langle I \rangle^2}{\sigma_{\text{shot}}^2 + \sigma_{\text{thermal}}^2 + \sigma_{\text{sig-spon}}^2 + \sigma_{\text{spon-spon}}^2} \]

\[ \approx \frac{\langle I \rangle^2}{\sigma_{\text{sig-spon}}^2} \approx \frac{(R \cdot G \cdot P)^2}{4R^2 P \cdot G(G-1) \cdot n_{sp} \cdot hf_c \cdot B_e} \]

- Signal-Spontaneous beat noise dominates the output noise of the amplifier
Amplifier Noise

\[ F_n = \frac{SNR_i}{SNR_o} \approx 2n_{sp} = 2 \frac{N_2}{N_2 - N_1} \]

- Even for ideal amplifiers, population inversion factor = 1, the noise figure is 3dB.
- For EDFA, NF is around 4-7dB.
- With coupling loss at the beginning, NF is worse.
Quantum noise

Quantum noise is associated with photons: it is a number-phase uncertainty. The two cannot be determined simultaneously with the same accuracy.

No photons (n=0) is vacuum EM noise (top left); As the number of coherent photons increases, the classical EM wave forms and becomes perfectly defined: this is laser light.

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As defined, the ASE is only the mean noise power:

$$N \leftrightarrow < n_{ASE} >$$

The actual characterization of noise is its variance:

$$\sigma_{ASE}^2 = < n_{out}^2 > - < n_{out} >^2$$

the root of the variance, $$\sigma$$, defines the signal fluctuation or uncertainty about its mean level.
Since optical amplification by stimulated emission perfectly clones photons, could we expect the number-phase uncertainty to disappear?

\[ \Delta n_{in} \Delta \phi \geq 1 \quad \text{output:} \quad G \Delta n_{in} \Delta \phi \geq 1 \quad \rightarrow \quad \Delta n_{in} \Delta \phi \geq \frac{1}{G} \approx 0 \]

Answer: this cannot be! The output would violate the uncertainty principle!

In order to satisfy to the principle, amplification must bring extra uncertainty under the form of noise, called amplified spontaneous emission (ASE).

The ASE corresponds to the amplification of equivalent input noise photons at a minimum rate of \( n_{eq} \approx 1 \) photon per polarization mode:

\[
n_{out} = Gn_{in} + N \quad \text{with} \quad N = n_{eq} G \quad \text{and} \quad n_{eq} = n_{sp} \frac{G - 1}{G}
\]
Spontaneous emission factor (1/2)  

The **spontaneous emission factor**, \( n_{sp} \), is an heritage concept from bulk and semiconductor lasers. It helps the understanding, but is **flawed** when it comes to fiber amplifiers!

It is defined as a relative degree of **laser inversion**: which has a unity limit at full inversion \((N_2 \to 1)\)  

\[
n_{sp} = \frac{N_2}{N_2 - N_1}
\]

At medium transparency \((N_2 = N_1)\) it is undefined or near-infinite.

It is meaningless to RFA, for which there is no upper level nor inversion.

It is meaningless to EDFA, since inversion varies with fiber coordinate. Yet, one can define **path-averaged populations**:

\[
\begin{align*}
\overline{N}_1 &= \frac{1}{L} \int N_1(z) \, dz \\
\overline{N}_2 &= 1 - \overline{N}_1 \\
\text{but} \quad n_{sp} &\neq \frac{\overline{N}_2}{\overline{N}_2 - \overline{N}_1} \\
\text{either} \quad n_{sp} &\neq \int \frac{N_2}{N_2 - N_1} \, dz
\end{align*}
\]
The correct and universal definition of ASE in fiber amplifiers is

\[ N = n_{eq} G \]

where \[ n_{eq} = \frac{1}{G} \frac{P_{ASE}^{total}}{2h\nu\delta\nu} \]

where \( n_{eq} \), the equivalent input noise, has a limit of unity in the high-inversion regime.

