1. Problem Set 4 Solution (Revised)

(a) Let \( D = \frac{\partial}{\partial x} \), \( N = \frac{\partial}{\partial \nu} \), \( D_n = \frac{\partial}{\partial \nu} \).

\[
\Phi(x) = -D_n \frac{\partial n}{\partial x} = -D_n n_0 \frac{\pi x}{\ell} 
\]

\[
\Phi(x) = \frac{\partial}{\partial \nu} \cos \frac{\pi x}{\ell} \left( \frac{\pi}{\ell} \sin \frac{\pi x}{\ell} \right) = \frac{\pi}{\ell} \ell \tan \frac{\pi x}{\ell}
\]

\[
\frac{d\Phi}{dx} = -E \Rightarrow \Phi(x) = \frac{Te}{\ell} \ln \cos \frac{\pi x}{\ell}
\]

\[
p = \frac{e_0}{\ell} \frac{d\Phi}{dx} = \frac{e_0}{\ell} \left( \frac{\pi}{\ell} \right) \frac{Te}{\ell} \sec^2 \frac{\pi x}{\ell} 
\]

\[
\frac{\rho}{\ell} = n_0 \left( \frac{\pi \lambda_{BE} e_0}{\ell} \right)^2 \frac{e_0^2}{\ell^2} 
\]

Ambipolar solution assumes \( N \approx 1 \); hence \( \rho/e < n \) for solution to be valid.

(b)

\[
\text{Graph of } p/e 
\]

(c) Equate \( n_0 \) to \( \rho/e \) at \( x = s/2 \) and solve for \( s \)

\[
x = \frac{s}{2} \Rightarrow \sec \frac{\pi x}{\ell} = \frac{1}{\sin \frac{\pi S}{\ell}} \approx \frac{\ell}{\pi S} \cos \frac{\pi x}{\ell} \approx \frac{s}{\ell} 
\]

\[
n_0 \frac{s}{\pi \ell} = n_0 \left( \frac{\pi \lambda_{BE} e_0}{\ell} \right)^2 \frac{e_0^2}{\ell^2} \Rightarrow s = \left( \frac{\lambda_{BE} e_0}{\pi} \right) \frac{1}{\ell} \]

(d) \( n_s^2 = \frac{n_0 \cos \frac{\pi x}{\ell}}{\ell} = \frac{n_0 \cos \frac{\pi (\frac{s}{2} - s)}{\ell}}{\ell} = \frac{n_0 \sin \frac{\pi s}{\ell}}{\ell} = n_0 \frac{\pi s}{\ell} \)

\[
\lambda_{BE}^2 = \frac{\epsilon_0 \frac{Te}{\ell}}{\pi n_s} \Rightarrow \lambda_{BE}^2 = \frac{\pi s}{\ell} \frac{\lambda_{BE}^2}{\ell} \]

\[
S^3 = \frac{\lambda_{BE}^2}{\pi} \text{ from (c)} \Rightarrow S^3 = \frac{\pi s^2 \ell}{\lambda_{BE}^2} \Rightarrow S = \lambda_{BE} \]
\( u_s = \frac{D_0 \pi \tan \frac{\pi}{2} (\frac{L}{s} - s)}{e} = \frac{\mu_i \mu_0 \pi}{L} \frac{1}{t} \tan \frac{\pi s}{L} \frac{L}{t} \frac{L}{\pi s} \)

\( u_s = \frac{\mu_i \mu_0}{s} = \frac{e \lambda_1 s \mu_0}{M u_s} = u_B \frac{1}{u_s} \frac{\lambda_1 s}{\lambda_2} \frac{1}{d s} \)

\( u_s = u_B \left( \frac{\lambda_1 s}{\lambda_2 d s} \right)^{1/2} \)
2. (a) \(-D_a \nabla^2 n_e = \nabla \cdot \nu_{iz} n_e\;
abla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\)

Let \(n_e = X(x) Y(y) Z(z)\) and use boundary conditions that \(n_e = 0\) at \(x = \pm \frac{a}{2}\), \(y = \pm \frac{b}{2}\) and \(z = \pm \frac{c}{2}\) to obtain \(n_e = N_0 \cos \frac{\pi x}{a} \cos \frac{\pi y}{b} \cos \frac{\pi z}{c}\)

(b) Put this \(n_e\) into the above diffusion eq!

\[D_a \left( \frac{\pi^2}{a^2} + \frac{\pi^2}{b^2} + \frac{\pi^2}{c^2} \right) = \nu_{iz}\]

(c) For wall at \(x = \frac{a}{2}\), \(\Gamma_x = -D_a \frac{\partial n_e}{\partial x}\)

\[\Gamma_x \left( \frac{a}{2} \right) = D_a \frac{\pi}{a} N_0 \cos \frac{\pi y}{b} \cos \frac{\pi z}{c}\]

The flux at \(x = -\frac{a}{2}\) is the negative of the above.

Similarly,

\[\Gamma_y \left( \frac{b}{2} \right) = D_a \frac{\pi}{a} N_0 \cos \frac{\pi x}{a} \cos \frac{\pi z}{c}\]

\[\Gamma_z \left( \frac{c}{2} \right) = D_a \frac{\pi}{a} N_0 \cos \frac{\pi x}{a} \cos \frac{\pi y}{b}\]

with the negative of these at \(y = -\frac{b}{2}\) and \(z = -\frac{c}{2}\).

(d) Integrate \(\Gamma_x\) over the face at \(x = \frac{a}{2}\) to get

\[I_x = D_a N_0 \frac{\pi}{a} \frac{2b}{\pi} \frac{2c}{\pi} = D_a N_0 \frac{4bc}{\pi a}\]

Similarly, \(I_y = D_a N_0 \frac{4ac}{\pi b}\) and \(I_z = D_a N_0 \frac{4ab}{\pi c}\)

\[I_{total} = 2(I_x + I_y + I_z) = D_a N_0 \frac{8abc}{\pi^3} \left( \frac{\pi^2}{a^2} + \frac{\pi^2}{b^2} + \frac{\pi^2}{c^2} \right)\]

(e) \(I_{total} = \nu_{iz} \int d\xi \int d\eta \int d\zeta n_e(x,\eta,\zeta)\)

\[-\frac{a}{2} \leq \xi \leq \frac{a}{2}, \quad \frac{b}{2} \leq \eta, \quad \frac{c}{2} \leq \zeta\]

\[I_{total} = \nu_{iz} N_0 \left( \frac{2a}{\pi} \right) \left( \frac{2b}{\pi} \right) \left( \frac{2c}{\pi} \right)\]

Using (b), we see that (d) and (e) give the same \(I_{total}\), as they must.