

# EE241B : Advanced Digital Circuits

## Lecture 18 – Power-Performance Tradeoffs 2

**Borivoje Nikolić**



**MarketWatch, March 28: Opinion: There's no returning to regular schooling as online learning goes mainstream, by Alex Hicks**



When in-person education resumes, online learning tools and methods will be entrenched in the system

# Announcements

- Project midterm reports due today, March 31
  - Please e-mail me the link to your web page
- Assignment 3 due Thursday, April 2.
  - Quiz next Tuesday
- Reading – req'd
  - Rabaey et al, LPDE, Ch. 4

# Outline

- **Module 5**
  - Power-performance tradeoffs



## 5.B Power-Performance Tradeoffs

# Know Your Enemy

- Where does power go in CMOS?
- Switching (dynamic) power
  - Charging capacitors
- Leakage power
  - Transistors are imperfect switches
- Short-circuit power
  - Both pull-up and pull-down on during transition
- Static currents
  - Biasing currents

# Summary of Power Dissipation Sources

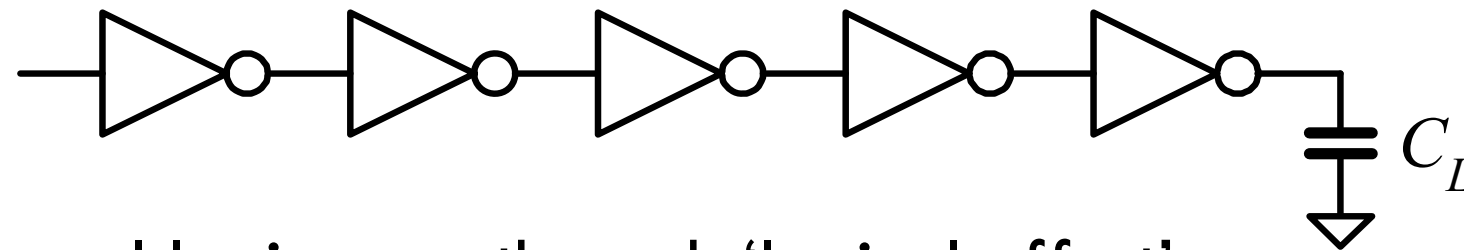
$$P \sim \alpha \cdot (C_L + C_{CS}) \cdot V_{swing} \cdot V_{DD} \cdot f + (I_{DC} + I_{Leak}) \cdot V_{DD}$$

- $\alpha$  – switching activity
- $C_L$  – load capacitance
- $C_{CS}$  – short-circuit “capacitance”
- $V_{swing}$  – voltage swing
- $f$  – frequency
- $I_{DC}$  – static current
- $I_{leak}$  – leakage current

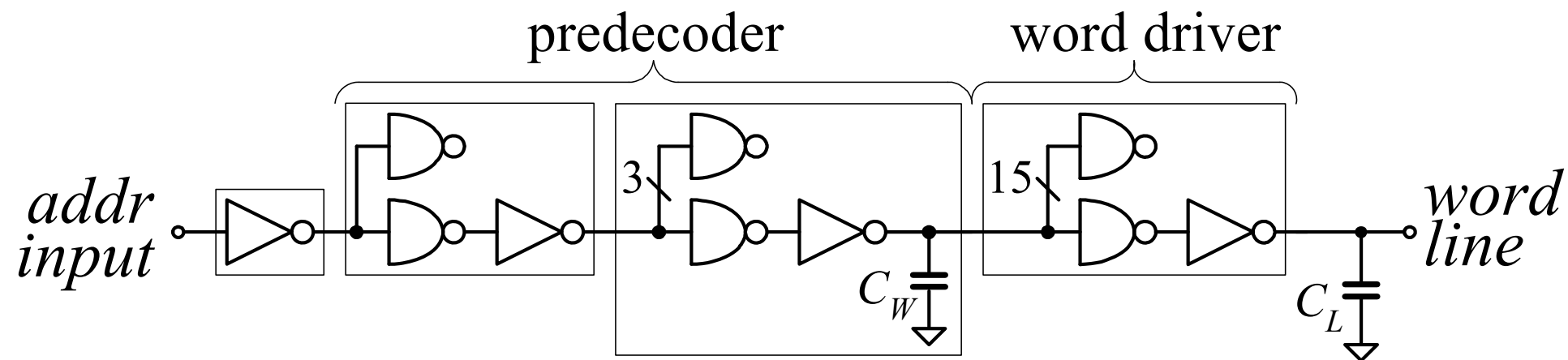
$$P = \frac{\text{energy}}{\text{operation}} \times \text{rate} + \text{static power}$$