\( n_{eq} \) is also well-defined in the transparency or non-inverted regimes (\( \bar{N}_2 = \bar{N}_1 \) or \( \bar{N}_2 < \bar{N}_1 \), respectively).

At small-signals, \( n_{eq} \) has an exact analytical definition (see Part II).

Concerning RFA, \( n_{eq} \) is also analytically-defined, with a small effect of temperature dependence (see Part III).
Amplified coherent light, noise variance

- **Coherent (laser) light** has a well-defined variance: \( \sigma_{in,coh}^2 = \langle n_{in} \rangle \)
  (also referred to as Poisson statistics)
- **Amplified coherent light**, resulting from laser light traversing an optical amplifier is noisier: \( \sigma_{out}^2 > G \langle n_{in} \rangle \)
- The exact definition of the output noise of ACL is the following

\[
\sigma_{out}^2 = G \langle n_{in} \rangle + N + 2G \langle n_{in} \rangle N + N^2 \equiv \sigma_{shot}^2 + \sigma_{beat}^2
\]

  \begin{align*}
  \text{Signal and ASE shot noises} & \quad \text{Signal-ASE and ASE-ASE beat noises}
  \end{align*}

- The concepts of **shot noise** and **beat noise** are “classical” ones based upon square-law detection of EM fields. They are not incorrect, but conceptually incomplete, because they represent “ad hoc” modeling not the actual physics.
Signal to noise ratio (SNR) 

(1/2)

Previous definitions of noise (mean and variance) were made with photon numbers. Let’s re-write them with optical and electrical powers (m= number of polarizations, namely m=2)

\[ < n_{in} > \leftrightarrow P_{in} \quad \sigma^2_{shot} \leftrightarrow \alpha (G P_{in} + m P_{ASE}) \]

\[ N \leftrightarrow P_{ASE} \quad \sigma^2_{beat} \leftrightarrow \beta^2 \left(2 G P_{in} P_{ASE} + m P^2_{ASE} \right) \]

with \( P_{ASE} = n_{eq} h \nu B_o G \) \((h \nu \delta B_o = \text{photon power in } B_o)\)

and \( B_o = 2 B_e = \text{optical bandwidth = } 2x \text{ electrical bandwidth} \)

and dimensional constants \(\alpha, \beta\) to be specified later.

Since the shot noise is in \( G \) and the beat noise is in \( G^2 \), we can neglect the first:

\[ \sigma^2_{out} \approx \sigma^2_{beat} \]
Figure 6.3: Three possible applications of optical amplifiers in lightwave systems: (a) as in-line amplifiers; (b) as a booster of transmitter power; (c) as a preamplifier to the receiver.
CASCADING FORMULA FOR NF

\[ \frac{F_1 G_1}{G_1 S_{IN}} \frac{F_2 G_2}{G_2 S_{IN}} \]

\[ N_1 = F_1 G_1 N_{IN} \]
\[ N_2 = G_2 N_1 + \Delta N \]
\[ = G_1 G_2 F_1 N_{IN} + (F_2 - 1) G_2 N_{IN} \]

\[ F = \frac{S_{IN}}{S_{OUT}} = \frac{N_{OUT}}{G_{IN}} = \frac{G_{IN} + \Delta N}{G_{IN}} \]

\[ S_{OUT} \text{ is amplifier added noise - independent of } N_{IN} \]
\[ G_{IN} \text{ is amplified input noise} \]

\[ \Delta N = (F - 1) G_{IN} \text{ - Note: } N_{IN} \text{ is generated in amplifier - independent of input noise} \]

\[ \text{Cascaded NF} = F_{TOT} = \frac{S_{IN}}{S_{OUT}} = \frac{N_{OUT}}{N_{IN}} \]
\[ = \frac{1}{G_1 G_2 F_1 + (F_2 - 1) G_2} \]

\[ \left[ F_T = F_1 + \frac{F_2 - 1}{G_1} \right] \]

\[ \left[ F_T = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \cdots + \frac{F_n - 1}{G_1 G_2 \cdots G_{n-1}} \right] \]
Note:

$F_2$ is divided by $G_1$, reducing its effect on $F_T$

...low noise, high gain preamp is placed in front of noisy, high gain amps.

