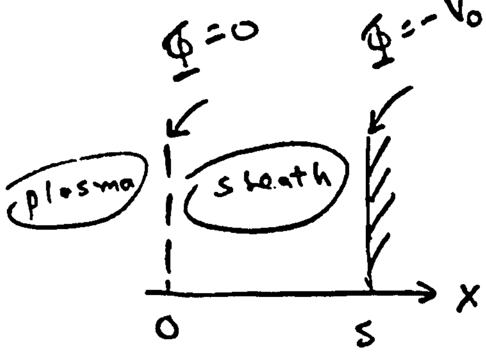
EE 239

M A Lieberman

Problem Set 5 Solution

Problem 6.4



(a) $\left.\begin{array}{l} v = \mu_i E \\ J_0 = env \end{array}\right\} \quad n = \dfrac{J_0}{ev} = \dfrac{J_0}{e\mu_i E}$

use $\dfrac{dE}{dx} = \dfrac{en}{\epsilon_0} = \dfrac{J_0}{\epsilon_0 \mu_i E}$

Integrate with b.c. that $E = 0$ at $x = 0$:

$$\frac{E^2}{2} = \frac{J_0}{\epsilon_0 \mu_i} x$$

Hence $E(x) = \left(\dfrac{2J_0}{\epsilon_0 \mu_i}\right)^{1/2} x^{1/2} = \dfrac{dV}{dx} \qquad (V = -\Phi)$

Integrate again to get $V(x)$ with b.c. that $V = 0$ at $x = 0$:

$$V(x) = \left(\frac{2J_0}{\epsilon_0 \mu_i}\right)^{1/2} \frac{x^{3/2}}{3/2}$$

At $x = s$, $V = V_0$

$$V_0 = \left(\frac{2J_0}{\epsilon_0 \mu_i}\right)^{1/2} \frac{2}{3} s^{3/2}$$

Solve for J_0: $\quad J_0 = \dfrac{9}{8} \epsilon_0 \mu_i \dfrac{V_0^2}{s^3}$ //

(b) $\lambda_i = \dfrac{1}{330(10)}$ cm $= 3.03 \times 10^{-4}$ cm

$u_B = \left(\dfrac{e T_e}{M}\right)^{1/2} = 2.19 \times 10^3$ m/s at $T_e = 2$ V

$\nu_{mi} = u_B / \lambda_i = 7.22 \times 10^8$ s^{-1}

$\mu_i = \dfrac{e}{M \nu_{mi}} = 3.31 \times 10^{-3}$ m^2/V-s

Now $n_s = 10^{15}$ m^{-3} so $\lambda_{De} = 740\sqrt{\dfrac{T_e}{n_s}} = 4.05 \times 10^{-2}$ cm

From (6.2.20), $u_s = \dfrac{u_B}{\left(1 + \dfrac{\pi \lambda_{De}}{2\lambda_i}\right)^{1/2}} = 1.51 \times 10^2$ m/s

$J_0 = e n_s u_s = 0.0241$ A/m^2

$s = \left(\dfrac{9}{8}\epsilon_0 \mu_i \dfrac{V_0^2}{J_0}\right)^{1/3} = 2.24 \times 10^{-3}$ m $= 0.224$ cm

For a collisionless Child law sheath

$s_{CL} = \left(\dfrac{4}{9}\epsilon_0 \left(\dfrac{2e}{M}\right)^{1/2} \dfrac{V_0^{3/2}}{J_0}\right)^{1/2} = 1.02 \times 10^{-2}$ m $= 1.02$ cm

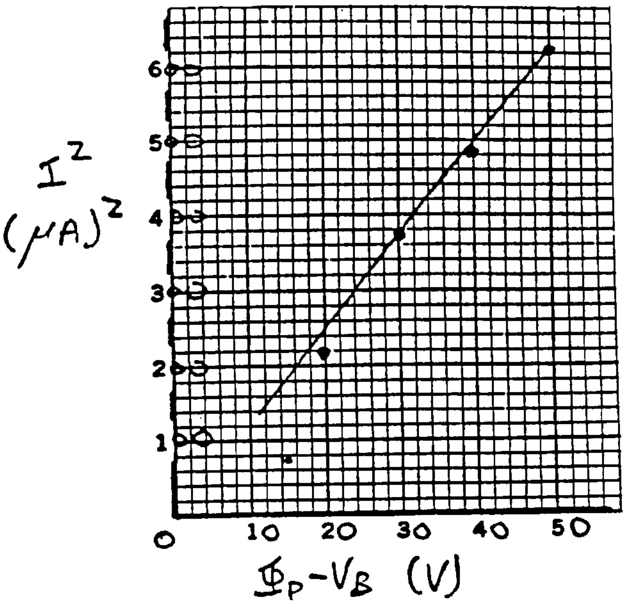
NOTE: If one does not use (6.2.20) but takes $J_0 = e n_s u_B = 0.350$ A/m^2