# CMOS Performance Optimization

- Reminder - sizing: Optimal performance with equal fanout per stage

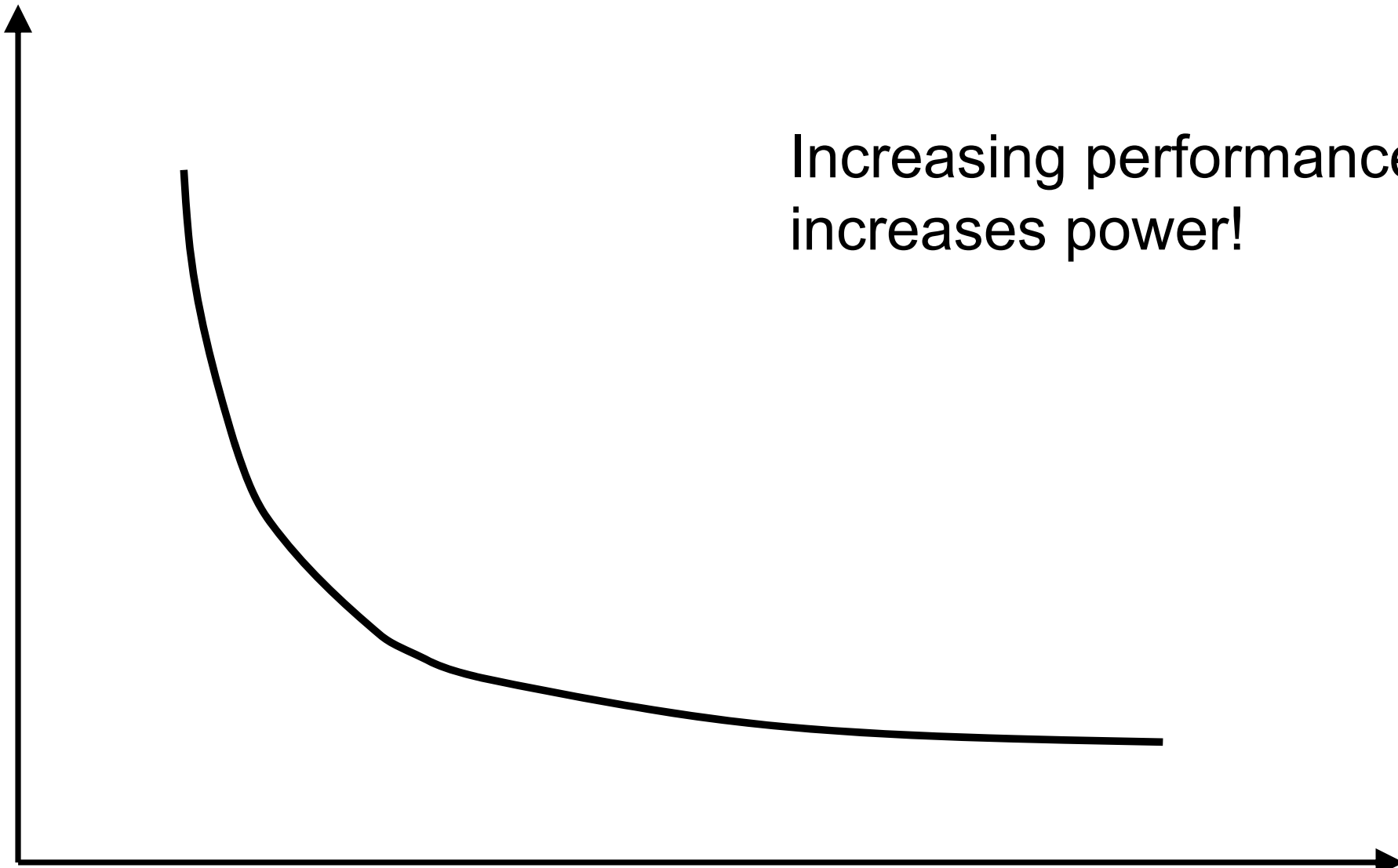


- Extendable to general logic cone through 'logical effort'
- Equal effective fanouts ( $g_i C_{i+1}/C_i$ ) per stage
- Optimal fanout is around 4



# Performance Optimization

Energy



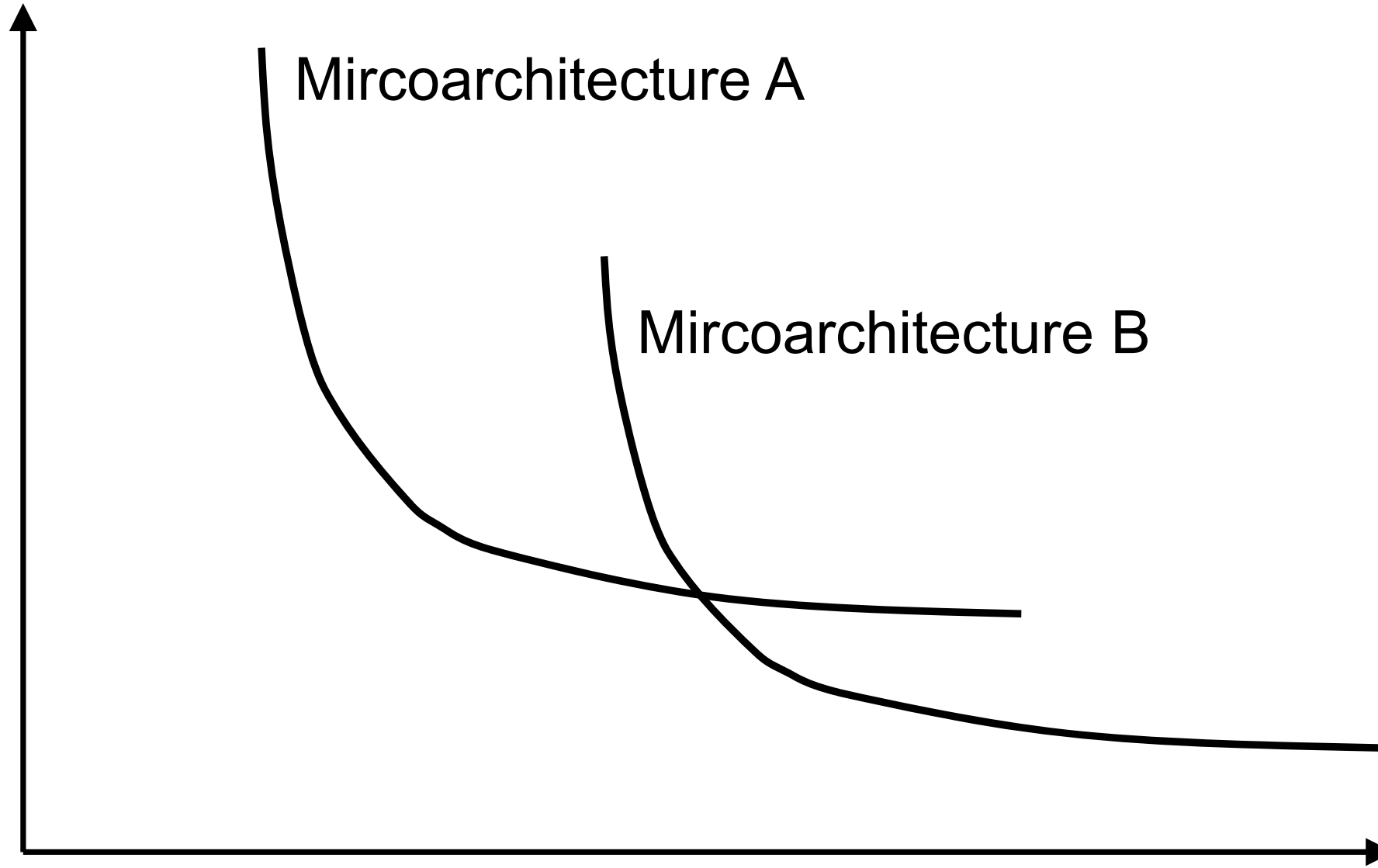
Increasing performance  
increases power!