Suppose $F_1 = 2 \text{ (dB)}$
$G_1 = 100 \text{ (20 dB)}$
$F_2 = 8 \text{ (dB)}$
$G_2 = 100 \text{ (20 dB)}$

$F_T = 2 + \frac{F_2 - 1}{100} = 2.07$

Now switch amps:

$F_T = 8 + \frac{2 - 1}{100} = 8.01$
SOA
Fig. 1 Typical configuration of a packaged SOA chip. The device has two fiber-chip couplings. Optional items like isolators may be placed in the light path.
Fig. 2 Schematic of carrier and light confinement in the active layer of a semiconductor optical amplifier. The smaller bandgap confines the electrons and holes, while the larger refractive index introduces optical waveguiding. The fraction of the mode inside the active layer is denoted by the confinement factor $\Gamma$. 

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Fig. 3  Reflections at the SOA facets will give rise to round-trip paths which will interfere constructively or destructively depending on the signal wavelength, leading to gain ripple as shown on the right.

Fig. 4  Top view of an amplifier with window region. The output light diverges before it is reflected back toward the waveguide, thereby reducing the optical power coupled back into it.
SOA: Semiconductor Optical Amplifier

\[ \lambda \text{n}_{\text{RIE}}, \lambda l = 11 \]

\( G \sqrt{R_1 R_2} < 0.17 \) - Traveling Wave (6.2.3)
\( \geq 0.17 \) - FP

\( G = 1000 \) (30 dB) \( \Rightarrow \sqrt{R_{12}} < 1.7 \times 10^{-4} \)

Fig. 6.4, Fig. 6.5, Fig. 6.6

- SOA has poor NF \( \sim 5 \text{dB (max)} \)
- Not ideal for preamp
- Difficult to make Travelling Wave
- Large interchannel cross-talk (cross-phase mod) - large \( \alpha \)-factor
- Four wave mixing

\( \alpha \equiv \beta_c \equiv \text{linewidth enhancement factor} \)
Figure 6.4: (a) Tilted-stripe and (b) buried-facet structures for nearly TW semiconductor amplifiers.
Figure 6.5: Amplifier gain versus signal wavelength for a semiconductor optical amplifier whose facets are coated to reduce reflectivity to about 0.04%. (After Ref. [3]; ©1987 IEEE; reprinted with permission.)
Figure 6.6: Three configurations used to reduce the polarization sensitivity of semiconductor optical amplifiers: (a) twin amplifiers in series; (b) twin amplifiers in parallel; and (c) double pass through a single amplifier.
Spontaneous and stimulated scattering

- Spontaneous and stimulated *scattering* are similar processes.

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**Spontaneous scattering**

- **Molecule**
  - **spontaneous (single) pump photon absorption**
  - **instant decay**
  - **fast decay**
  - **phonon**

**Virtual Level 2**

- **noise photon**

**Random Noise Photon**

**OUT**

**IN**

**signal photon emission**

**OUT**

---

**Stimulated scattering**

- **Molecule**
  - **sustained pump photon absorption**
  - **instant decay**

**Signal Photon**

**OUT**

**IN**

**signal photons**

**OUT**
STIMULATED RAMAN & BRILLOUIN

\[ \Omega \]

acoustic freq.  \[ \Omega_{\text{ac}} \]

K

acoustic wave vector

\[ \frac{V_{\text{phonon}}}{K} = \frac{\Omega}{K} \sim 3 \times 10^3 \text{ m/s} \sim 10^{-5} c \]

\[ \omega_s = \omega_p - \Omega_{\text{ac}} \]

K_{\text{acoustic}} \rightarrow K_{\text{optical}} \rightarrow K_{\text{acoustic}}

K_{\text{optical}} \rightarrow K_{\text{acoustic}} \rightarrow K_{\text{optical}}

\omega_s = \omega_p - \Omega_{\text{optical}} \sim 40 \text{ nm}

\text{Narrow band} \sim 20 \text{ MHz}

\text{at} \sim 11 \text{ GHz}

5G5 - backward scatter.

