Sp '09
EE 239

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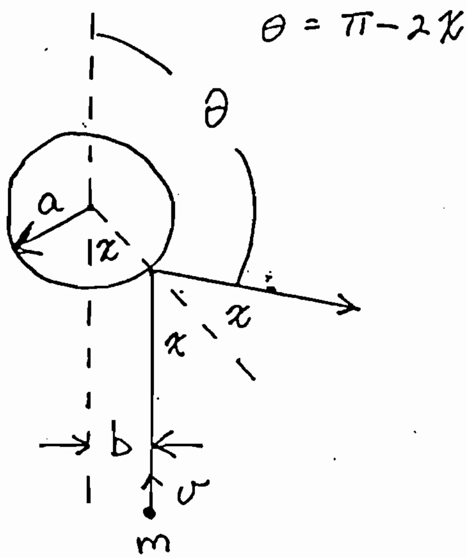
Problem Set 2 Solution

Problem 3.8

(a) $2\pi\, I(\theta)\sin\theta\, d\theta = -2\pi b\, db$

$\dfrac{b}{a} = \sin\chi$ and $\chi = \dfrac{\pi-\theta}{2}$



Hence $b = a\cos\frac{\theta}{2}$ and

$db = -\dfrac{a}{2}\sin\dfrac{\theta}{2}\, d\theta$

$I(\theta) = \dfrac{1}{\sin\theta}\dfrac{db}{d\theta} = a^2/4$ //

(b) $\sigma_{el} = 2\pi\displaystyle\int_0^\pi \sin\theta\, I(\theta)\, d\theta = \pi a^2$ //

$\sigma_m = 2\pi\displaystyle\int_0^\pi \sin\theta\,(1-\cos\theta)\, I(\theta)\, d\theta = \pi a^2$ //

They are the same because the scattering is isotropic.

(c) $n_g = 3.3\times 10^{16} \times 20\times 10^{-3} = 6.6\times 10^{14}$ atoms / cm^3

$a = \alpha_R^{1/3} a_0 = (11.08)^{1/3}(0.53)\times 10^{-8}$ cm $= 1.2\times 10^{-8}$ cm

$\sigma = \pi a^2 = 4.4\times 10^{-16}$ cm^2

Hence $\lambda_{el} = \dfrac{1}{n_g \sigma} = 3.45$ cm //

A 5 V electron has a velocity v given by

$\frac{1}{2} m v^2 = 5\ \mathrm{V} \times 1.6\times 10^{-19}\ \dfrac{\mathrm{J}}{\mathrm{V}}$ or $v = 1.33\times 10^{6}$ m/sec

or $v = 1.33\times 10^{8}$ cm/sec.

$\nu_{el} = n_g \sigma v = \dfrac{v}{\lambda_{el}} = 3.86\times 10^{7}$ sec^{-1}. //

Problem 2.

$$K = \left(\frac{m}{2\pi e T_e}\right)^{3/2} 4\pi \int_0^{\infty} v^2\, dv \cdot \sigma(v)\, v\, e^{-mv^2/2eT_e}$$

Let $\quad \mathcal{E} = \frac{1}{2}\frac{mv^2}{e} \qquad d\mathcal{E} = \frac{m}{e}\, v\, dv$

$\quad v^2 = \frac{2e\mathcal{E}}{m}$

$$K = \left(\frac{m}{2\pi e T_e}\right)^{3/2} \cdot 4\pi\, \frac{2e}{m}\,\frac{e}{m}\,\sigma_0 \underbrace{\int_{\mathcal{E}_0}^{\mathcal{E}_1} \mathcal{E}\, d\mathcal{E}\, e^{-\mathcal{E}/T_e}}_{-T_e^2\, e^{-\mathcal{E}/T_e}\left(1+\frac{\mathcal{E}}{T_e}\right)\Big|_{\mathcal{E}_0}^{\mathcal{E}_1}}$$

$$K = \left(\frac{2\pi e T_e}{m}\right)^{1/2} \frac{\cancel{m^2}}{\cancel{4}\pi^{\cancel{2}}\cancel{e^2}\cancel{T_e^2}} \cdot \cancel{4\pi}\, \frac{2e^2}{\cancel{m^2}}\,\sigma_0\, \cancel{T_e^2} \left[\left(1+\frac{\mathcal{E}}{T_e}\right) e^{-\mathcal{E}/T_e}\right]_{\mathcal{E}_1}^{\mathcal{E}_0}$$

