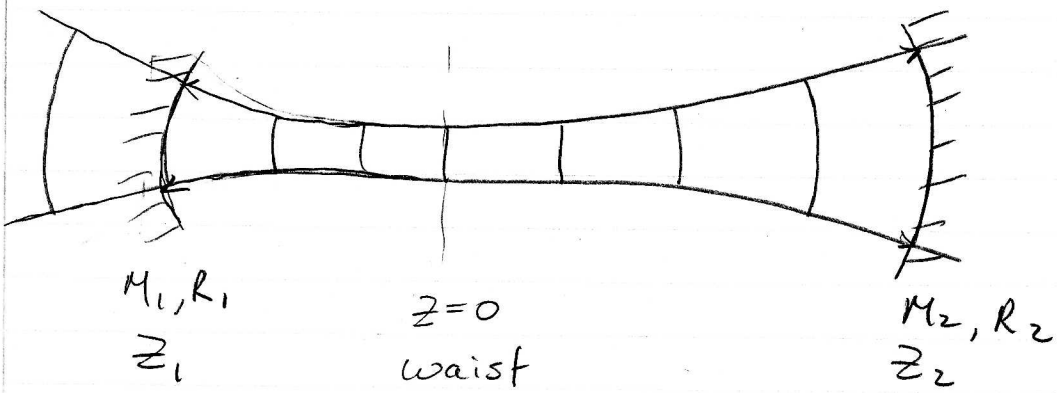
Optical Resonators

Reading assignment: Svelto 5.1-5.5

Suppose we have a certain Gaussian beam

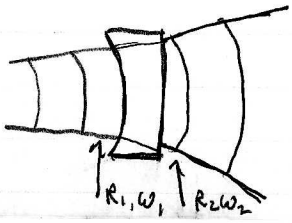


place mirror $M_{1,2}$ at $z_{1,2}$ with radius $R_{1,2}$ to match Gaussian beam radii.

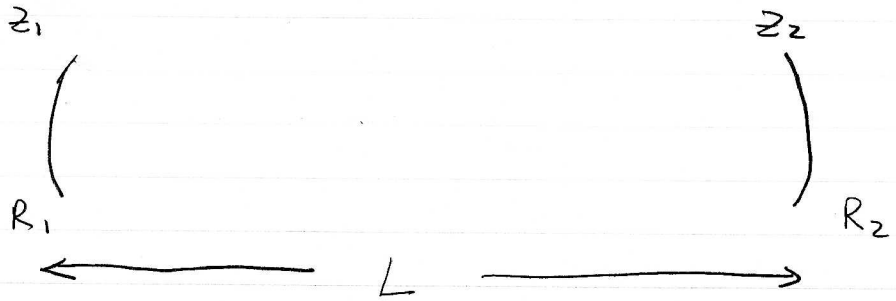
Since mirror surfaces are normal to energy propagation direction, the beam reflects perfectly back on itself.

Beam is <u>trapped</u>. "Reproducing stable field" configuration results.
(Mirrors must be large enough so spillover past them is negligible).

It follows that if mirror is partially transmitting, a Gaussian beam emerges.
(Mirror might act as a lens to transform output beam)



How about if we choose mirror radii and positions: Can we find the Gaussian beam that fits inside?



If we can find z_1, z_2 and ω_0 (or z_0) (relative to z_0 – position of beam waist, we know everything)

$$R(z_1) = z_1 + \frac{z_R^2}{z_1} = -R_1$$

$$R(z_2) = z_2 + \frac{z_R^2}{z_2} = R_2$$

solve for z_1, z_2, z_R

define $g_1 \equiv 1 - \frac{L}{R_1}$; $g_2 = 1 - \frac{L}{R_2}$ "resonator g-parameters"

solutions are:

$$z_R^2 = \frac{g_1 g_2 (1 - g_1 g_2)}{(g_1 + g_2 - 2 g_1 g_2)^2} L^2$$

$$z_1 = \frac{g_2 (1 - g_1)}{g_1 + g_2 - 2 g_1 g_2} L$$

You check!