\text{Forward}

\text{and backward}

\text{wideband} \sim 40 \text{ nm}
6.3. RAMAN AMPLIFIERS

Figure 6.10: Schematic of a fiber-based Raman amplifier in the forward-pumping configuration.
Figure 6.11: Raman-gain spectra (ratio $g_R/\alpha_p$) for standard (SMF), dispersion-shifted (DSF), and dispersion-compensating (DCF) fibers. Normalized gain profiles are also shown. (After Ref. [30]; ©2001 IEEE; reprinted with permission.)
Raman Gain

\[ P(\omega_z) = e^{\frac{\omega(z)}{\Delta \nu}} \]

(6.1.6)

\[ \omega(z) = \frac{\omega_0}{\Delta \nu} \cdot \frac{p_{\text{pump}}}{a_{\text{pump}}} \]  (6.1.7)

\[ a_{\text{pump}} = \text{pump cross-section} \]

\[ \Delta \nu \sim 6 \text{THz} \]

DWD channels spacing

\[ 25 \sim 100 \text{GHz} \]

\[ \Delta \nu \sim 10 \sim 40 \text{ per THz} \]

\[ G = \frac{e^{\frac{\omega_0 \cdot p_{\text{p}}}{a_{\text{p}} \cdot L}}}{e^{\frac{\omega_0}{\Delta \nu}}} \] (6.1.7)

\[ \begin{align*}
\omega_0 & \sim 6 \times 10^{-14} \text{ m/W} \\
A & \sim 1550 \text{ nm} \\
ap & \sim 50 \mu m^2 \\
g \cdot L & \sim 6.7 \rightarrow G = 30 \text{ dB} \\
p_{\text{p}} & \sim 5 \text{ W} \quad \text{for} \quad L = 1 \text{ km}
\end{align*} \]

Fig. 6.11
RAMAN AMPLIFIER CHARACTERISTICS

Forward pumping coupled eqs.

(6.3.2) \[ \frac{dP_s}{dz} = -\alpha_s P_s + [g_{pe}(\alpha_p)P_p] P_s \]

(6.3.3) \[ \frac{dP_p}{dz} = -\alpha_p P_p - (\omega_p/\omega_s)[g_{pe}(\alpha_p)P_p] P_s \]

\( \alpha_s, \alpha_p = \) fiber loss

\( (\omega_p/\omega_s) = \) photon energy ratio

Assume small-signal amplification, i.e. neglect 2nd term in (6.3.3) — no pump depletion

\[ P_p (z) = P_p (0) e^{-\alpha_p z} \]

\[ P_s (z) = P_s (0) \exp \left[ g_{pe}(\alpha_p)L_{eff}/\alpha_p - \alpha_s z \right] \]

\[ P_0 = P_p (0) = \text{input pump power} \]

(6.3.5) \[ L_{eff} = 1 - e^{-\alpha_p L}/\alpha_p \]

\[ e^{-x} \approx 1 - x/3 \]
Figure 6.12: Variation of amplifier gain $G_0$ with pump power $P_0$ in a 1.3-km-long Raman amplifier for three values of the input power. Solid lines show the theoretical prediction. (Adapted from Ref. [31]; ©1981 Elsevier; reprinted with permission.)
\[ G_A = \exp(g_0L), \text{ unsat} \]

\[ G_S = \frac{(1+r)}{(r+G_A^{-1+r})}, \text{ sat} \]

\[ r = \left(\frac{\omega_p}{\omega_s}\right)\left(\frac{P_s(0)}{P_p(0)}\right) \]

Figure 6.13: Gain–saturation characteristics of Raman amplifiers for several values of the unsaturated amplifier gain \( G_A \).
The RFA bandwidth (dB) is strictly that defined by the Raman gain coefficient in linear scale (slide #1/3), i.e. 6THz or 50nm.

