EE 239 sp '09 M A Lieberman

Problem Set 9 Solution

Problem 1

(a) $\lambda_i = 3.0 \times 10^{-3}$ m, $h_\ell = 0.19$
and $h_R = 0.096$ give
$d_{eff} = 0.206$ m, $n_g d_{eff} = 6.8 \times 10^{19}$ m^{-2}
From Fig 10.1, $T_e \approx 2.5$ V

(b) From Fig 3.17, $\mathcal{E}_c \approx 82$ V
$\mathcal{E}_{ih} = 400 + \frac{T_e}{2} = 401$ V
$\mathcal{E}_{i\ell} = 5.2\, T_e = 12.9$ V
$u_B = 2.4 \times 10^3$ m/s

(c) $$P_{abs} = e n_0 u_B \left[(2\pi R \ell h_R + \pi R^2 h_\ell)(\mathcal{E}_c + 2T_e + \mathcal{E}_{i\ell}) + \pi R^2 h_\ell (\mathcal{E}_c + 2T_e + \mathcal{E}_{ih}) \right]$$

$$\Rightarrow \quad n_0 = 3.3 \times 10^{17} \text{ m}^{-3}$$

$$\Gamma_{is} = n_0 u_B h_\ell = 1.6 \times 10^{20} \text{ m}^{-2}\text{-s}^{-1}$$

2. Problem 10.13

(a) $$V \frac{dn_O}{dt} = -\Gamma_O \, 2A\, p_{rec} \, ; \quad V = Al \text{ and } \Gamma_O = \frac{1}{4} n_O \bar{v}_O$$

$$\frac{dn_O}{dt} = -\frac{1}{4} \bar{v}_O \frac{2p_{rec}}{l} = \underbrace{\frac{\bar{v}_O \, p_{rec}}{2l}}_{K_{loss}} n_O$$

$$\left.\begin{array}{l} \bar{v}_O = 6.3 \times 10^4 \text{ cm/s} \\ 2l = 20 \text{ cm} \\ p_{rec} = 10^{-4} \end{array}\right\} \Rightarrow K_{loss} = 0.315 \ s^{-1}$$

$$K_{loss} \ll K_O$$

(b) $$\frac{dn_O}{dt} = 2K_3 n_e n_{O_2} + K_{11} n_e n_{O_2} - K_{12} n_e n_O - K_O n_O = 0$$

$$\frac{dn_{O_2^+}}{dt} = K_4 n_e n_{O_2} - K_{O_2^+} n_{O_2^+} = 0$$

$$\frac{dn_{O^+}}{dt} = K_{11} n_e n_{O_2} + K_{12} n_e n_O - K_{O^+} n_{O^+} = 0$$

(c) From 1st equation

$$\frac{n_O}{n_{O_2}} = \frac{(2K_3 + K_{11}) n_e}{K_{12} n_e + K_O} \quad //$$

For $K_{12} n_e \gg K_O$, $$\frac{n_O}{n_{O_2}} = \frac{(2K_3 + K_{11})}{K_{12}}$$

At $T_e = 3$ V, $K_0 = 30\ s^{-1}$, $K_3 = 6.5 \times 10^{-10}\ \frac{cm^3}{s}$,
$K_4 = 2.3 \times 10^{-11}\ \frac{cm^3}{s}$, $K_{11} = 1.8 \times 10^{-12}\ \frac{cm^3}{s}$,
and $K_{12} = 2.1 \times 10^{-10}\ \frac{cm^3}{s}$.

For high density limit, require $n_e \gg \frac{K_0}{K_{12}}$

or $n_e \gg 1.4 \times 10^{11}$ //

Then $\frac{n_0}{n_{O_2}} = 6.2$ //

(d) $$\frac{n_{O^+}}{n_{O_2^+}} = \frac{\cancel{n_e}(K_{11} n_{O_2} + K_{12} n_0) K_{O_2^+}}{K_{O^+} K_4 \cancel{n_e} n_{O_2}}$$

But $K_{12} n_0 = (2K_3 + K_{11}) n_{O_2}$ at high density

Hence

$$\frac{n_{O^+}}{n_{O_2^+}} = \frac{2(K_3 + K_{11})}{K_4} \frac{K_{O_2^+}}{K_{O^+}}$$

$$\frac{K_{O_2^+}}{K_{O^+}} = \frac{1}{\sqrt{2}}$$

Hence $\frac{n_{O^+}}{n_{O_2^+}} = 40$ //