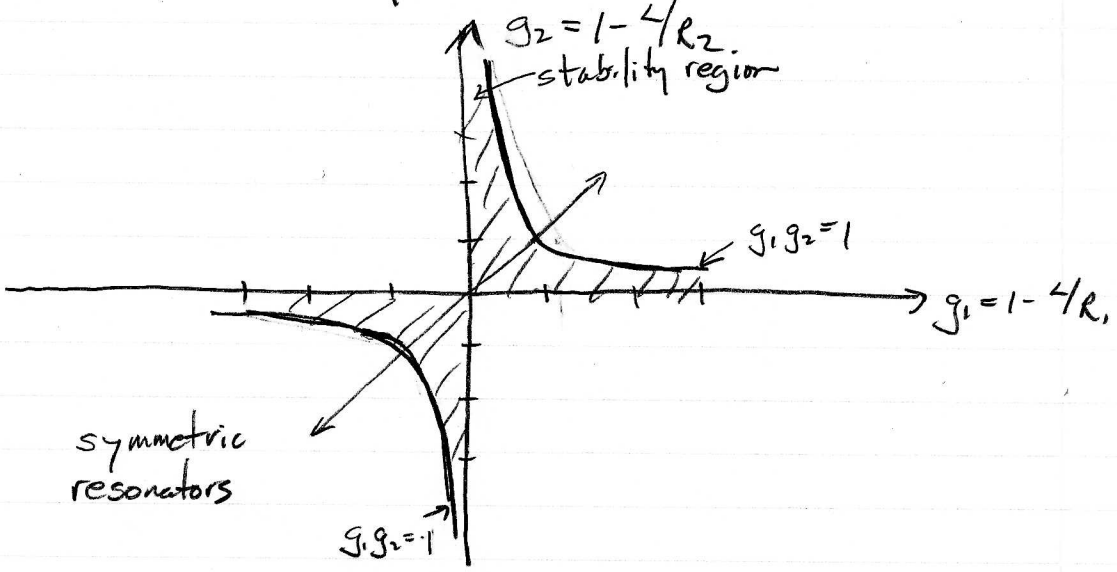
also $\omega_0^2 = \dfrac{\lambda z_R}{\pi}$

$$= \frac{L\lambda}{\pi}\sqrt{\frac{g_1 g_2(1-g_1 g_2)}{(g_1+g_2-2g_1 g_2)^2}}$$

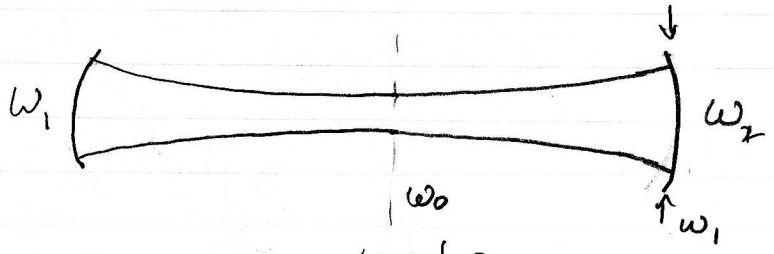
$$\omega_1^2 = \frac{L\lambda}{\pi}\sqrt{\frac{g_2}{g_1(1-g_1 g_2)}} \quad ; \quad \omega_2^2 = \frac{L\lambda}{\pi}\sqrt{\frac{g_1}{g_2(1-g_1 g_2)}}$$

for ω_0, ω_1 and ω_2 to be real + finite clearly

resonator stability diagram:

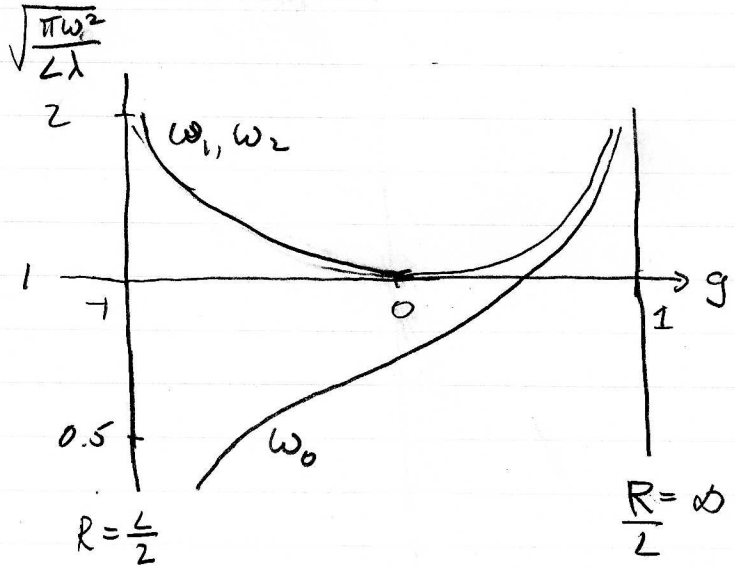


example: symmetric resonator $R_1 = R_2$

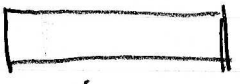


waist in center

$$\omega_1^2 = \omega_2^2 = \frac{L\lambda}{\pi}\sqrt{\frac{1}{1-g^2}}$$

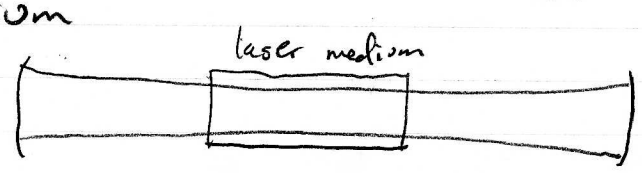


confocal resonator

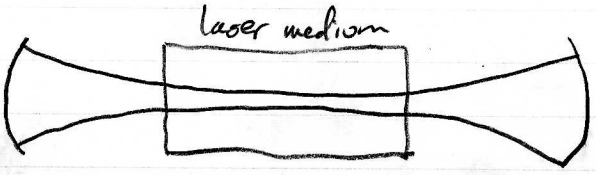


plane parallel resonator

Often desire large mode volume in laser to extract maximum energy from gain medium



