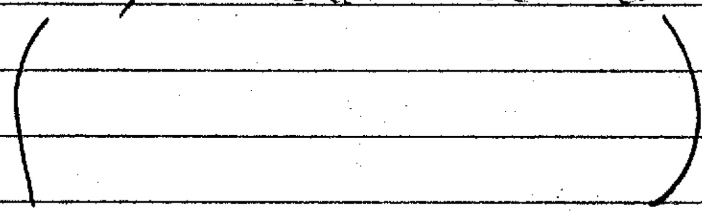
EE236
Fall 2004
Problem Set 7
Solution

1) Yariv problem 7.2

Notice that Yariv's notation here is inconsistant, acording to table 6.1 (4), a negative radius would half to correspond to a convex mirror (diverging) which is also the usual convention, but in part (d) he states that for a confocal cavity $R_1 = -R_2 = l$ which would change the sign convention.

If we use the convention of section 7.1, however, then the curvature of mirror 1 (on the left) should be negative and the radius of mirror 2 should be positive — if they are concave toward each other (and therefore a stable cavity.

So let's use the conventions of section 7.1, where we have



$R_1 = -20$ cm $\qquad R_2 = 32$ cm

$l = 16$ cm

we then have

$$z_0^2 = \frac{l(-R_1 - l)(R_2 - l)(R_2 - R_1 - l)}{(R_2 - R_1 - 2l)^2}$$

$$z_0^2 = \frac{16(4)(16)(52-16)}{(52-32)^2}$$

$$z_0^2 = 92.16 \text{ cm}^2$$

$$z_0 = 9.6 \text{ cm}$$

$$w_0 = \left(\frac{\lambda z_0}{\pi n}\right)^{1/2}$$

$\lambda = 10^{-4}$ cm
$n = 1$ (air or vacuum)

$$w_0 = (3.05 \times 10^{-4})^{1/2}$$
$$w_0 = 1.74 \times 10^{-2} \text{ cm}$$

b) the distance to the mirrors from this waist are:

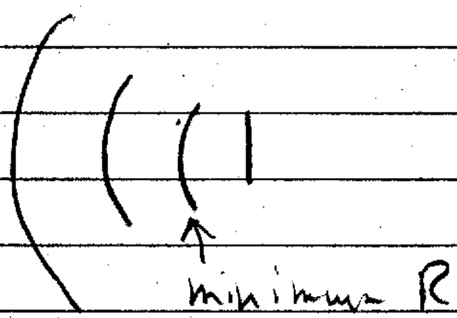
$$z_1 = \frac{R_1}{2} \pm \frac{1}{2}\sqrt{R_1^2 - 4z_0^2}$$

$$z_2 = \frac{R_2}{2} \pm \frac{1}{2}\sqrt{R_2^2 - 4z_0^2}$$

$$z_1 = -10 \pm \frac{1}{2}\sqrt{400 - 4(92.16)}$$

$$z_1 = -10 \pm 2.8$$

notice that there are two possible solutions because, tracking back from the waist, the radius starts at $\infty$, goes through a minimum, and then goes up




To decide which one to use, we will need to solve for $z_2$

$$z_2 = \frac{R_2}{2} \pm \frac{1}{2}\sqrt{R_2^2 - 4z_0^2}$$

$$z_2 = 16 \pm \frac{1}{2}\sqrt{32^2 - 4(92.16)}$$

$$z_2 = 16 \pm 12.8 \text{ cm}$$

Since we know the cavity is only 16 cm long, $z_2 = 3.2$ cm and so $z_1 = 16 - 3.2 = 12.8$ which corresponds to the negative sign above

c) we have the spot size for a Gaussian beam:

$$w(z) = w_0\left[1 + \left(\frac{z}{z_0}\right)^2\right]^{1/2}$$

$$z_0 = 9.6 \text{ cm}$$

$$w_0 = 1.74 \times 10^{-2} \text{ cm}$$

$$w(z_1) = w(12.8) = 1.74 \times 10^{-2}\left[1 + \left(\frac{12.8}{9.6}\right)^2\right]^{1/2}$$

$$= 1.74 \times 10^{-2} \times 1.667$$

$$= 2.9 \times 10^{-2} \text{ cm}$$

$$w(z_2) = 1.74 \times 10^{-2}\left[1 + \left(\frac{3.2}{9.6}\right)^2\right]^{1/2} = 1.83 \times 10^{-2} \text{ cm}$$