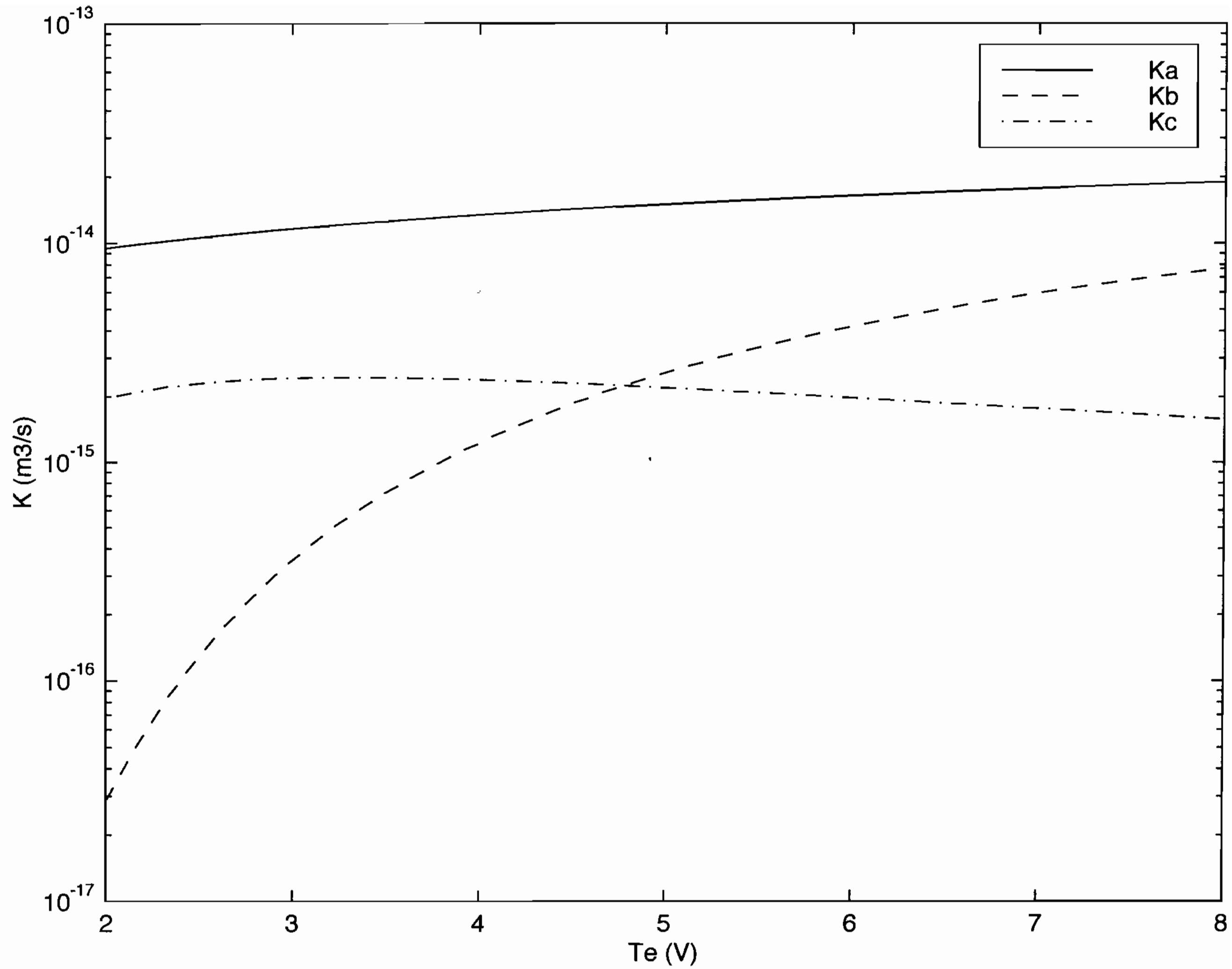
$$\boxed{K(T_e) = \left(\frac{8 e T_e}{\pi m}\right)^{1/2} \sigma_0 \left[\left(1+\frac{\mathcal{E}}{T_e}\right) e^{-\mathcal{E}/T_e}\right]_{\mathcal{E}_1}^{\mathcal{E}_0}}$$

For (a), $\mathcal{E}_0 = 0$, $\mathcal{E}_1 = \infty$

For (b), $\mathcal{E}_0 = 16$, $\mathcal{E}_1 = \infty$

For (c), $\mathcal{E}_0 = 4$, $\mathcal{E}_1 = 6$

(3)



Problem 3.15

$$\begin{cases} m_A v_A = (m_A + m_B) v_{AB*} & \text{mom. cons.} \\ \frac{1}{2} m_A v_A^2 = \frac{1}{2}(m_A + m_B) v_{AB*}^2 + e\,\mathcal{E}_{ex} & \text{energy cons.} \end{cases}$$

$$v_{AB*} = \frac{m_A}{m_A + m_B} v_A \quad //$$

use in energy cons to get

$$\frac{1}{2} m_A v_A^2 = \frac{1}{2}(m_A + m_B) \frac{m_A^2}{(m_A + m_B)^2} v_A^2 + e\,\mathcal{E}_{ex}$$

$$e\,\mathcal{E}_{ex} = \frac{1}{2} m_R v_A^2 \quad // \qquad m_R = \frac{m_A m_B}{m_A + m_B}$$

For $v_A > 0$, then $e\,\mathcal{E}_{ex} > 0$.

Hence two bodies can not collide elastically to form one body.