Delay = 1/Performance



# Performance Optimization

Energy



Microarchitecture A

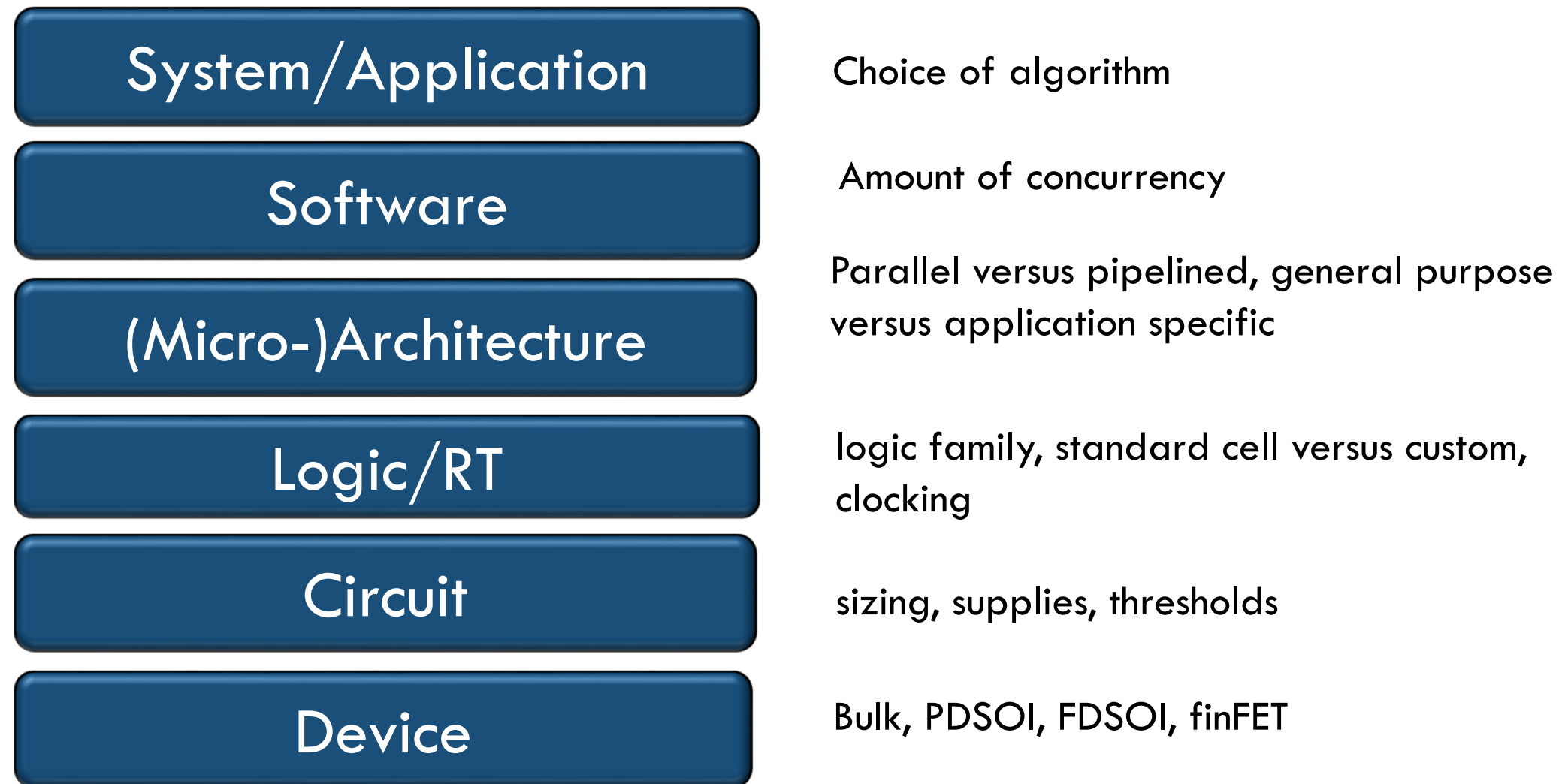
Microarchitecture B

Delay = 1/Performance

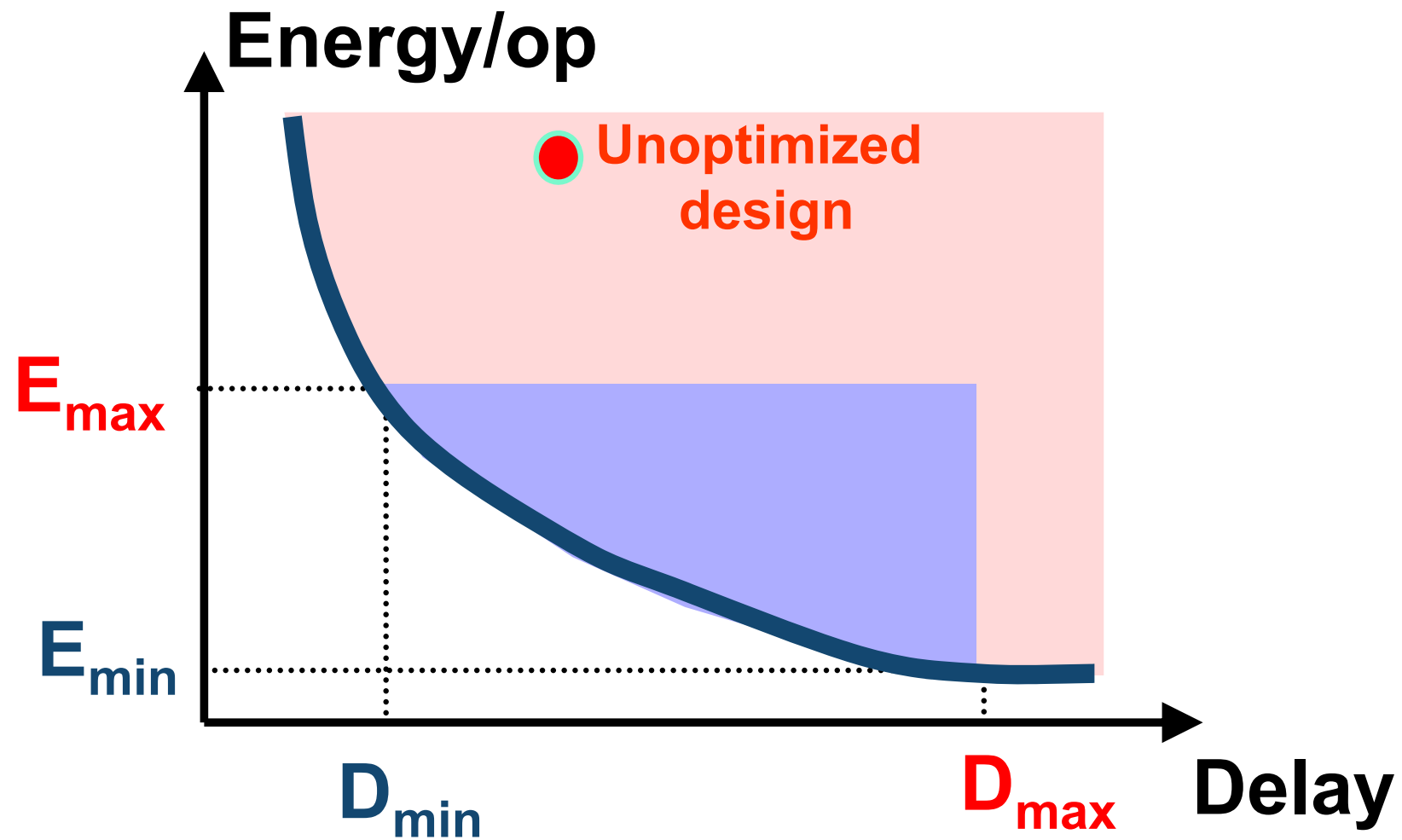
# The Design Abstraction Stack

A very **rich set of design parameters** to consider!

