Two mirror resonators

Two mirror resonators are used to make lasers & very narrow filters called Fabry-Perot filters. We can also use them to model resonators with more complex cavities.

If we have two high reflectivity mirrors, we can consider putting a beam of monochromatic light through one of the mirrors. Some light would go through the mirror and start bouncing back and forth.

\[
\begin{align*}
Ae^{-i(wt-kx)} & \xrightarrow{=} Ce^{-i(wt-kx)} \\
\leftarrow Be^{-i(wt+kx)} & \xrightarrow{=} De^{-i(wt+kx)} \\
\end{align*}
\]
and we will define the reflection

\[ r_1' \rightarrow r_2' \]

\[ r_1' \rightarrow t_1 \]

\[ t_1' \rightarrow t_2 \]

We then have

\[ \beta = r, A + t, D \]

\[ C = t, A + r, D \]

\[ E = C e^{-iKz} t_2 \quad (r = 0) \]

\[ D = C e^{-iKz} r_2 e^{+iKz} \]

We can now solve for \( C(A) \)

\[ C = t, A + r, (C e^{-iKz} r_2 e^{+iKz}) \]

\[ A = (1 - r, r, e^{-2iKz}) \frac{1}{t_1} C \]

\[ C = A t_1 \]

\[ \frac{1}{1 - r', r_2 e^{-2iKz}} \]

\[ E = \frac{1}{1 - r', r_2 e^{-2iKz}} \]
So we can see that the amount of energy inside the cavity is a strong function of
\[-\frac{2iKz^2}{e},\text{ especially where } r_1, r_2 \approx 1\]
this equation has maxima where
\[-\frac{2iKz^2}{e} = +1\]
\[\Rightarrow 2Kz^2 = \eta 2\pi\]

Note that \(2Kz^2\) is just the phase change in one round trip in the cavity.

\[\Phi_{\text{one round trip}} = \eta 2\pi\]

(Yariv (7.3))

the round trip phase found for our gaussian beam solution was just

\[2(Kg - (m + n + 1)) \left( \tan^{-1} \frac{r_1}{z} - \tan^{-1} \frac{r_2}{z} \right) \]
\[= \eta 2\pi\]

(Yariv 7.3+3, but for one round trip)
If we let \( k_z = \delta \)

Find the ratio of the intensities

\[
\left( \frac{E}{A} \right) \left( \frac{E'}{A} \right) = \frac{I_i (2\pi)^2}{(4\pi r^4 - 2r^2 \cos \delta)}
\]

where we take \( r_1 = r_2 = r \)

and then plot \( r \) vs \( \delta \)

increasing reflectivity

If \( r \) is very close to 1, this becomes a comb of spikes at \( n(2\pi) \)

A useful measure of the Fabry Perot is the Finesse \((F)\)

\[
F = \frac{\text{Free spectral range}}{\text{Full width at } \frac{1}{2} \text{ maximum}}
\]

For the asymmetric Fabry Perot \( F = \frac{\pi (R_1 R_2)^{1/4}}{1 - (R_1 R_2)^{1/4}} \)

which will be a homework problem
The narrowness of a resonance is related to the / lossiness of a resonator, \( \Rightarrow \) concept of \( Q \) & photon lifetime.

Photon lifetime will be the first of several rate equations which will be important for solving laser resonators.

\[ \text{Small loss of photons} \]

If we reverse time, our equations can represent light trapped in a cavity & slowly leaking out.

The total ratio of photons lost after hitting both mirrors is

\[ (1 - R_1 R_2) \]

We can convert that into a loss of photons per unit time.
by dividing by the time $2\pi d$
(should be group velocity, but assume slow \( n(w) \) for now)

\[
\frac{\text{ratio of photons lost}}{\text{time}} = \frac{1 - R_1 R_2}{2\pi d}
\]

we can then write a rate equation for photon loss:

\[
\frac{dN_p}{dt} = \frac{1 - R_1 R_2}{2\pi d} N_p
\]

which has the solution

\[
N_p(t) = N_p(0) e^{-\frac{t}{\tau_p}}
\]

where \( \tau_p \) is the photon lifetime

\[
\tau_p = \frac{2\pi d/\epsilon}{1 - R_1 R_2} = \frac{\text{round trip time}}{\text{fractional loss per pass}}
\]

Remember the definition of \( Q \):

\[
Q = W_0 \left( \frac{\text{energy stored}}{\text{power lost}} \right)
\]

\[
Q = W_0 \left( \frac{W}{dW/dt} \right)
\]

\[
N_p = Q/W_0 \quad \text{or} \quad Q = W_0 N_p
\]
Note that for optical frequencies \( \Omega \) can become very large (10^8 or larger) and can easily be made larger by simply lengthening the cavity — but the free spectral range goes down — so finesse is the more useful quantity.

If we have a pair of curved mirrors, then the modes are the Hermite Gaussian modes, so we must take our input to be the same H-G mode. What if it isn’t? 

The H-G modes of the resonator are a complete set.

1) Expand the input over the H-G modes.
2) Propagate each of them through the cavity.
3) Recombine at exit.

Going back to the G.D. phase condition:

\[
K, \beta = (n + m + 1) \left( \pi \tan^{-1} \frac{\beta}{2\Omega} - \tan \frac{\beta}{2\Omega} \right) = \theta \pi
\]
The resonance of the cavity will occur at frequencies which correspond to all three indices \( g, m, + n \).

The relative spacing depends on the Gaussian beam shape.

For a confocal resonator, the mode spacing for the transverse mode is \( \frac{\lambda}{2} \) of that for the longitudinal modes.

Reading for next lecture:

Yariv Chapter 8: 8.0 \( \rightarrow \) 8.2.