EE 239

Sp '09

Problem Set 1 Solution

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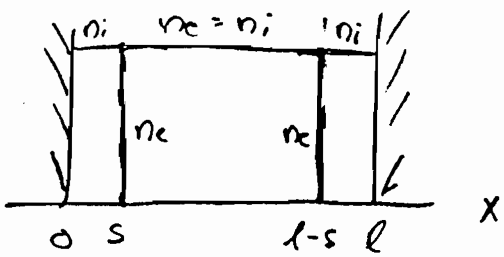
1. Problem 2.2



(a) $$\frac{d^2\Phi}{dx^2} = 0 \qquad s < x < l-s$$

$$= -\frac{e n_0}{\epsilon_0} \qquad 0 < x < s \quad \text{and} \quad l-s < x < l$$

Φ must be symmetric with $\Phi = 0$ at $x = 0, l$.

Hence

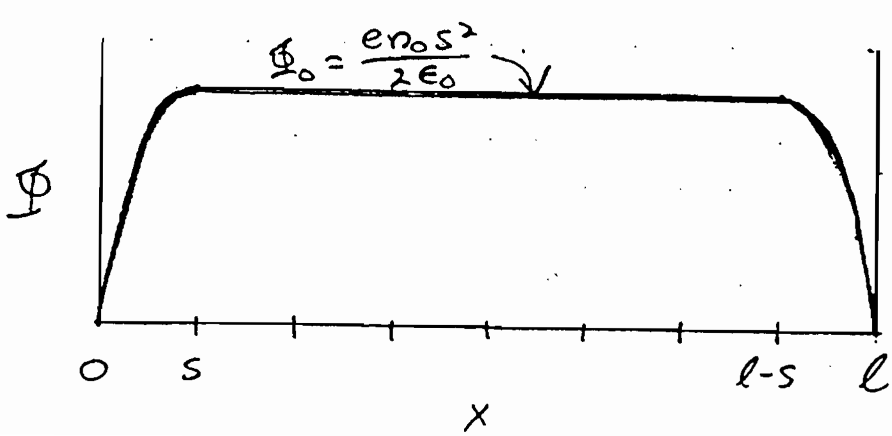
$$\Phi = \Phi_0 \qquad s < x < l-s$$

$$\Phi = \frac{e n_0 s^2}{2\epsilon_0}\left(1 - \left(\frac{s-x}{s}\right)^2\right) \qquad 0 < x < s$$

$$\Phi = \frac{e n_0 s^2}{2\epsilon_0}\left(1 - \left(\frac{x-(l-s)}{s}\right)^2\right) \qquad l-s < x < l$$

At $x = s$ and $x = l-s$, Φ must be continuous

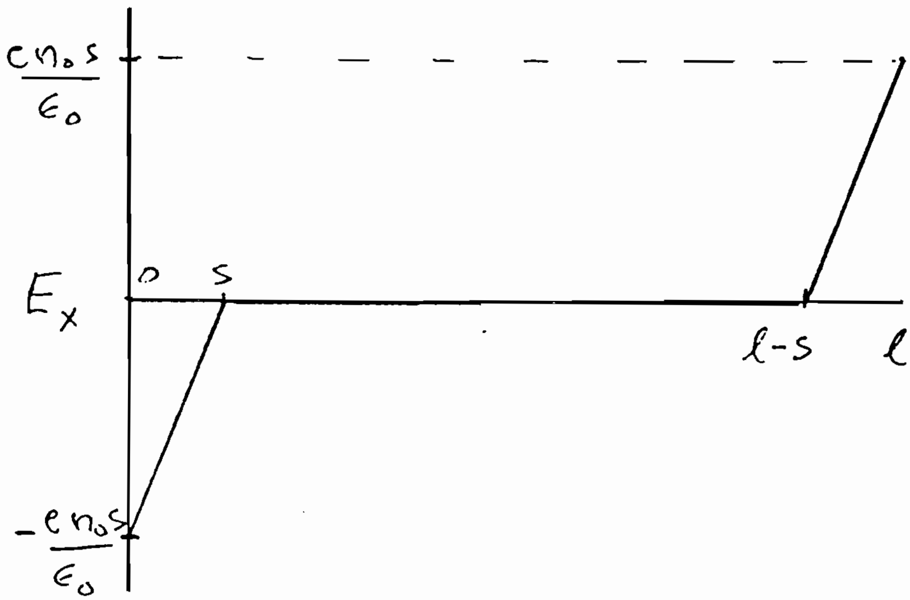
Hence $\Phi_0 = \dfrac{e n_0 s^2}{2\epsilon_0}$ //