good



bad

Numbers: assume $\lambda = 600\,nm$, $L = 10\,cm$.
Suppose we have a 3 mm diameter laser rod. design for $d \cong 3\omega_0$ or $\omega_0 = 1\,mm$

$$\mathcal{K}^2$$

$$4\mathcal{K}^2(1-g) = 1+g$$

$$\frac{4\mathcal{K}^2-1}{4\mathcal{K}^2+1} = g$$

$$\mathcal{K}^2 = 2742, \quad g = 1 - 1.82\times10^{-4}$$

planar

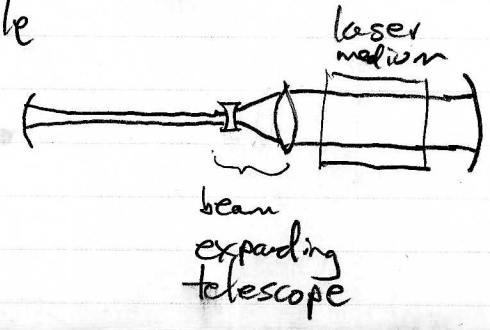
~planar resonators very unstable alignment

Practical $R = 10\,m \Rightarrow \omega_0 = 0.04\,cm$, $.4\,mm$

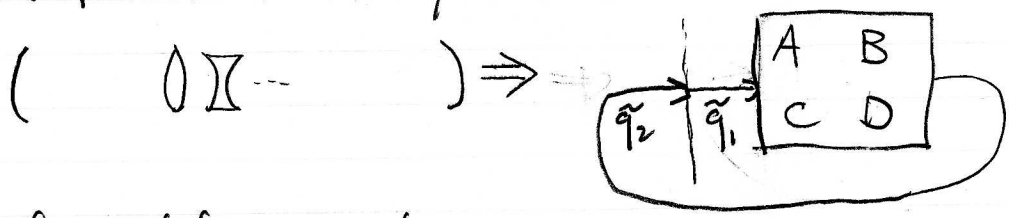
mirror diameter should be large to cover full beam $d \gtrsim 3\omega_0$

To more efficiently fill laser mode volume, put additional optics inside cavity.

example



General resonator analysis



Model complete resonator by single ABCD matrix. Include laser mirrors, any intra-cavity optics, and free space propagation $\rightarrow$ one complete round trip

Choose reference plane. Input Gaussian beam with parameter $\tilde{q}_1$. After one full round trip, beam comes back around with parameter $\tilde{q}_2$.

Resonator mode is self-consistent $\tilde{q}_2 = \tilde{q}_1 \equiv \tilde{q}$

$$\tilde{q} = \frac{A\tilde{q} + B}{C\tilde{q} + D}$$

$$\frac{1}{\tilde{q}^2} + \frac{(A-D)}{B}\frac{1}{\tilde{q}} - \frac{C}{B} = 0$$

$$\therefore \frac{1}{q_{1,2}} = \frac{(D-A)}{2B} \pm \frac{1}{2B}\sqrt{(A-D)^2 + 4BC}$$

Using $AD - BC = 1$

$$\frac{1}{q_{1,2}} = \frac{D-A}{2B} \pm \frac{1}{2B}\sqrt{(A+D)^2 - 4}$$

Recall $\frac{1}{q} = \frac{1}{R} - i\frac{\lambda}{\pi w^2}$.

$\frac{1}{q}$ must have a negative imaginary part

so $(A+D)^2 < 4$, or $\boxed{}$

generalized resonator stability condition.

Must take $\ominus$ root. Only <u>one</u> unique stable Gaussian mode.

[$\oplus$ root $\rightarrow$ unstable mode

Mode frequencies

Mode must also reproduce itself in phase.
Total round trip phase shift $= 2\pi$.
Condition sets mode frequencies.
(Recall simple analysis giving $\boxed{\nu_{ax} = n\frac{c}{2L}}$)

Go back to simple 2-mirror resonator.

Phase shift from 1 mirror to the other

$$\phi(z_2 - z_1) = kL - (n+m+1)[\eta(z_2) - \eta(z_1)]$$

$\boxed{}$

phase shift depends on higher order mode #s

It is possible to show (tedious algebra),

$$\eta(z_2) - \eta(z_1) = \cos^{-1} \pm\sqrt{g_1 g_2}$$

+ sign $\rightarrow$ pos branch
− sign $\rightarrow$ neg branch

round trip phase shift is just twice this.

So mode frequencies determined by:

$$\nu_{snm} = \frac{\omega_{snm}}{2\pi} = \frac{c}{2L}\left[s + (n+m+1)\,\frac{\cos^{-1}(\pm\sqrt{g_1 g_2})}{\pi} \right]$$

as before

mode frequencies for higher order modes are shifted