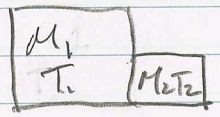
Interaction of matter and radiation

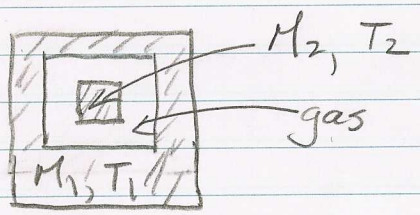
[Reading assignment, Svelto, Ch. 2]

- Thermal radiation. Thermal equilibrium between 2 masses can be achieved by several processes

Conduction

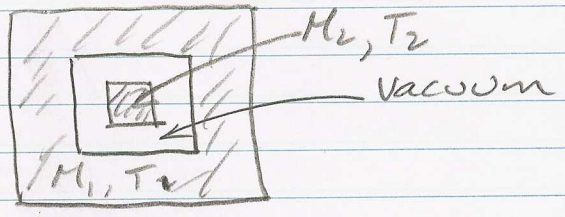


convection



gas transports energy to reach equilibrium

radiation



to reach equilibrium, masses must radiate energy + absorb energy.

hot bodies radiate light. Think of glowing coals. Boltzmann's law says that for two energy levels of a system, in thermal equilibrium, the population ratio must be:

for a continuous spectrum of energy levels (i.e. a solid)

$$\frac{N_2(E)\,dE}{N_1(E)\,dE} = \frac{g_2(E)}{g_1(E)} e^{-(E_2-E_1)/kT}$$

where here, $g(E)$ is the density of states function.

To get visible light (~2 eV photons), consider Boltzmann ratio.

at $T = 300K$, $kT = .026\,eV$

$e^{-2/0.026} \cong 4 \times 10^{-34}$ very small population!

at $T = 1000K$, $kT = .087\,eV$

$e^{-2/.087} \cong$

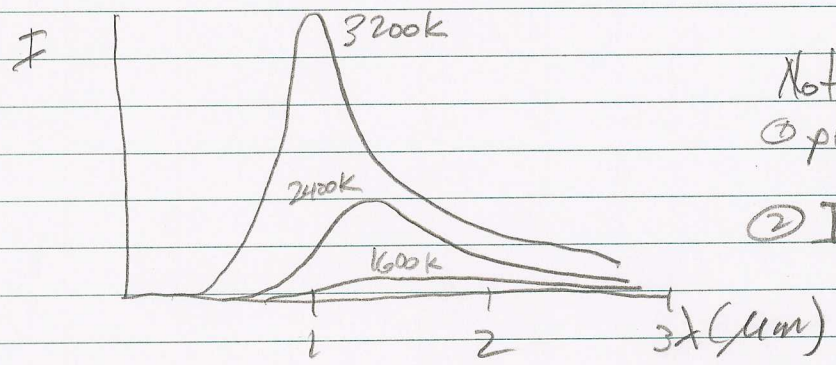
at $T = 5000K$, $kT = 0.43\,eV$

$e^{-2/0.43} \cong$

Thermal radiation - total radiation increases sharply with temperature, and peak shifts to shorter λ with T.

Stefan-Boltzmann law:
total intensity radiated by surface of a hot body

Stefan-Boltzmann constant $\sigma = 5.67 \times 10^{-4}\ W/cm^2 \cdot K^4$
emissivity: e_m - dimensionless $0 \le e_m \le 1$
ability of body to emit + absorb energy.



Notice:
① pk λ shifts shorter with T
$\lambda_m T = \text{const}$
② Intensity increases at all λ with T

If M_1 is initially at higher T, it radiates more and heats up M_2. But eventually, M_2 must radiate as much as it absorbs to reach equilibrium.

Power/area incident on M_2 : I_2 "irradiance"
radiated by M_2 : H_2 "radiance"

b_2 is fraction of incident power that is absorbed

If several masses are put inside the cavity, the intensity would be the same for all, so

$$I = \frac{H_2}{b_2} = \frac{H_3}{b_3} = \ldots$$

This says the ratio of power radiated to fraction absorbed is a constant, independent of material. Since $H_2 = e_{m_2} \sigma T^4$ and in equilibrium, all masses reach the same T, we can find $e_m = b$

Perfect reflector $\rightarrow e_m = 0$
Lampblack $\rightarrow e_m = 0.95$

For the cavity, inside $I = \sigma T^4$. If we open a small hole and allow I to come out, we have a perfect blackbody radiator at the hole.