It helps to consider options in relation to their abstraction layer

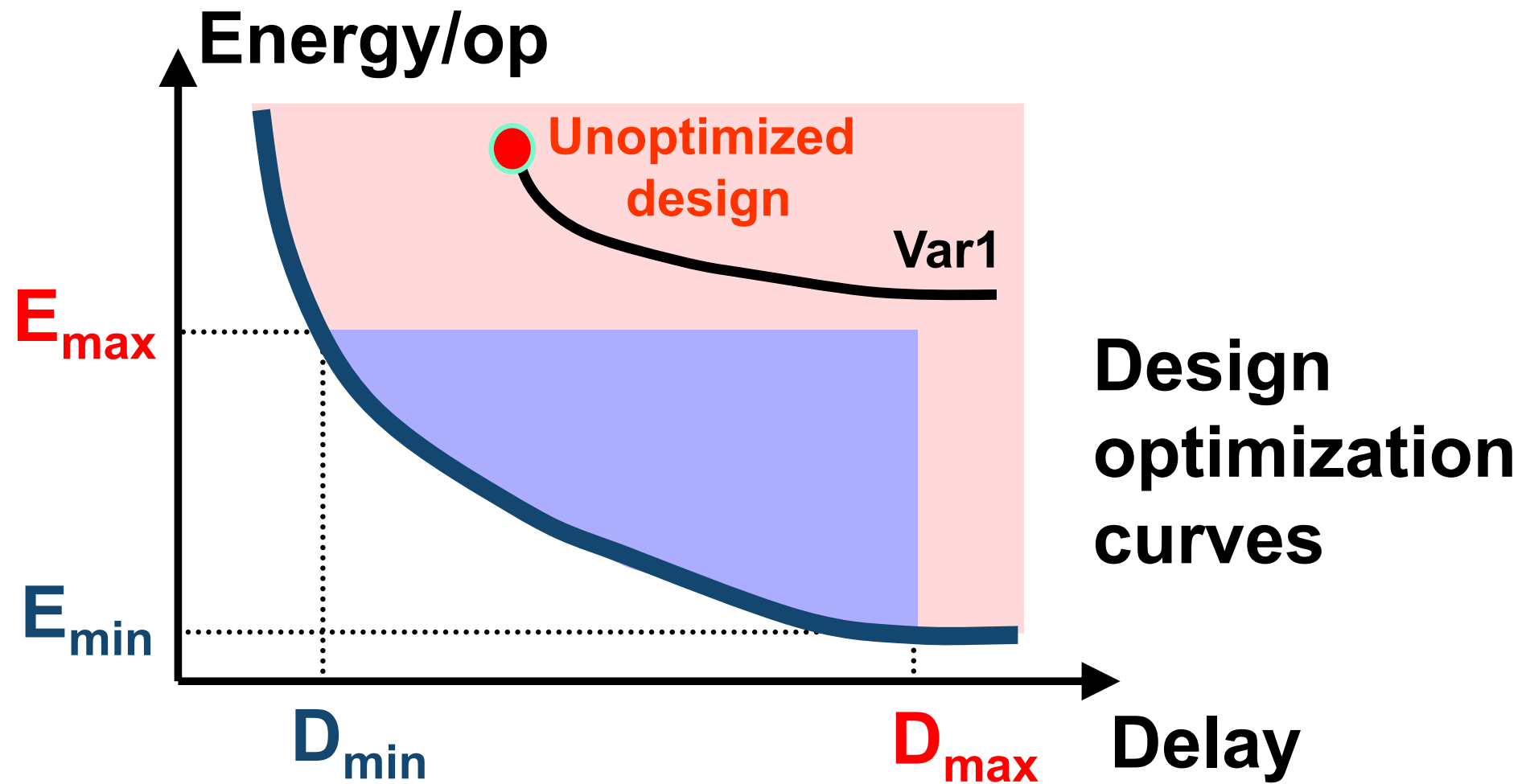


# Power-Performance Optimization



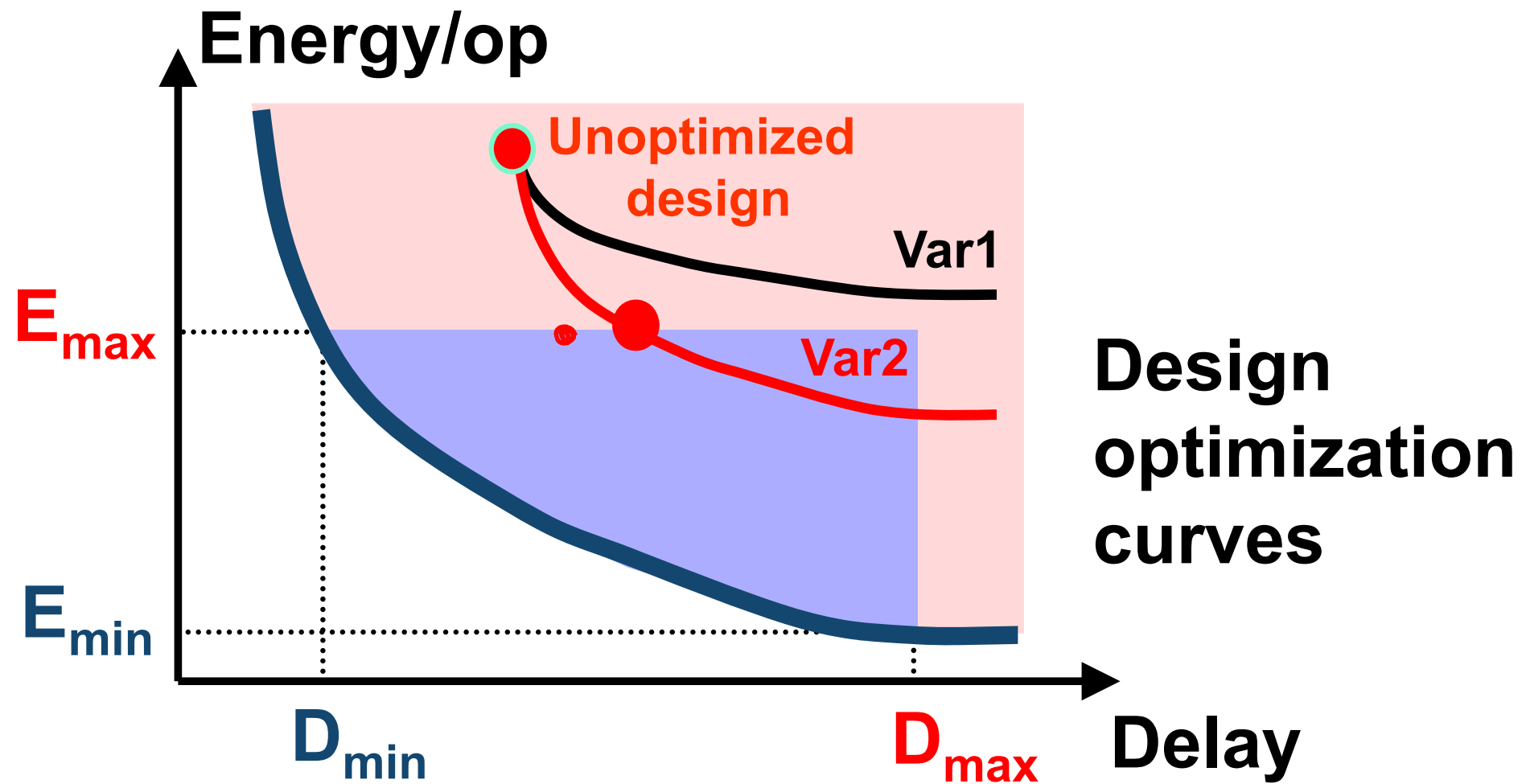
**Achieve the highest performance  
under the power cap**