Bandwidth can be enhanced (e.g. 100nm) by using multiple pumps:

Gain flatness to be optimized by adequate choice of pump powers and wavelength in each channel, taking into account pump/pump SRS gains.
Tunable range

Desurvire

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Figure 6.4-zg
Distributed RFA: four pumping configurations:

- **forward**
  - signal excursion
  - transfers pump noise to signal

- **backward**
  - better, but higher NF

- **bi-directional**
  - lowest NF
  - lowest excursion

- **EDFA/hybrid**
  - THE compromise

Desurvire
Hybrid RFA/EDFA, detailed configuration

Backward pumping (+ POL-MUX pumps):
- reduces LD power
- suppresses PDG
- suppresses pump-to-signal noise conversion
EDFA
Rare Earths
Einstein's principle of spontaneous and stimulated emission in LASER media.

**Spontaneous emission**

- Atom 
- Random noise photon

**Stimulated emission**

- Signal photon
- Signal photons
Spontaneous/stimulated emission vs. scattering

- Both generate (spontaneous) noise photons and (stimulated) coherent signal photons. More to come next.

- Both processes act as laser systems, the first with rare-earth ions as fiber dopants (e.g. erbium, thullium, praseodymium) , the second with molecules forming the glass fiber host (e.g. Si-O-Si, or P-O-P), which is the Raman effect.

- Both stimulated-emission processes (RE doping and Raman) clone input signal photons (same output polarization and phase); the avalanche effect generates coherent signal wave with power amplified by gain factors between 10 and $10^4$ (10dB to 40dB)

- With RE-doped amplifiers, the energy levels are fixed, thus determining fixed pump (absorption) and fixed signal (emission) bands; with Raman amplifiers, only the frequency difference between pump and signal is fixed, making the signal band tunable.
Figure 4.11: Energy level structure of Er$^{3+}$. The wavelength scale corresponds to the wavelength of the transition from a given energy level to the ground state.
Figure 4.12: Experimentally measured absorption spectrum of an Er$^{3+}$-doped germano-alumino-silica fiber. The absorption in the 400–600 nm region has been divided by a factor of 10. The small oscillatory structure near 1100 nm corresponds to the cutoff of the second-order mode of the fiber.
4.4. FUNDAMENTAL PROPERTIES

Figure 4.9: Left: a homogeneously broadened line for a collection of ions with identical transition frequencies and lifetimes. Right: inhomogeneously broadened line made up of a collection of homogeneously broadened lines with different center frequencies and linewidths.

Figure 4.10: Gain saturation for a broadened line (solid line: unsaturated gain; dotted line: saturated gain in the presence of a strong signal). Left: gain saturation for a homogeneously broadened line. Right: gain saturation for an inhomogeneously broadened line (the spectral position of the narrow band strong signal is indicated by the arrow).
Gain coefficient and spectrum

Case of EDFA: picture a bit more complex!

- energy levels 1 and 2 split into sublevel manifolds:
  - there are as many laser sub-transitions possible
  - sublevels populations fixed by thermal equilibrium (Boltzmann’s exponential law)
- the gain coefficient now takes the form:

\[
g(\lambda) = \rho \left[ \sigma_e(\lambda) N_2 - \sigma_a(\lambda) N_1 \right]
\]

\[
\sigma_a(\lambda) \quad \sigma_e(\lambda)
\]

there exists two different cross-sections:
one for absorption, one for emission
Gain spectrum: inversion-dependent, with two regions of absorption/gain
Figure 6.15: (a) Energy-level diagram of erbium ions in silica fibers; (b) absorption and gain spectra of an EDFA whose core was codoped with germania. (After Ref. [64]; ©1991 IEEE; reprinted with permission.)
EDFA configurations for WDM systems

The basic configuration and its components (e.g. forward pumping)

Signal in ($\lambda=1.53-1.56\mu m$)