<u>Planck theory of cavity radiation</u>

Calculate the spectral energy density for radiation in thermal equilibrium with walls of cavity.

Total energy density in the e-m field : ρ
Spectral energy density : ρ_ν

$$\rho = \int_0^\infty \rho_\nu \, d\nu$$

Spectral Intensity (for light leaking out a hole in cavity)

(note: for a beam travelling in a specific direction, $I_\nu = \frac{c}{n} \rho_\nu$. But integrating light coming out of the hole over 2π solid angle, taking into account $\cos\theta$ factor for obliquity, we get the factor of 4.)

For cavity radiation, ρ_ν is calculated as product of

[# of modes/unit volume - unit frequency][average energy/mode]

similar to density of states

<u>Mode counting</u> solve wave equation in a rectangular box

$$E(x,y,z) \sim e^{i(k_x x + k_y y + k_z z)}$$

$$K^2 = k_x^2 + k_y^2 + k_z^2 \qquad K = \frac{2\pi}{\lambda} = \frac{2\pi\nu}{c}$$

$n_{x,y,z}$: integer

Total # of modes with frequency from $0 \to \nu$ is Volume of one octant of sphere in k-space

$$N(\nu) = \frac{\frac{1}{8}\cdot\frac{4}{3}\pi\left(\frac{2\pi\nu}{c}\right)^3}{(\pi/L_x)(\pi/L_y)(\pi/L_z)} \times 2 = \frac{8\pi\nu^3}{3c^3}V$$

2 polarizations per mode

We want #modes/freq-vol:

Average energy/mode = $\dfrac{h\nu}{\text{occupation probability}}$

for photons: Bose-Einstein distribution

$$\langle E \rangle = \frac{h\nu}{e^{(h\nu/kT)}-1}$$

So:

$$\rho_\nu = \frac{8\pi\nu^2}{c^3}\frac{h\nu}{e^{(h\nu/kT)}-1}$$

$$I_\nu = \frac{2\pi h\nu^3}{c^2}\frac{1}{e^{(h\nu/kT)}-1}$$

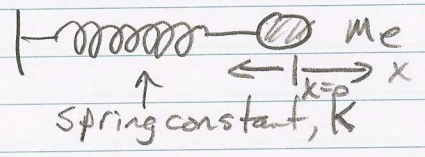
(HW: derive S-B law, find σ)

Atomic radiative transitions

- Classical electron oscillator model

radiation from transitions occurs at discrete frequencies. Atom appears to behave as a simple oscillator

model - electron on a spring



Assuming <u>no damping</u>, simple spring eqn:

$$m_e \frac{d^2x}{dt^2} + Kx = 0$$

$$x = x_0 e^{-j\omega_0 t}$$

Oscillating electron <u>radiates</u>. Total power radiated by a dipole is

μ: dipole moment $= e x_0$

$$P = \frac{dE}{dt} = -\gamma_0 E \qquad E\text{: total energy in the spring system}$$

$$E = E^0 e^{-\gamma_0 t} = E_0 e^{-t/\tau_0}$$

Radiative damping - add term $\gamma_0 \frac{dx}{dt}$ to eqn of motion

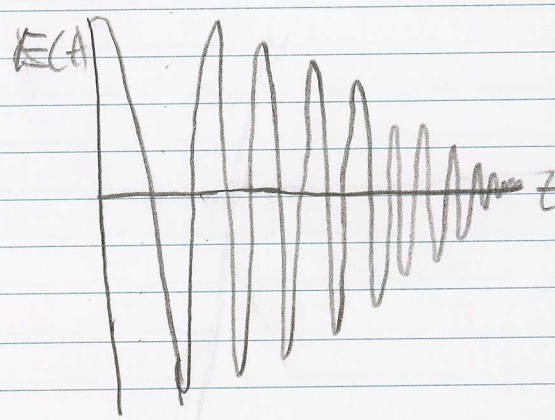
$$m \frac{d^2x}{dt^2} + \gamma_0 \frac{dx}{dt} + m_e \omega_0^2 x = 0$$