# Power-Performance Optimization



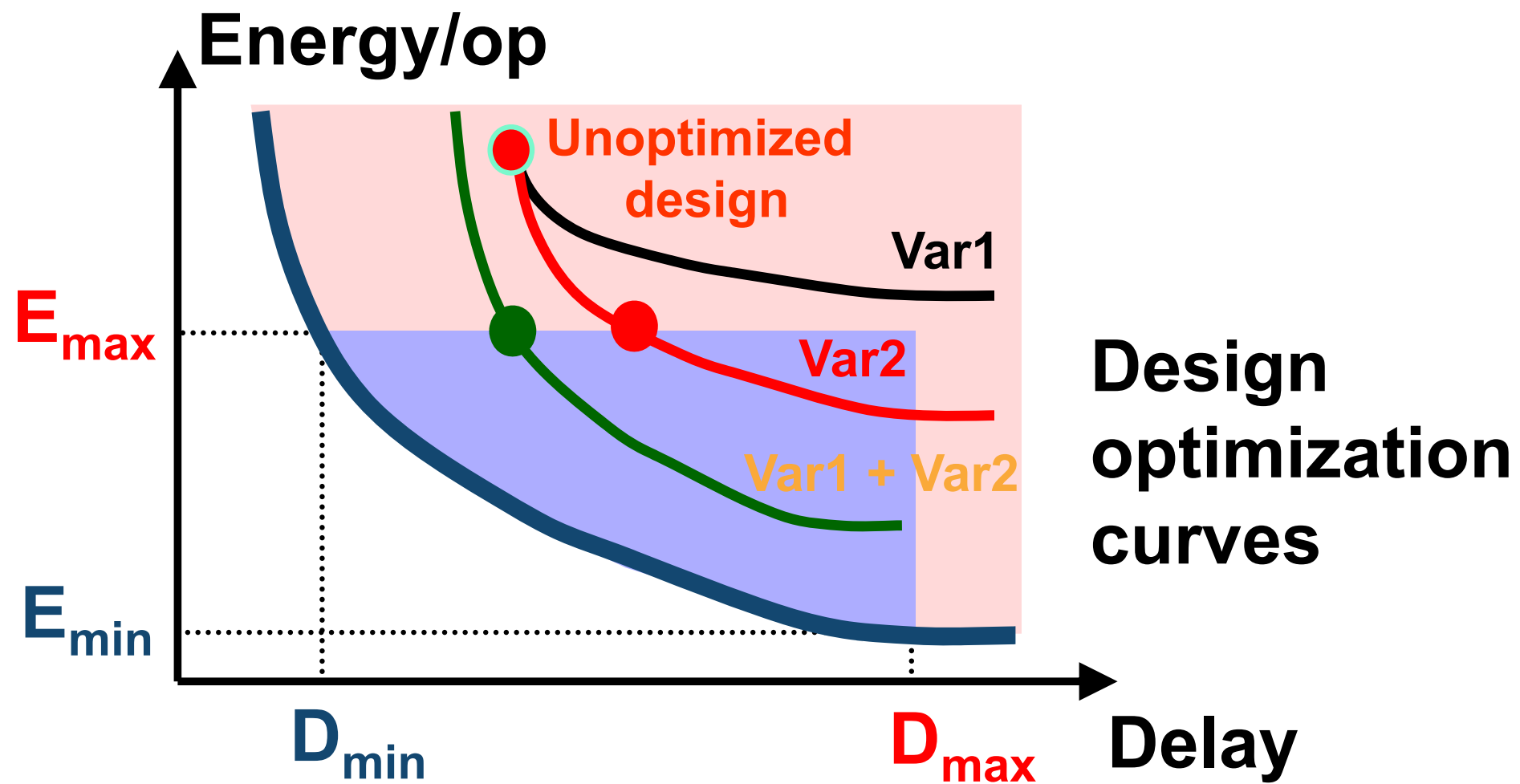
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# Power-Performance Optimization



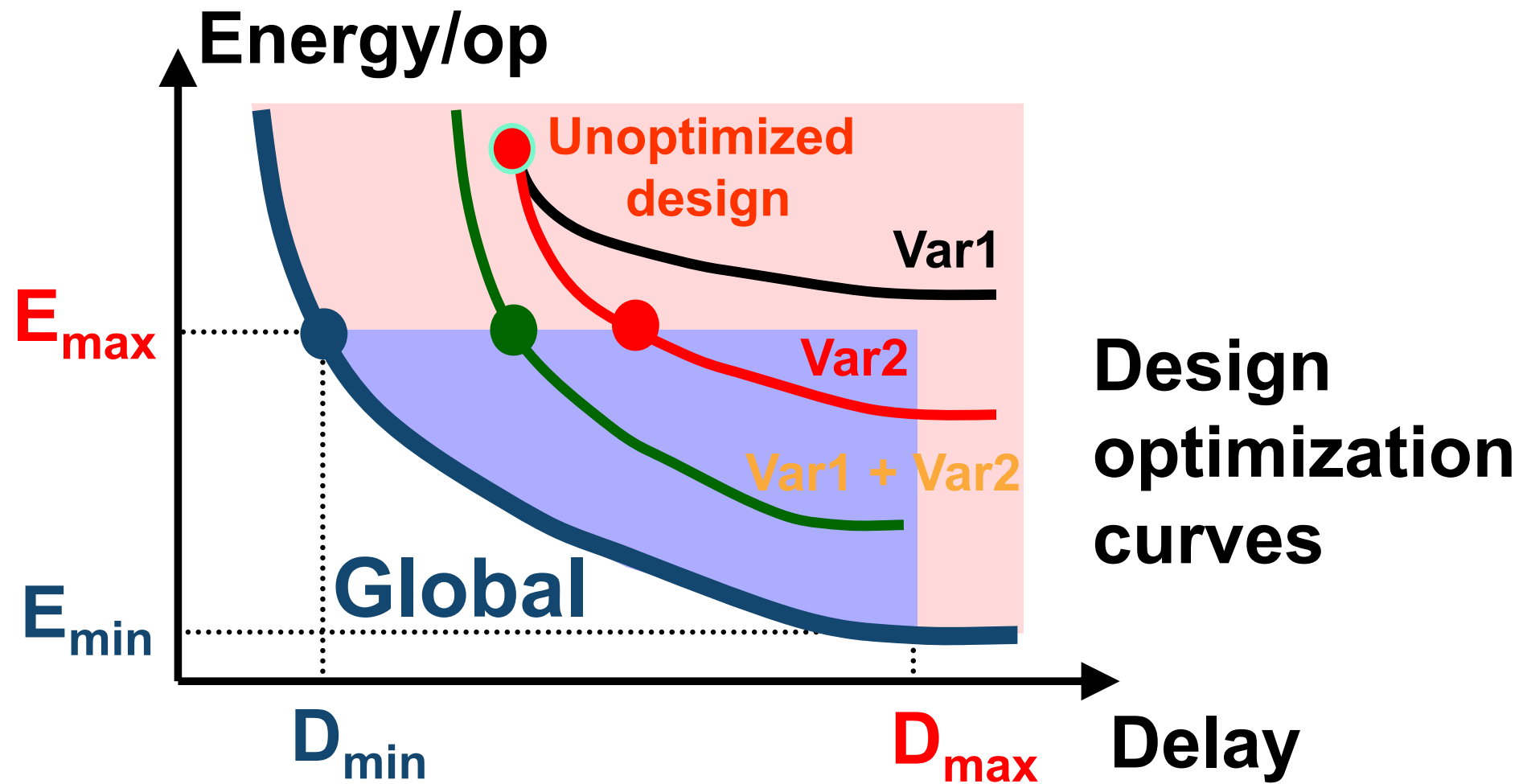
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# Power-Performance Optimization



