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Problem Set 7 Solution

1. Problem 8.8

$$\sigma_{rec} = \pi \left(\frac{e}{4\pi\epsilon_0}\right)^2 \frac{1}{\mathcal{E}\,\mathcal{E}_{affA}}$$

$\mathcal{E}_{affA} = 1.463$ V for O.

$$K = 4\pi \int_0^\infty v^3\,dv\, f_m(v)\, \sigma_{rec}(v)$$

$$f_m = \left(\frac{m_R}{2\pi e \mathrm{T}}\right)^{3/2} e^{-\frac{m_R v^2}{2e\mathrm{T}}}$$

Let $\mathcal{E} = \frac{m_R}{2e} v^2$; $d\mathcal{E} = \frac{m_R}{e} v\,dv$

$$K = 4\pi \cdot \left(\frac{m_R}{2\pi e \mathrm{T}}\right)^{3/2} \frac{2e}{m_R}\frac{e}{m_R}\, \pi\left(\frac{e}{4\pi\epsilon_0}\right)^2 \frac{1}{\mathcal{E}_{affA}} \underbrace{\int_0^\infty \mathcal{E}\, d\mathcal{E} \frac{1}{\mathcal{E}} e^{-\mathcal{E}/\mathrm{T}}}_{\mathrm{T}}$$

$m_R = \frac{16 \cdot 32}{16+32} = 10.7$ amu $= 1.78\times10^{-26}$ kg

$e\mathrm{T} = kT$; k = Boltzmann's constant $= 1.38\times10^{-23}$ J/K

$$\Rightarrow K = 1.32\times10^{-13}\left(\frac{300}{T}\right)^{1/2} \ \mathrm{m^3/s}$$

$$= 1.32\times10^{-7}\left(\frac{300}{T}\right)^{1/2} \ \mathrm{cm^3/s}$$

Table 8.2 gives

$$K = 1\text{–}2 \times 10^{-7}\left(\frac{300}{T}\right)^{1/2}$$

2. Problem 8.9

$$K = 4\pi \int_{\left(\frac{2e\mathcal{E}_a}{m_A}\right)^{1/2}}^{\infty} v_R^3\, \sigma(v_R) \left(\frac{m_R}{2\pi kT}\right)^{3/2} e^{-\frac{m_A v_R^2}{2kT}}\, dv_R$$

where

$$\frac{1}{2} m_A v_R^2 = \frac{1}{2} m_R' v_R'^2 + e\mathcal{E}_a$$

Hence $\quad m_R v_R\, dv_R = m_R' v_R'\, dv_R'$

$$K = 4\pi \frac{m_R'}{m_R} \left(\frac{m_R}{2\pi kT}\right)^{3/2} e^{-\frac{e\mathcal{E}_a}{kT}} \int_0^{\infty} v_R'\, dv_R'\, \underbrace{v_R^2\, \sigma(v_R)}_{\frac{g_C g_D}{g_A g_B} \frac{m_R'^2}{m_R^2} v_R'^2 \sigma(v_R')} e^{-\frac{m_R' v_R'^2}{2kT}}$$

$$K = 4\pi \frac{m_R'}{m_R} \left(\frac{m_R}{2\pi kT}\right)^{3/2} \frac{m_R'^2}{m_R^2} \frac{g_C g_D}{g_A g_B} e^{-\mathcal{E}_a/T} \int_0^{\infty} v_R'^3\, \sigma(v_R')\, e^{-\frac{m_R' v_R'^2}{2kT}}\, dv_R'$$

$$K = \left(\frac{m_R'}{m_R}\right)^{3/2} \frac{g_C g_D}{g_A g_B} e^{-\mathcal{E}_a/T} K' \qquad //$$

3. Problem 8.10

(a)
$$e + B \underset{K_q}{\overset{K_{ex}}{\rightleftarrows}} e + B^*$$

$$\frac{K_{ex}}{K_q} = \frac{\bar{g}_*}{\bar{g}_B} e^{-\mathcal{E}/T_e}$$

$$K_q = \frac{\bar{g}_B}{\bar{g}_*} K_{ex} e^{+\mathcal{E}/T_e}$$

$$K_q = \frac{\bar{g}_B}{\bar{g}_*} K_0 e^{(\mathcal{E}-\mathcal{E}_0)/T_e}$$

(b) From Table 8.1 $\mathcal{E} = 1.97$ V

$B = {}^3P$ $\quad * = {}^1D$

$g_B = (2L+1)(2S+1) = 3 \cdot 3 = 9$

$g_* = (2L+1)(2S+1) = 5 \cdot 1 = 5$

From Table 8.2 reaction #23

$$K_{ex} = 4.54 \times 10^{-9} \exp(-2.36/T_e)$$

$$\Rightarrow K_q = \frac{9}{5} 4.54 \times 10^{-9} \exp \frac{1.97 - 2.36}{T_e} \text{ cm}^3/\text{s}$$

$$= 8.17 \times 10^{-9} e^{-0.39/T_e} \text{ cm}^3/\text{s}$$

checks with reaction #24 ✓

(c) Yes, since K_{ex}/K_q is set by thermodynamics so if K_{ex} is correct, then K_q is also