$$x \cong x_0 e^{-\gamma_0 t/2} e^{-j\omega_0 t}$$

$$E(t) = E_0 e^{-\gamma_0 t/2} e^{-i\omega_0 t}$$

electric field radiated has same time dependence

$$I(t) = |E(t)|^2 = I_0 e^{-\gamma_0 t}$$



Emission spectrum of radiating electron

Fourier transform

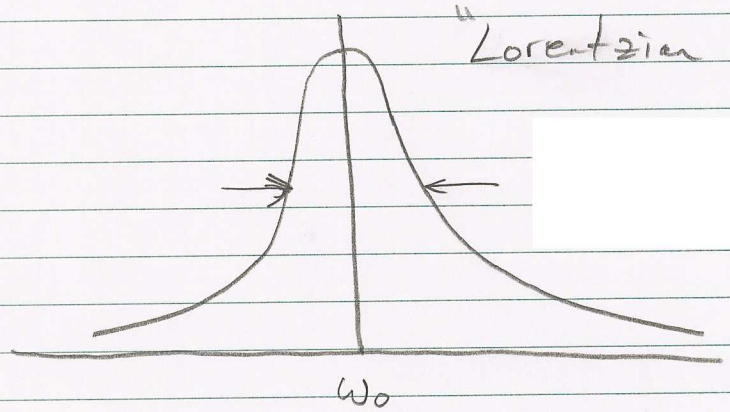
$$= \frac{E_0}{\sqrt{2\pi}} \int_0^\infty e^{j[(\omega-\omega_0) + j\gamma_0/2]t}$$

$$= -\frac{E_0}{\sqrt{2\pi}} \frac{1}{j[(\omega-\omega_0) + j\gamma_0/2]}$$

Intensity spectrum $\quad I(\omega) = |E(\omega)|^2$

$$I(\omega) = I_0 \frac{\gamma_0/2\pi}{[(\omega-\omega_0)^2 + \gamma_0^2/4]} \quad ; \quad I_0 \equiv \int_0^\infty I(\omega)\,d\omega \quad \text{total intensity}$$

"Lorentzian lineshape"



Quantum mechanical treatment

- Rigorous QM treatment is beyond our scope. Need QM theory for radiation field.
- Semiclassical treatment. QM for atoms, classical for field.

Radiation occurs during transition from level 2 to level 1. The atom develops a quantum mechanical <u>dipole moment</u> that radiates.

$$\psi_1(r,t) = u_1(r)\, e^{-j\frac{E_1}{\hbar}t} \qquad \psi_2 = u_2(r)\, e^{-j\frac{E_2}{\hbar}t}$$

$u_{1,2}$ are stationary eigenfunctions of the atom

During the transition, $\psi = a_1(t)\psi_1 + a_2(t)\psi_2$

- Electric dipole moment of atom

$$= -e\int \vec{r}\,|a_1|^2 |u_1|^2 dV - e\int \vec{r}\,|a_2|^2 |u_2|^2 dV$$

$$-e\int \vec{r}\left[a_1 a_2^* u_1 u_2^* e^{j\omega_0 t} + cc\right] dV \qquad \omega_0 = \frac{E_2 - E_1}{\hbar}$$

oscillating dipole $\mu_{osc} = Re\left[2a_1 a_2^* \mu_{21} e^{j\omega_0 t}\right]$

$\mu_{21} = \int u_2^* r\, u_1\, dV$ electric dipole matrix element

If we write the QM power radiated per atom:

A_{21}: spontaneous emission rate

Compare to classical expression:

$$P_R = \frac{\mu^2 \omega_0^4}{12\pi\epsilon_0 c^3}$$

Identify μ with $2\mu_{21}$: dipole matrix element, then

$$A_{21}\hbar\omega_{21} = \frac{4\mu_{21}^2\omega_{21}^4}{12\pi\epsilon_0 c^3}$$

or

$$A_{21} = \frac{\mu_{21}^2\omega_{21}^3}{3\pi\epsilon_0\hbar c^3} \equiv \frac{1}{\tau_{spon}}$$