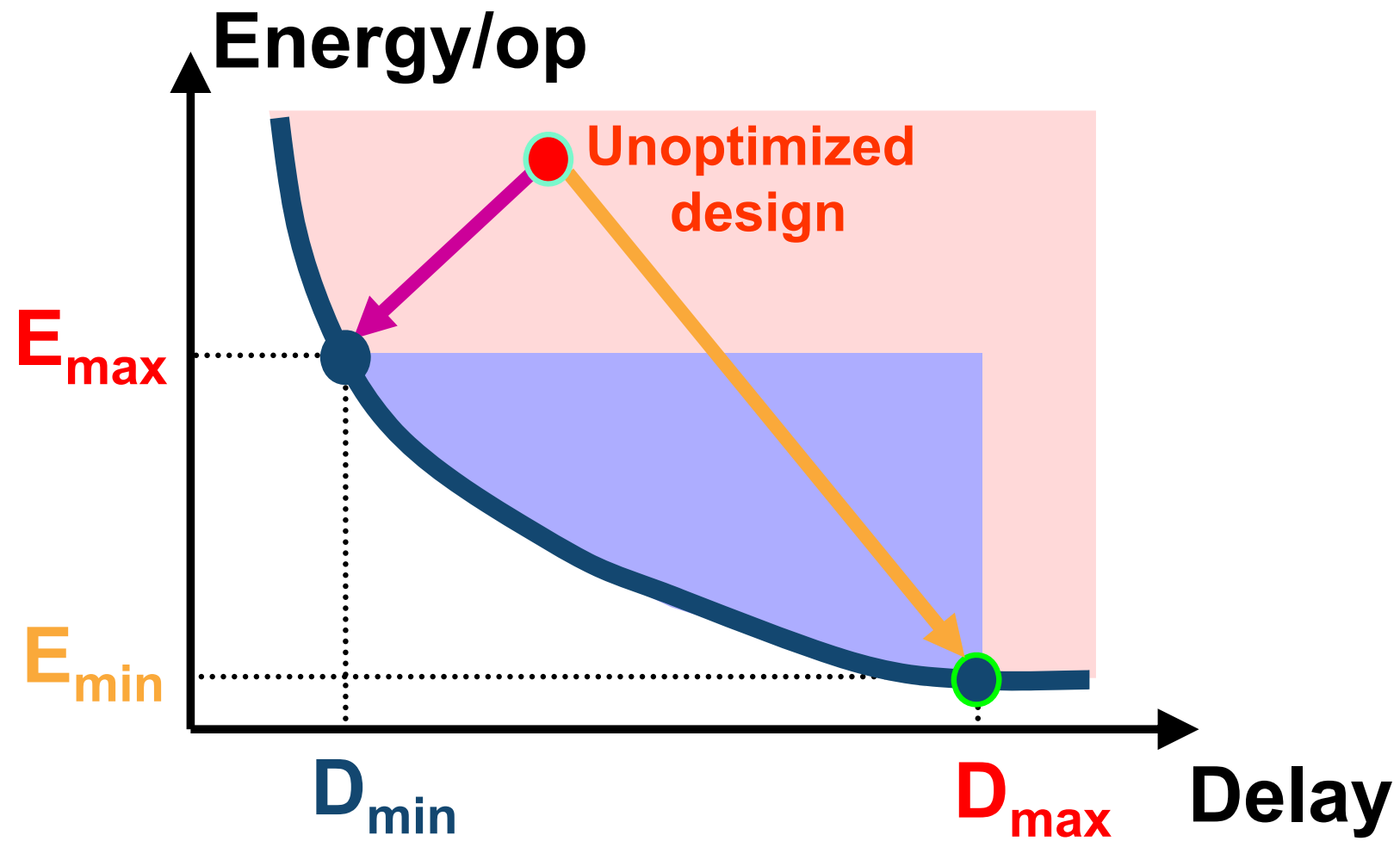
**How far away are we from the optimal solution?**

