Laser pumping

4-level laser rate equations

How to achieve population inversion

1. Pumping process excites atoms from ground state to state 4
2. Fast decay 4 → 3
3. Slow decay 8 → 2
4. Fast decay 2 → 1

⇒ build-up of inversion on 3 → 2

Rate equation analysis

\[ \frac{dN_4}{dt} = R_p (N_1 - N_4) - (\gamma_{43} + \gamma_{42} + \gamma_{41}) N_4 \]

+ pump rate

\[ \frac{1}{T_4} = \gamma_{43} + \gamma_{42} + \gamma_{41} \]

in steady state, \( \frac{dN_4}{dt} = 0 \)

\[ N_4 = \frac{R_p z_4}{1 + R_p z_4} \]

\[ N_1 = \frac{R_p z_4 N_1}{1 + R_p z_4} \]

(\text{pump process not saturated})

\[ \frac{dN_3}{dt} = \gamma_{43} N_4 - (\gamma_{32} + \gamma_{31}) N_3 = \frac{N_4}{T_{43}} - \frac{N_3}{T_3} \]

⇒ 2

\[ \frac{T_3}{T_{43}} \gg \gamma_{43} \]

\[ N_3 > N_4 \]
\[
\frac{dN_2}{dt} = Y_{42} N_4 + Y_{32} N_3 - Y_{24} N_2 = \frac{N_4}{\tau_{42}} + \frac{N_3}{\tau_{32}} - \frac{N_2}{\tau_{24}}
\]

"at steady state, use \( \circ \)"

\[= \beta N_3 \quad \beta = \frac{\tau_{21}}{\tau_{32}} + \frac{\tau_{42} \tau_{21}}{\tau_{3} \tau_{42}} \]

For a good laser, \( Y_{42} \approx 0 \), level 4 feeds primarily into level 3.

\[\beta = \frac{\tau_{21}}{\tau_{32}} = \frac{N_2}{N_3} \quad \text{(3)} \]

If \( \tau_{32} > \tau_{21} \), \( \beta < 1 \rightarrow N_2 < N_3 \) \textit{inversion}

depends only on \textit{lifetime ratios}. Easy to look for good laser candidates.

Magnitude of inversion density \( (N_3 - N_2) \)?

\(\circ \) \( N_3 = \frac{\tau_{24}}{\tau_{43}} N_4 \)

\(\circ \) \( N_4 = \frac{R_p \tau_4 N_1}{N_2} = \beta N_3 \)

\[N_1 + N_2 + N_3 + N_4 = N \quad \text{conservation of atoms} \]

\[N_1 = \frac{1}{R_p \tau_4} \frac{\tau_{43}}{\tau_3} N_3 \quad \text{using} \ (\circ) \]

\[N_3 \left[ \frac{1}{R_p \tau_4} \frac{\tau_{43}}{\tau_3} + \beta + 1 + \frac{\tau_{43}}{\tau_3} \right] = N \]
\[
\frac{N_3 - N_2}{N} = \frac{(1 - \beta) R_p \tau_{43}}{\left[ \frac{\tau_{43}}{\tau_3} + (1 + \beta + \frac{\tau_{43}}{\tau_3}) R_p \tau_{43} \right]}
\]

Quantum efficiency \( \eta \) - \( \frac{\text{# of photons emitted and laser trans}}{\text{# of pump photons absorbed}} \)

\( \gamma = \frac{\eta_{43}}{\eta_3} \times \frac{\eta_{32}}{\eta_4} \)

fraction of 4 that relaxes to 3
fraction of 3 that radiates

\[
\frac{N_3 - N_2}{N} = \frac{(1 - \beta) R_p \eta \tau_{43}}{1 + (1 + \beta + \frac{\tau_{43}}{\tau_3}) \eta R_p \tau_{43}}
\]

Good laser \( \tau_{43} \ll \tau_3 \), \( \eta \approx 1 \)

\[
\frac{N_3 - N_2}{N} = \frac{(1 - \beta) R_p \eta \tau_{43}}{1 + (1 + \beta) R_p \eta \tau_{43}}
\]

large \( R_p \eta \tau_{43} \Rightarrow \beta \ll 1 \)

\[
\Delta N = \frac{(1 - \beta)}{(N + \beta)} N 
\]

\( \rightarrow \) \( R_p \tau_{43} \)
Review conditions for large inversion
1. Level 4 efficiently feeds level 3
   \[ \gamma_{42} \approx 0 \quad \gamma_4 \approx \gamma_{43} \]
   \[ \Rightarrow \beta \approx \gamma_{21} / \gamma_{32} \]
2. \( \gamma_{21} \) fast, \( \gamma_{32} \) slow \( \Rightarrow \beta \ll 1 \)
3. High fluorescence efficiency

4. Pump rate \( R_p \gg \gamma_{rad} \)
   \[ R_p \gg \gamma_{32} \]

Three level laser

\[ \frac{dN_3}{dt} = R_p (N_1 - N_3) - \frac{N_3}{\tau_3} \]

\[ N_1 + N_2 + N_3 = N \]  
conservation of atoms
Fluorescence quantum efficiency

For high efficiency, \( \tau_{32} \approx \tau_3 \), \( \tau_{21} \approx \tau_{21} \)

\[ \eta \approx 1 \]

Relaxation ratio, \( \beta \approx \frac{\tau_{32}}{\tau_{21}} \)

\( \tau_{32} \) fast, \( \tau_{21} \) slow, \( \beta \ll 1 \)

Steady state solution

\[ \frac{N_2 - N_1}{N} = \frac{(1-\beta)\eta R_p \tau_{21} - 1}{(1+2\beta)\eta R_p \tau_{21} + 1} \]

even if \( \beta < 1 \), also need minimum pump rate to get inversion

Compare 3-level + 4 level inversion vs pump

Ruby is best example of 3-level laser has close to ideal conditions.
- Broad level 3: get high \( \eta \) using flash lamps
- \( \eta \approx 1 \)
- \( \tau_{21} \approx 4 \text{ ms} \) - nearly purely radiative