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EE239

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Problem Set 4 Solution (Revised)

1. Problem 5.6

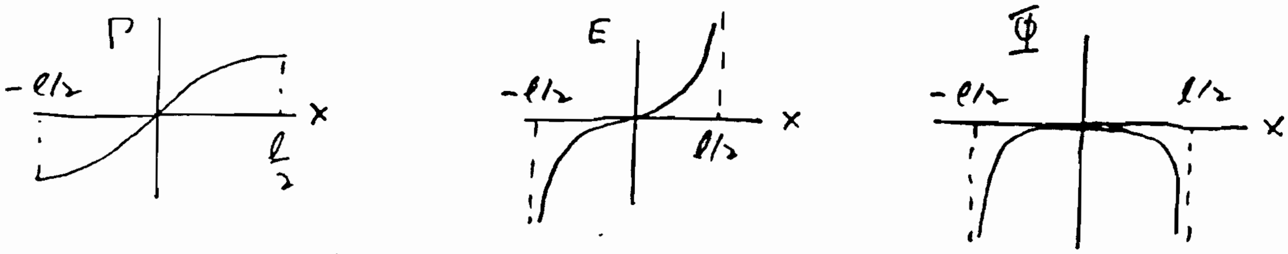
(a) Let $D_i \ll D_e$, $\mu_i \ll \mu_e$, $D_a = \mu_i T_e$

$$\Gamma(x) = -D_a \frac{dn}{dx} = D_a n_0 \frac{\pi}{\ell} \sin \frac{\pi x}{\ell}$$

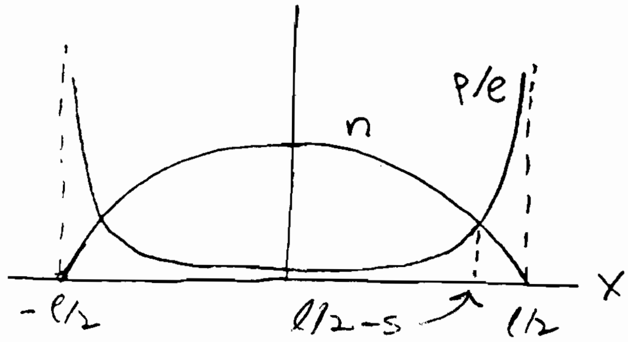
$$E(x) = \frac{D_i - D_e}{\mu_i + \mu_e} \frac{1}{n} \frac{dn}{dx} = -T_e \frac{1}{\cos \frac{\pi x}{\ell}} \left(-\frac{\pi}{\ell} \sin \frac{\pi x}{\ell}\right) = \frac{\pi}{\ell} T_e \tan \frac{\pi x}{\ell}$$

$$\frac{d\Phi}{dx} = -E \Rightarrow \Phi(x) = T_e \ln \cos \frac{\pi x}{\ell}$$

$$\rho = \epsilon_0 \frac{dE}{dx} = \epsilon_0 \left(\frac{\pi}{\ell}\right)^2 T_e \sec^2 \frac{\pi x}{\ell} \;; \quad \rho/e = n_0 \left(\frac{\pi \lambda_{De}}{\ell}\right)^2 \sec^2 \frac{\pi x}{\ell}$$



(b)



Ambipolar solution assumes $n_e \approx n_i$; hence $\rho/e \ll n$ for solution to be valid.

(c) Equate n_0 to ρ/e at $x = s$ and solve for s

$$x = \frac{\ell}{2} - s \Rightarrow \sec \frac{\pi x}{\ell} = \frac{1}{\sin \frac{\pi s}{\ell}} \approx \frac{\ell}{\pi s} \;; \quad \cos \frac{\pi x}{\ell} \approx \frac{s}{\pi \ell}$$

$$n_0 \frac{s}{\pi \ell} = n_0 \left(\frac{\pi \lambda_{De}}{\ell}\right)^2 \frac{\ell^2}{\pi^2 s^2} \Rightarrow s = \left(\frac{\lambda_{De}^2 \ell}{\pi}\right)^{1/3}$$