# Power-Performance Optimization



**Global optimum – best performance**

# Power-Performance Optimization

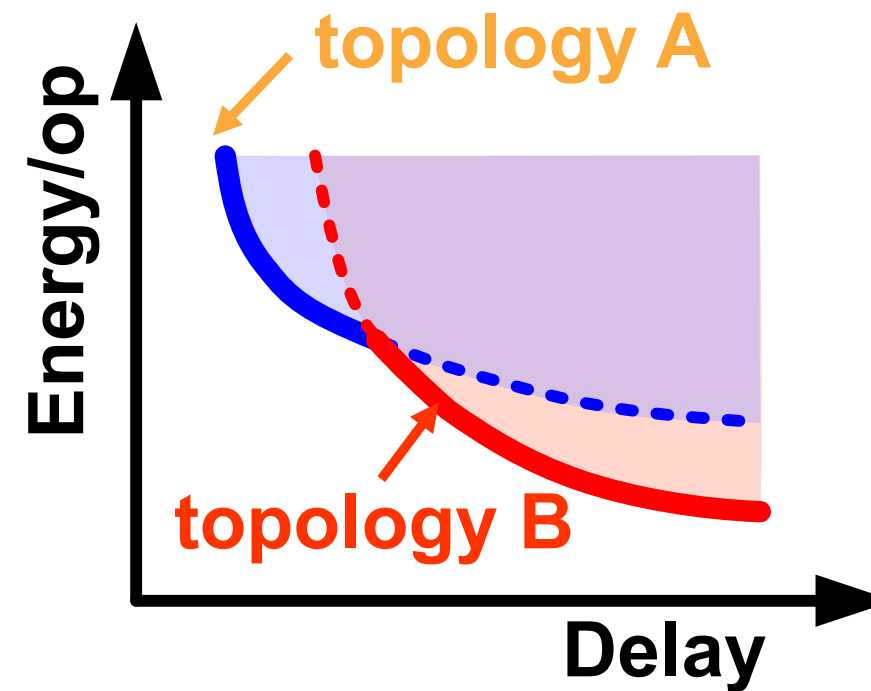


**Maximize throughput for given energy or**  
**Minimize energy for given throughput**



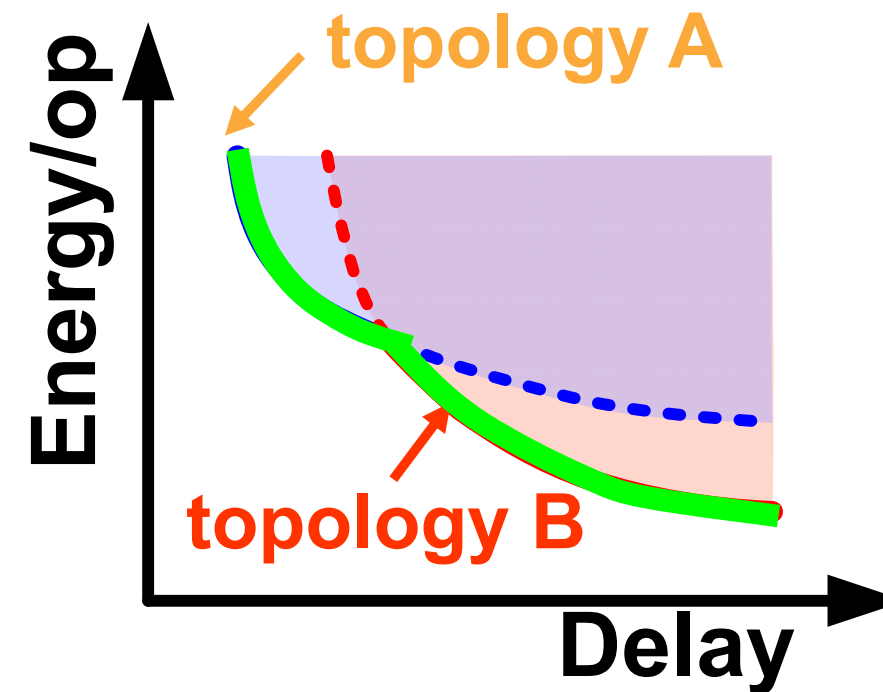
# Power-Performance Optimization

- There are many sets of parameters to adjust
  - Tuning variables
  - Circuit  
(sizing, supply, threshold)
  - Logic style  
(std. cells, custom , ...)
  - Block topology  
(adder: CLA, CSA, ...)
  - Micro-architecture  
(parallel, pipelined)



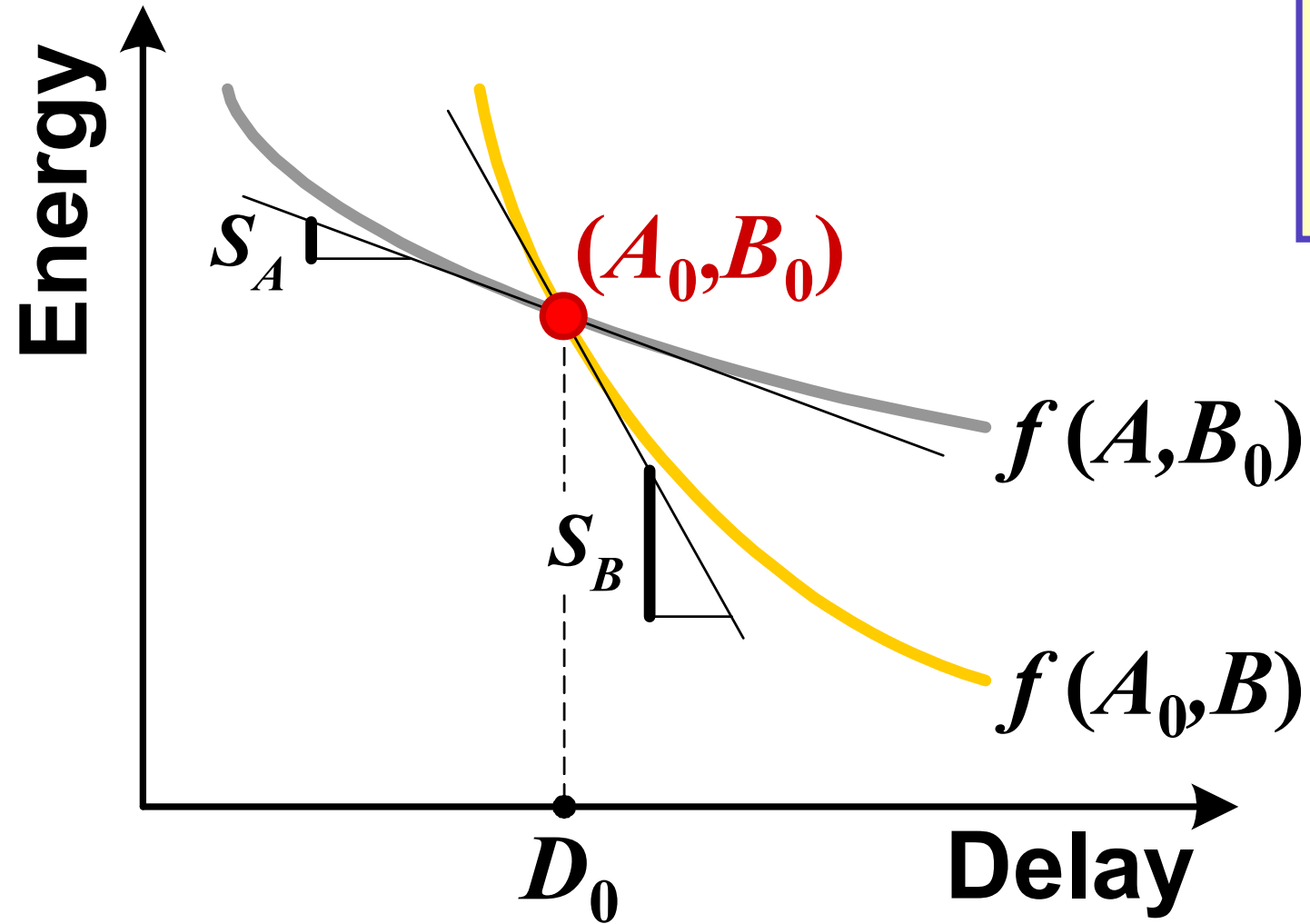
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**Globally optimal power-performance curve for a given function**