(d) $(W_0)_{conf} = \left(\frac{\lambda l}{2\pi n}\right)^{1/2} = \left(\frac{10^{-4}\cdot 16}{2\pi}\right)^{1/2}$

$(W_0)_{conf} = 1.60 \times 10^{-2}\,cm$

$(W_{1,2})_{conf} = \sqrt{2}\,(W_0)_{conf}$

$(W_{1,2})_{conf} = 2.25 \times 10^{-2}\,cm$

$$\frac{W_0}{(W_0)_{conf}} = 1.088$$

$$\frac{W_1}{(W_{1,2})_{conf}} = 1.29$$

$$\frac{W_2}{(W_{1,2})_{conf}} = .81$$

2) Yariv problem 7.3

Each mode looses 0.5 % on each reflection, or 1 % per pass.

Therefore we want 2 % loss (more) per pass for the $TE_{01}$ mode, or 1% per mirror

from figure 7.7, this comes out to

$$\left(\frac{a^2 n}{\lambda d}\right) \approx .9$$

$$a = (\lambda \ell \cdot .9)^{1/2}$$

$$a \approx (10^{-4} \times 16 \times .9)^{1/2}$$

$$a \approx 3.8 \times 10^{-2} \text{ cm}$$

for this aperature, the loss due to diffraction is less than $10^{-3}$ per mirror, so the total loss is $\approx$ 1 % per pass

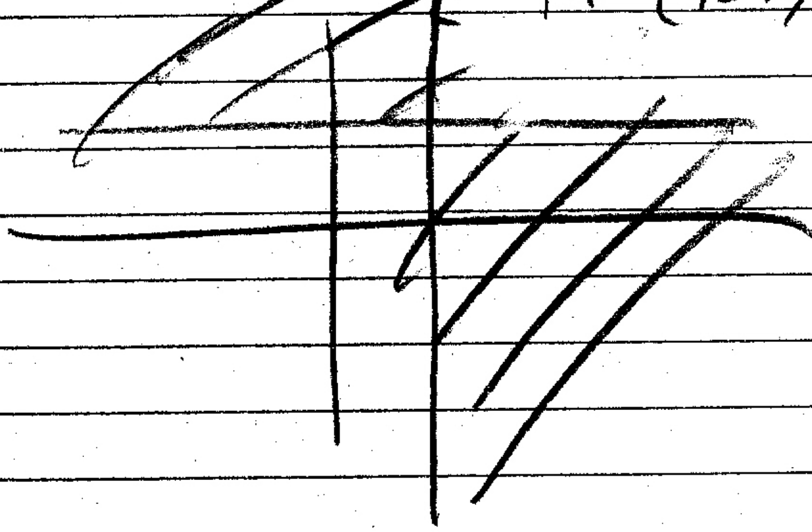
Using a small mirror cause higher losses for the higher order modes

3) Yariv 7.4)

take the inequality

$$0 \le [1-(\ell/R_1)][1-(\ell/R_2)]$$

this gives us $\begin{cases} \text{if } [1-(\ell/R_1)] > 0 \quad [1-\ell/R_2] > 0 \\ \text{if } (1-\ell/R_1) < 0 \quad [1-\ell/R_2] < 0 \end{cases}$