(d) $$n_s = n_0 \cos \frac{\pi x}{\ell} = n_0 \cos \frac{\pi}{\ell}\left(\frac{\ell}{2} - s\right) = n_0 \sin \frac{\pi s}{\ell} \approx n_0 \frac{\pi s}{\ell}$$

$$\lambda_{Ds}^2 = \frac{\epsilon_0 T_e}{e n_s} = \lambda_{De}^2 \frac{\ell}{\pi s} \Rightarrow \lambda_{De}^2 = \frac{\pi s}{\ell} \lambda_{Ds}^2$$

$$s^3 = \frac{\lambda_{De}^2 \ell}{\pi} \text{ from (c)} \Rightarrow s^3 = \frac{\pi s}{\ell} \lambda_{Ds}^2 \frac{\ell}{\pi} \Rightarrow s = \lambda_{Ds} \,//$$

(e) $$u_s = \frac{D_a \pi}{\ell} \tan \frac{\pi\left(\frac{\ell}{2}-s\right)}{\ell} = \frac{\mu_i T_e \pi}{\ell} \frac{1}{\tan \frac{\pi s}{\ell}} \approx \frac{\mu_i T_e \cancel{\pi}}{\cancel{\ell}} \frac{\cancel{\ell}}{\cancel{\pi} s}$$

$$u_s = \frac{\mu_i T_e}{s} = \frac{e \lambda_{is}}{M u_s} \frac{T_e}{s} = u_B^2 \frac{1}{u_s} \frac{\lambda_{is}}{\lambda_{Ds}}$$

$$u_s = u_B \left(\frac{\lambda_{is}}{\lambda_{Ds}}\right)^{1/2}$$

2.(a) $-D_a \nabla^2 n_e = \nu_{iz} n_e$; $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

Let $n_e = X(x)\, Y(y)\, Z(z)$ and use boundary conditions that $n_e = 0$ at $x = \pm a/2$, $y = \pm b/2$ and $z = \pm c/2$ to obtain

$$n_e = n_0 \cos\frac{\pi x}{a} \cos\frac{\pi y}{b} \cos\frac{\pi z}{c} \qquad //$$

(b) Put this n_e into the above diffusion eq:

$$D_a\left(\frac{\pi^2}{a^2} + \frac{\pi^2}{b^2} + \frac{\pi^2}{c^2}\right) = \nu_{iz} \qquad //$$

(c) For wall at $x = \frac{a}{2}$, $\Gamma_x = -D_a \frac{\partial n_e}{\partial x}\Big|_{x=a/2}$

$$\Gamma_x\left(\frac{a}{2}\right) = D_a \frac{\pi}{a} n_0 \cos\frac{\pi y}{b} \cos\frac{\pi z}{c} \qquad //$$

The flux at $x = -\frac{a}{2}$ is the negative of the above //

Similarly, $\Gamma_y\left(\frac{b}{2}\right) = D_a \frac{\pi}{b} n_0 \cos\frac{\pi x}{a} \cos\frac{\pi z}{c}$ //

$\Gamma_z\left(\frac{c}{2}\right) = D_a \frac{\pi}{c} n_0 \cos\frac{\pi x}{a} \cos\frac{\pi y}{b}$ //

with the negative of these at $y = -\frac{b}{2}$ and $z = -\frac{c}{2}$ //

(d) Integrate Γ_x over the face at $x = a/2$ to get

$$I_x = D_a n_0 \frac{\pi}{a} \frac{2b}{\pi} \cdot \frac{2c}{\pi} = D_a n_0 \frac{4bc}{\pi a}$$

Similarly $I_y = D_a n_0 \frac{4ac}{\pi b}$; $I_z = D_a n_0 \frac{4ab}{\pi c}$

$$I_{total} = 2(I_x + I_y + I_z) = D_a n_0 \frac{8abc}{\pi^3}\left(\frac{\pi^2}{a^2} + \frac{\pi^2}{b^2} + \frac{\pi^2}{c^2}\right) \qquad //$$

(e) $$I_{total} = \nu_{iz} \int_{-a/2}^{a/2} dx \int_{-b/2}^{b/2} dy \int_{-c/2}^{c/2} dz\, n_e(x,y,z)$$

$$I_{total} = \nu_{iz} n_0 \frac{2a}{\pi} \cdot \frac{2b}{\pi} \cdot \frac{2c}{\pi} \qquad //$$

Using (b), we see that (d) and (e) give the same I_{total}, as they must. //