(b) $E_x = -\dfrac{d\Phi}{dx} = 0 \,, \qquad s < x < \ell - s$

$= -\dfrac{e n_0}{\epsilon_0}(s - x) \,, \qquad 0 < x < s$

$= \dfrac{e n_0}{\epsilon_0}\{x - (\ell - s)\} \,, \qquad \ell - s < x < \ell$



(c) $4 T_e = \dfrac{e n_0 s^2}{2 \epsilon_0}$

$s^2 = 8 \dfrac{\epsilon_0 T_e}{e n_0} = 8 \lambda_{De}^2$

$s = 2\sqrt{2}\, \lambda_{De}$ //

(3)

2. Problem 2.7

(a)
$$\bar{v}_e = \frac{1}{n_e}\int_0^\infty 4\pi v^2 dv \cdot v\, f_e(v)$$

$$f_e(v) = n_e\left(\frac{m}{2\pi e T_e}\right)^{3/2} e^{-\frac{mv^2}{2eT_e}}$$

let $x = \left(\frac{m}{2eT_e}\right)^{1/2} v$

$$\bar{v}_e = \left(\frac{m}{2\pi e T_e}\right)^{3/2} 4\pi \left(\frac{2eT_e}{m}\right)^2 \underbrace{\int_0^\infty x^3 e^{-x^2} dx}_{1/2}$$

$$= \left(\frac{8eT_e}{\pi m}\right)^{1/2} \quad //$$

(b)
$$\Gamma_e = \int_0^\infty dv_z \int_{-\infty}^\infty dv_x \int_{-\infty}^\infty dv_y\, v_z\, n_e \left(\frac{m}{2\pi e T_e}\right)^{3/2} e^{-\frac{m(v_x^2+v_y^2+v_z^2)}{2eT_e}}$$

$$= \left(\frac{m}{2\pi e T_e}\right)^{1/2} n_e \left(\frac{2eT_e}{m}\right) \underbrace{\int_0^\infty x\, dx\, e^{-x^2}}_{1/2}$$

$$= n_e \left(\frac{eT_e}{2\pi m}\right)^{1/2}$$

$$= \frac{1}{4} n_e \bar{v}_e \quad //$$

(c)
$$S_e = \int_0^\infty dv_z \int_{-\infty}^\infty dv_x \int_{-\infty}^\infty dv_y\, v_z \frac{1}{2} m (v_x^2+v_y^2+v_z^2)\, n_e \left(\frac{m}{2\pi e T_e}\right)^{3/2} e^{-\frac{m(v_x^2+v_y^2+v_z^2)}{2eT_e}}$$

$$= S_{ex} + S_{ey} + S_{ez}$$

$$S_{ex} = \int_0^\infty dv_z \int_{-\infty}^\infty dv_x \int_{-\infty}^\infty dv_y\, v_z \frac{1}{2} m v_x^2\, n_e \left(\frac{m}{2\pi e T_e}\right)^{3/2} e^{-\frac{m(v_x^2+v_y^2+v_z^2)}{2eT_e}}$$

$$= \frac{eT_e}{2} \Gamma_e$$

$$S_{ez} = \left(\frac{m}{2\pi e T_e}\right)^{1/2} n_e \left(\frac{2eT_e}{m}\right)^2 \frac{1}{2} m \underbrace{\int_0^\infty x^3 \, dx \, e^{-x^2}}_{1/2}$$

$$= e T_e \Gamma_e$$

Hence $S_e = 2 e T_e \Gamma_e$ or $W_e = 2 e T_e$ //

Problem 2.8 We now write in place of (2.4119)

$$\frac{d^2\Phi}{dx^2} = \frac{e n_0}{\epsilon_0}\left(e^{\Phi/T_e} - e^{\Phi/T_i}\right).$$

Linearizing, we obtain

$$\frac{d^2\Phi}{dx^2} = \frac{e n_0}{\epsilon_0}\left(\frac{1}{T_e} + \frac{1}{T_i}\right)\Phi$$

yielding the solution ($\Phi \to 0$ as $x \to \infty$)

$$\Phi = \Phi_0 e^{-x/\lambda_D}$$

where $\displaystyle \frac{1}{\lambda_D^2} = \underbrace{\frac{e n_0}{\epsilon_0 T_e}}_{\frac{1}{\lambda_{De}^2}} + \underbrace{\frac{e n_0}{\epsilon_0 T_i}}_{\frac{1}{\lambda_{Di}^2}}$ //

Note for $T_e \gg T_i$, $\lambda_D \approx \lambda_{Di} = \left(\frac{\epsilon_0 T_i}{e n_0}\right)^{1/2}$

Hence λ_D depends on the ions alone.