so the above cross checked area are unstable

now Take $[1-\ell/R_1][1-\ell/R_2] \le 1$

take the limiting case $[1-\ell/R_1][1-\ell/R_2] = 1$

$$[1-\ell/R_1] = [1-\ell/R_2]^{-1}$$

$x = \ell/R_1 \qquad y = \ell/R_2$

$$[1-x] = [1-y]^{-1}$$

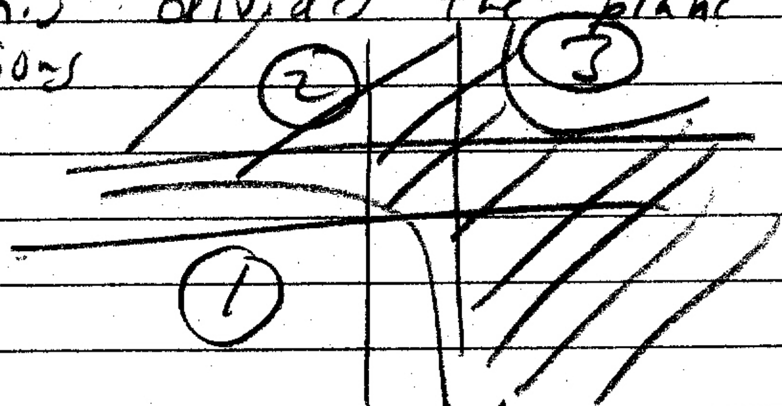
$$1-y=[1-x]^{-1}$$

defining $y'=1-y$
$x'=1-x$

$$y'=\frac{1}{x'}$$

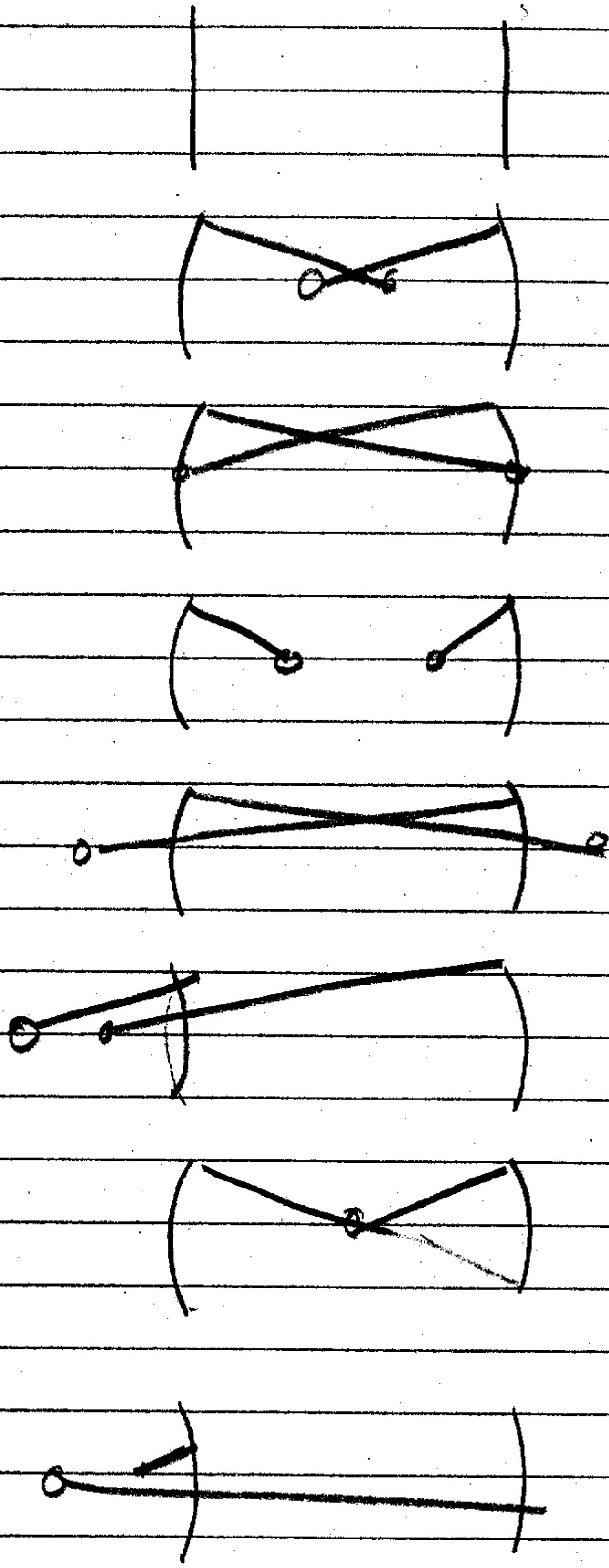
so the limiting cases are a plot of $\frac{1}{x}$ shifted by $(1,1)$