Pump module, $\lambda=980\text{nm}$ or $1.48\mu m$

NB: two LD pumps can be polarization-multiplexed

Front-end elements (WSC, S) with combined transmission $T$ increase noise figure by $1/T$
Energy Levels

• Stark splitting
  – $T_{32}$ ~ 1us, $T_{21}$ ~ 10ms
  – Can’t modulate laser faster than 100Hz

• gain bandwidth
  1525nm - 1570 nm
  ~ 45 nm ~ 5600THz

• gain peak at 1532nm
Pump Source

- 980 nm
  - low ASE, low noise amplifier
- 1480 nm
  - higher power pump laser
  - high output power
  - not as efficient
  - degree of population inversion is lower
Figure 6.19: Schematic of an L-band EDFA providing uniform gain over the 1570–1610-nm bandwidth with a two-stage design. (After Ref. [92]; ©1999 IEEE; reprinted with permission.)
**SIMPLE TOFA THEORY**

Assume 2-level model — all pump states decay immediately to $N_2$.

- $N_2$
- $N_1$

**Neglect ASE**

\[ \frac{\partial N_2}{\partial t} = \left( \sigma^a_{p,1} N_1 - \sigma^e_{p,1} N_2 \right) \Phi_p + \left( \sigma^a_{e,1} N_2 - \sigma^e_{e,1} N_2 \right) \Phi_e - \frac{N_2}{\tau}, \]

\[ \frac{\partial N_1}{\partial t} = \left( \sigma^a_{p,1} N_1 - \sigma^e_{p,1} N_1 \right) \Phi_p + \left( \sigma^a_{e,1} N_2 - \sigma^e_{e,1} N_1 \right) \Phi_e + \frac{N_1}{\tau_1}, \]

$\sigma^a_{s,p}$ = absorption/emission cross-sections at pump/signal

$\tau_1$ = spontaneous lifetime of excited state (2) \( \approx 10 \text{ ms} \).

$\Phi_{p,s}$ = photon flux pump/signal

~ $P_{p,s} (k_{p,s} h \nu_{p,s})$, $k$ = cross-section fiber mode.
and $R_2$ vary due to absorption and stimulated emission.

Neglecting spontaneous emission:

$$\frac{\partial P_s}{\partial z} = \Gamma_s \left( \sigma_s^{e} N_s - \sigma_s^{a} N_i \right) P_s - \alpha_s P_s \quad (6.4.4)$$

$$\frac{s \partial P_p}{\partial z} = \Gamma_p \left( \sigma_p^{e} N_s - \sigma_p^{a} N_i \right) P_p - \alpha_p P_p \quad (6.4.5)$$

$\alpha_s, \alpha_p$ losses at signal, pump.

$\alpha_s \rightarrow 0$ for typical $L \sim 10$-20m

$\Gamma_s, \Gamma_p$ accounts for overlap of mode and doped core.

$s = \pm 1$ for forward/backward pump.

With $N_s = N_i + N_e$ and $\sigma_s^{a} = 0$ and steady-state, $\frac{\partial}{\partial t} = 0$

$$N_s(3) = -\frac{T_i}{\alpha_i h \nu} \frac{\partial P_s}{\partial z} - \frac{s T_i}{\alpha_i h \nu} \frac{\partial P_p}{\partial z} \quad (6.4.6)$$

with

$$\alpha_i = \Gamma_s a_i = \Gamma_p a_p$$

Substitute (6.4.6) in (6.4.4) and (6.4.5) and integrate over $z \rightarrow$
Total amplifier gain

\[ G = \Gamma_5 \exp \left[ \int_0^L (\sigma_e N_2 - \sigma_a N_1) \, dz \right]. \]

see Fig. 6.16 (a) - saturation
(b) - peak vs \( L \).

At 1.55 \( \mu \)m signal, 1.48 \( \mu \)m pump

\( P_s = 5 \) mW, \( L = 30 \) m

\[ G = 35 \mathrm{dB} \]

Note: \( G \) is not affected by bit pattern for \( B \geq 100 \) b/s
because of \( T_1 \approx 10 \) ms for \( N_2 \rightarrow N_1 \) spontaneous emission.