Then $s = 0.098$ cm and

$s_{CL} = 0.496$ cm

Problem 6.7

(a) Graphing I^2 $(\mu A)^2$ versus $\Phi_p - V_B$, we obtain



$$I_i^2 \approx 12.5 \times 10^{-12} (\Phi_p - V_B) \ \text{A}^2$$

Using (6.6.29) with

$A_p = 2\pi a d \approx 1.98 \times 10^{-6}$ m^2

and $M = 6.7 \times 10^{-26}$ kg (Ar),

we obtain $n_s \approx 1.6 \times 10^{16}$ m^{-3} //

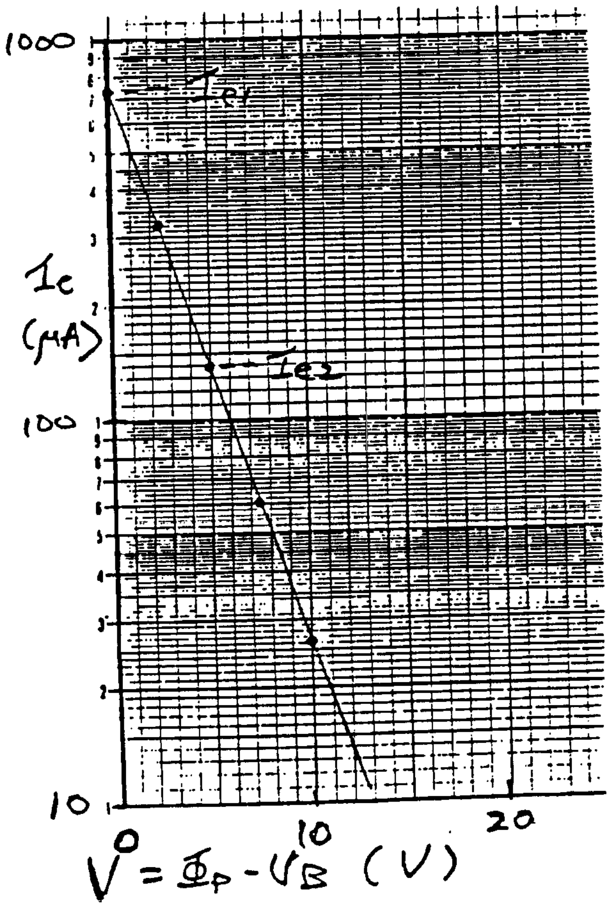
(b) $$\frac{1}{T_e} = -\frac{\ln I_{e1} - \ln I_{e2}}{V_1 - V_2}$$

$T_e \approx 3.0$ V //

At $V = 0$, $I_e = \frac{1}{4} e n_s \bar{v}_e A_p$

$$\bar{v}_e = \left(\frac{8 e T_e}{\pi m}\right)^{1/2} \approx 1.16 \times 10^6 \text{ m/s}$$

$\Rightarrow n_s \approx 0.8 \times 10^{16}$ m^{-3} //



As described in the text (p. 195), the ion saturation current tends to overestimate the density.

I_e	0	0.4	0.1	1.0	5.0	26.2	60.2	138.9	318.6	733
I_i	−25	−22.4	−19.4	−15.8	−13.7	−11.2	−9.7	−7.9	−5.6	0
$I(\mu A)$	−25	−22	−19.3	−14.8	−8.7	15	50.5	131	313	733
V_B(V)	−20	−10	0	10	15	20	22.5	25	27.5	30
$\Phi_p - V_B$	50	40	30	20	15	10	7.5	5	2.5	0

Problem 3

From (6.5-5) and (6.5-6) evaluated at $x = s$, we obtain the electric field E at the wall

$$E = \frac{5}{3}\frac{V_0}{s} \qquad (1)$$

The ion velocity at the wall is

$$u_i = \mu_i E$$

where μ_i is given in (6.5.2)

$$u_i = \frac{2e\lambda_i}{\pi M u_i} E \qquad (2)$$

From (1) and (2)

$$u_i^2 = \frac{2e\lambda_i}{\pi M}\frac{5}{3}\frac{V_0}{s}$$

Hence

$$\mathcal{E}_{ic} = \frac{1}{2} M u_i^2 / e = \frac{5}{3\pi}\frac{\lambda_i}{s} V_0 \quad //$$