this divides the plane into three regions

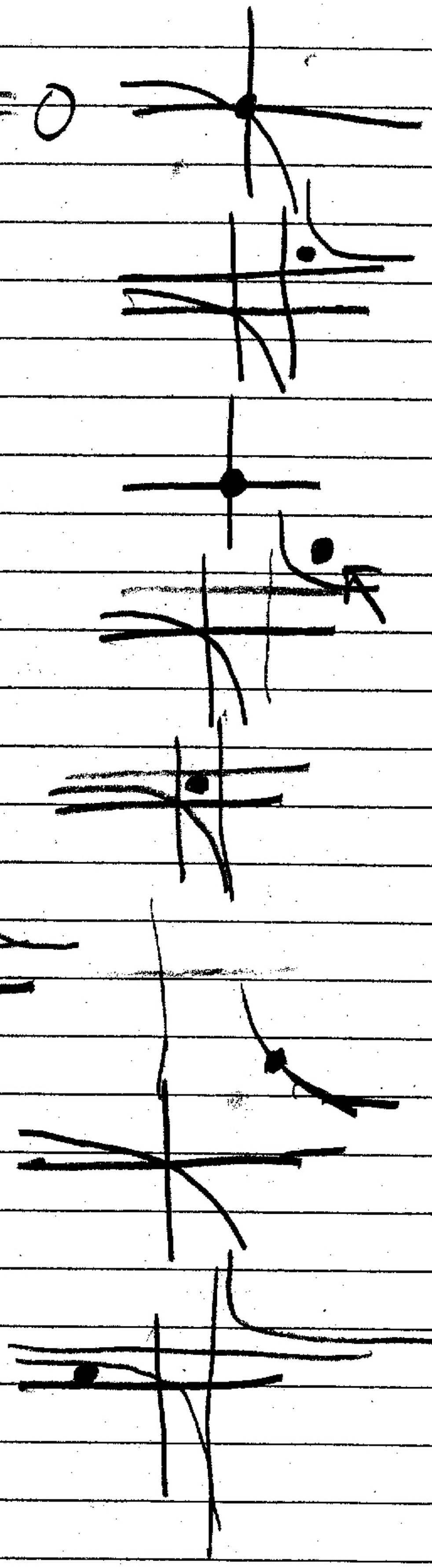


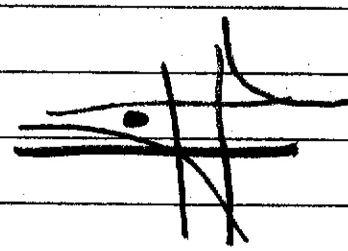

For the case $\frac{l}{R_1}\to 1$ & $\frac{l}{R_2}\to 1$,

this is in the middle of region 2, so region 1 & 3 are unconfined



$$\frac{l}{R_1} - \frac{l}{R_2} = 0$$





4) Yariv 7.7

Replacing any two wavefronts of a gaussian beam results in a two-mirror cavity. To show stability, we must show that there is a value of $q$ which reproduces itself after one round trip

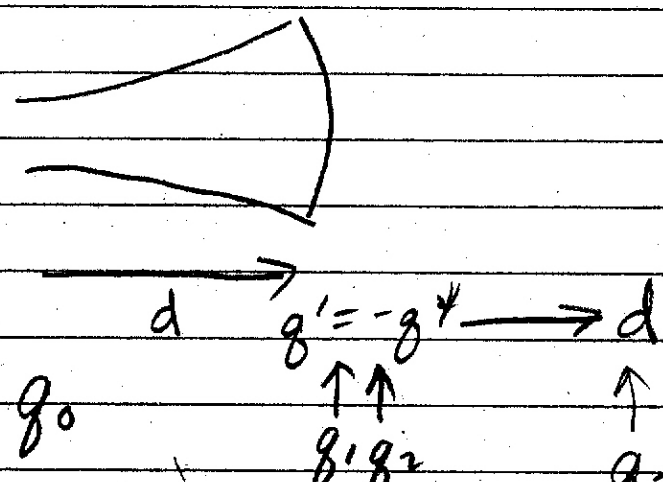
$$q = \frac{Aq+B}{Cq+D} \quad \text{where } \begin{pmatrix} A & B \\ C & D \end{pmatrix} \text{ is for one round trip}$$

To show this, we will break this into two peices $q \to q_- \to q$ where $q'$ is the value of $q$ that is taken on when the solution goes back through the reference plane, but in the opposite direction

we also have $\dfrac{1}{q(z)} = \dfrac{1}{R(z)} - i\dfrac{\lambda}{\pi n w^2(z)}$

When a Gaussian beam hits a mirror with the same curvature $R$, the beam keeps the same beam radius $w^2(z)$, but the radius $R$ changes sign so $q' = -q^*$

so we have




$$q_1 = q_0 + d$$

$$q_2 = -q_0^* - d)$$

$$q_3 = -q_0^*$$

So when the beam returns from the bounce off of the second mirror,

$$q_{\text{round trip}} = -(-q_0^*)^* = q_0$$

so the G.B. we started with is the solution to the cavity, and it is stable