Each signal is much less than saturation energy (\( \approx 10 \mu \)J), EDFI gain responds to average \( P_s \).

For SOA, \( T_1 \approx 100 \) ps and \( G \) can be modulated by injection current or by bit pattern at \( \approx 10 \) GHz.
CHAPTER 6. OPTICAL AMPLIFIERS

Figure 6.16: Small-signal gain as a function of (a) pump power and (b) amplifier length for an EDFA assumed to be pumped at 1.48 μm. (After Ref. [64]; ©1991 IEEE; reprinted with permission.)
Optical amplification
(2/6)

EDFA implementation

• End-pumping through compact laser diode (possibly) directly in $^4I_{13/2} - ^4I_{15/2}$ transition
• Dopant confined in fiber core

![Diagram]

Gain ($G(\lambda_s)$) = $\sigma_e(\lambda_s)N_2 - \sigma_a(\lambda_s)N_1$

1.451-1.48\(\mu\)m or 980nm (alternatively)

signal 1.53-1.56\(\mu\)m

Er\(^{3+}\) doped fibre

coupler

isolator

optimal doped-fiber length

amplified signal
Figure 6.17: (a) noise figure and (b) amplifier gain as a function of the length for several pumping levels. (After Ref. [74]; ©1990 IEE; reprinted with permission.)
Figure 6.3: Three possible applications of optical amplifiers in lightwave systems: (a) as in-line amplifiers; (b) as a booster of transmitter power; (c) as a preamplifier to the receiver.
Figure 6.20: Receiver sensitivity versus optical-filter bandwidth for several values of the noise figure $F_n$ when an optical amplifier is used for preamplification of the received signal.
MULTICHANNEL AMPLIFICATION

- FWM and XPM are not a problem in EDFAs due to $t_1 \sim 10^{-6}$.

- Spectral non-uniformity is main problem in amplifier cascade. 0.1 dB gain difference between WDM components becomes 20 dB in 100 amplifier chain.

- Use gain-flattening filters + active gain-equalization.

- Note: fiber amp (Raman or EDFAs) has minimum coupling loss vs EDFAs.

  Coupling & loss
  Coupling loss degrades S/N can't recover.
Figure 6.21: Variation of the signal power $P_s$ and the ASE power $P_{ASE}$ along a cascaded chain of optical amplifiers. The total power $P_{TOT}$ becomes nearly constant after a few amplifiers. (After Ref. [119]; ©1991 IEEE; reprinted with permission.)
EDFA configurations for WDM systems

forward pumping

backward pumping

bidirectional (an hybrid) pumping

redundancy pumping

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Figure 6.4-zc

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EDFA configurations for WDM systems

Combined conventional band (C) and long band (L) operation

L-band: longer EDF length, higher pump, similar performance

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Parallel Type EDFA

1.55 μm-band EDFA

First Sub-Unit
WDM Fiber Coupler
Optical Isolator
Silica-based EDF (4 m)
980 nm LD

Second Sub-Unit
Bulk-type WDM Coupler
Fluoride-based EDF (8 m)
1480 nm LD

C-Band
Input Isolator
1.55/1.58 μm WDM Coupler

1.58 μm-band EDFA

First Sub-Unit
WDM Fiber Coupler
Optical Isolator
Silica-based EDF (35 m)
980 nm LD

Second Sub-Unit
Bulk-type WDM Coupler
Silica-based EDF (200 m)
1480 nm LD

L-Band
Output Isolator
1.55/1.58 μm WDM Coupler

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Parallel Type EDFA

Signal Gain (dB)

Wavelength (nm)

Noise Figure (dB)

Bandwidth: 30 nm (1530-1560 nm)

Bandwidth: 30 nm (1570-1600 nm)

1.55 μm-band amplification unit

1.58 μm-band amplification unit
The End