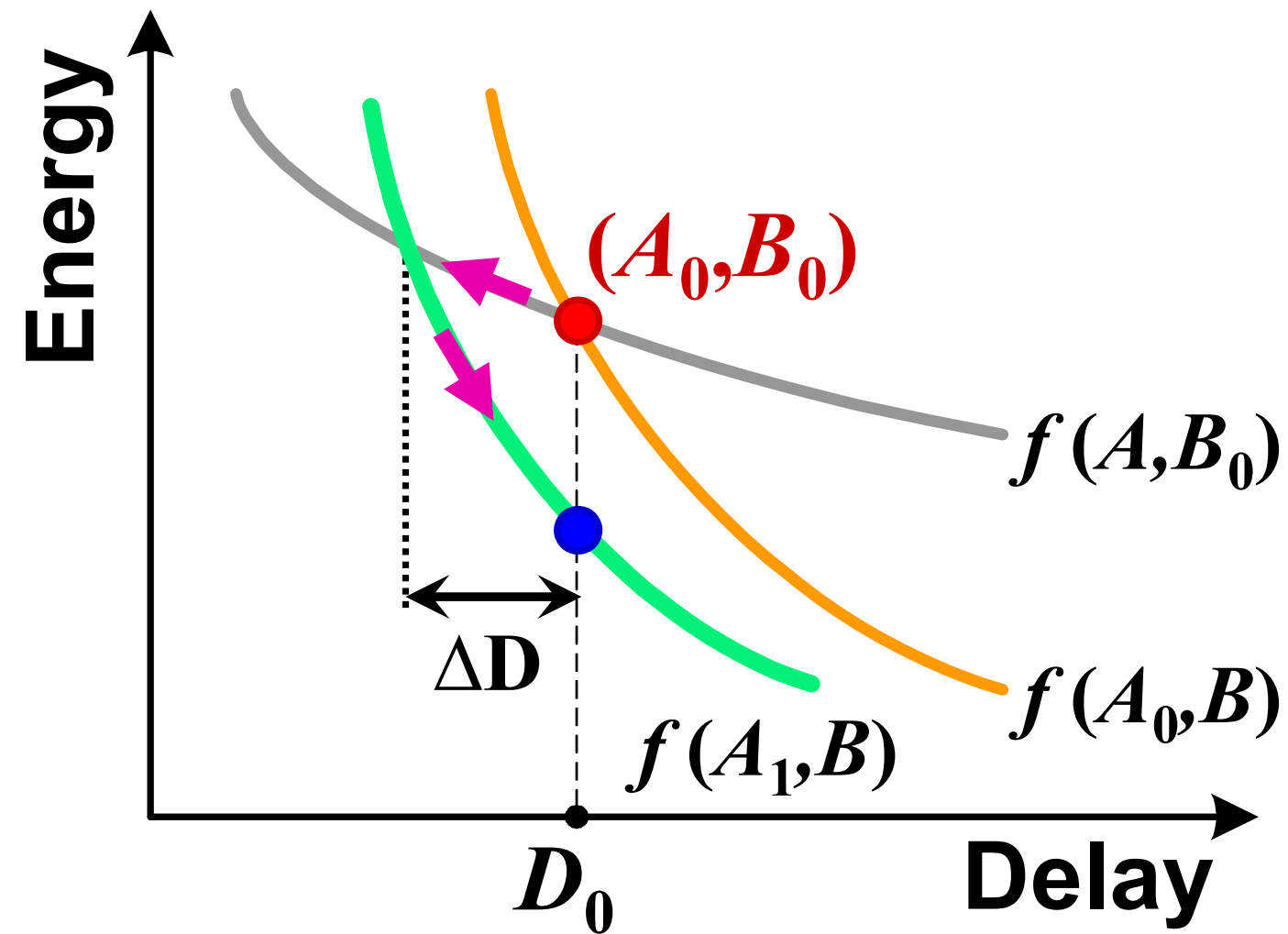
# Energy-Delay Sensitivity



$$S_A = \left. \frac{\partial E / \partial A}{\partial D / \partial A} \right|_{A=A_0}$$

## Solution: Equal Sensitivities

$$\Delta E = S_A \cdot (-\Delta D) + S_B \cdot \Delta D$$



At the solution point all sensitivities should be equal



## 5. C Architectural Optimization

# Optimal Processors

- Processors used to be optimized for performance
  - Optimal logic depth was found to be 8-11 FO4 delays in superscalar processors
  - 1.8-3 FO4 in sequentials, rest in combinatorial
    - Kunkel, Smith, ISCA'86
    - Hrishesh, Jouppi, Farkas, Burger, Keckler, Shivakumar, ISCA'02
    - Harstein, Puzak, ISCA'02
    - Sprangle, Carmean, ISCA'02
- But those designs are have very high power dissipation
  - Need to optimize for both performance and power/energy

# From System View: What is the Optimum?

- How do sensitivities relate to more traditional metrics:
  - Power per operation (MIPS/W, GOPS/W, TOPS/W)
  - Energy per operation (Joules per op)
  - Energy-delay product
- Can be reformatted as a goal of optimizing power x delay<sup>n</sup>
  - $n = 0$  – minimize power per operation
  - $n = 1$  – minimize energy per operation
  - $n = 2$  – minimize energy-delay product
  - $n = 3$  – minimize energy-(delay)<sup>2</sup> product

# Optimization Problem

- Set up optimization problem:
  - Maximize performance under energy constraints
  - Minimize energy under performance constraints
- Or minimize a composite function of  $E^n D^m$ 
  - What are the right  $n$  and  $m$ ?
- $n = 1, m = 1$  is EDP – improves at lower  $V_{DD}$
- $n = 1, m = 2$  is invariant to  $V_{DD}$ 
  - $E \sim C V_{DD}^2$
  - $D \sim 1/V_{DD}$



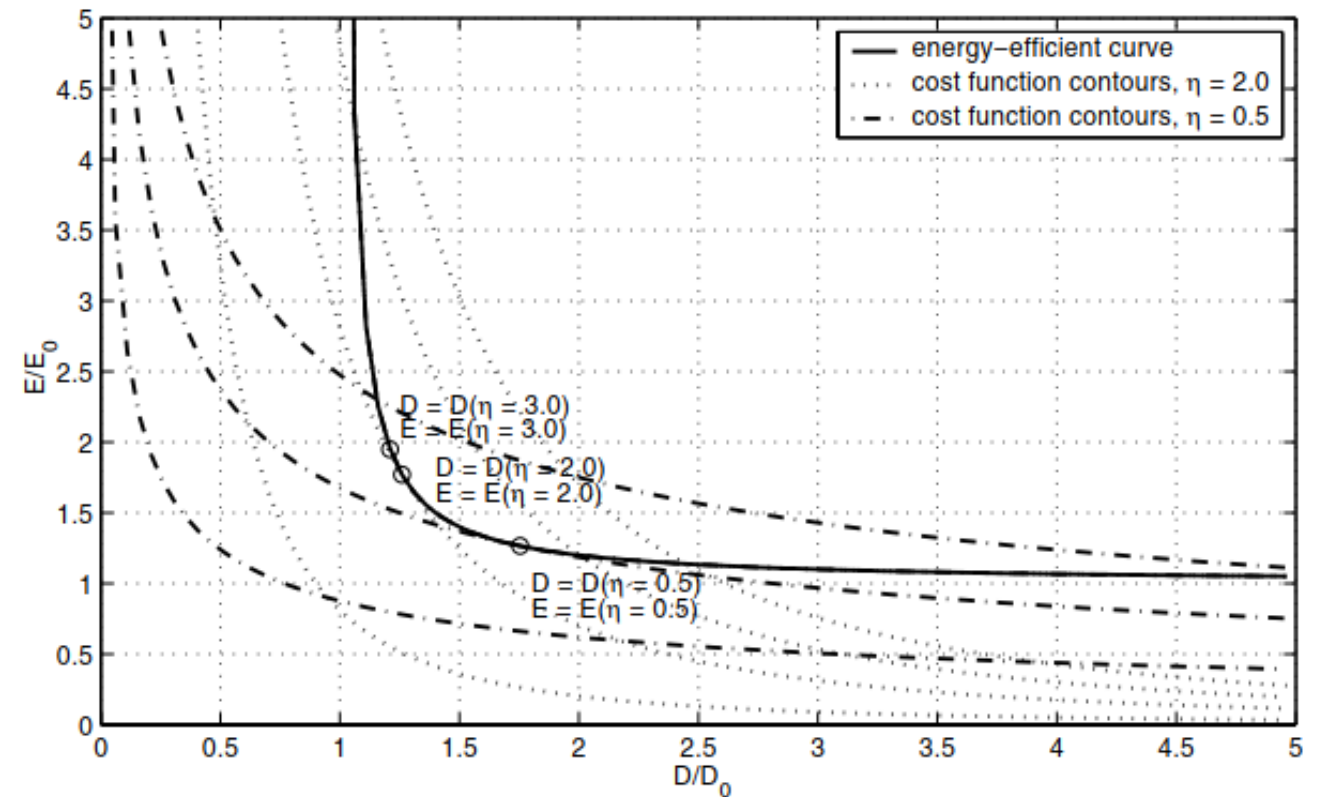
# Hardware Intensity

- Introduced by Zyuban and Strenski in 2002.
- Measures where is the design on the Energy-Delay curve
- Parameter in cost function optimization

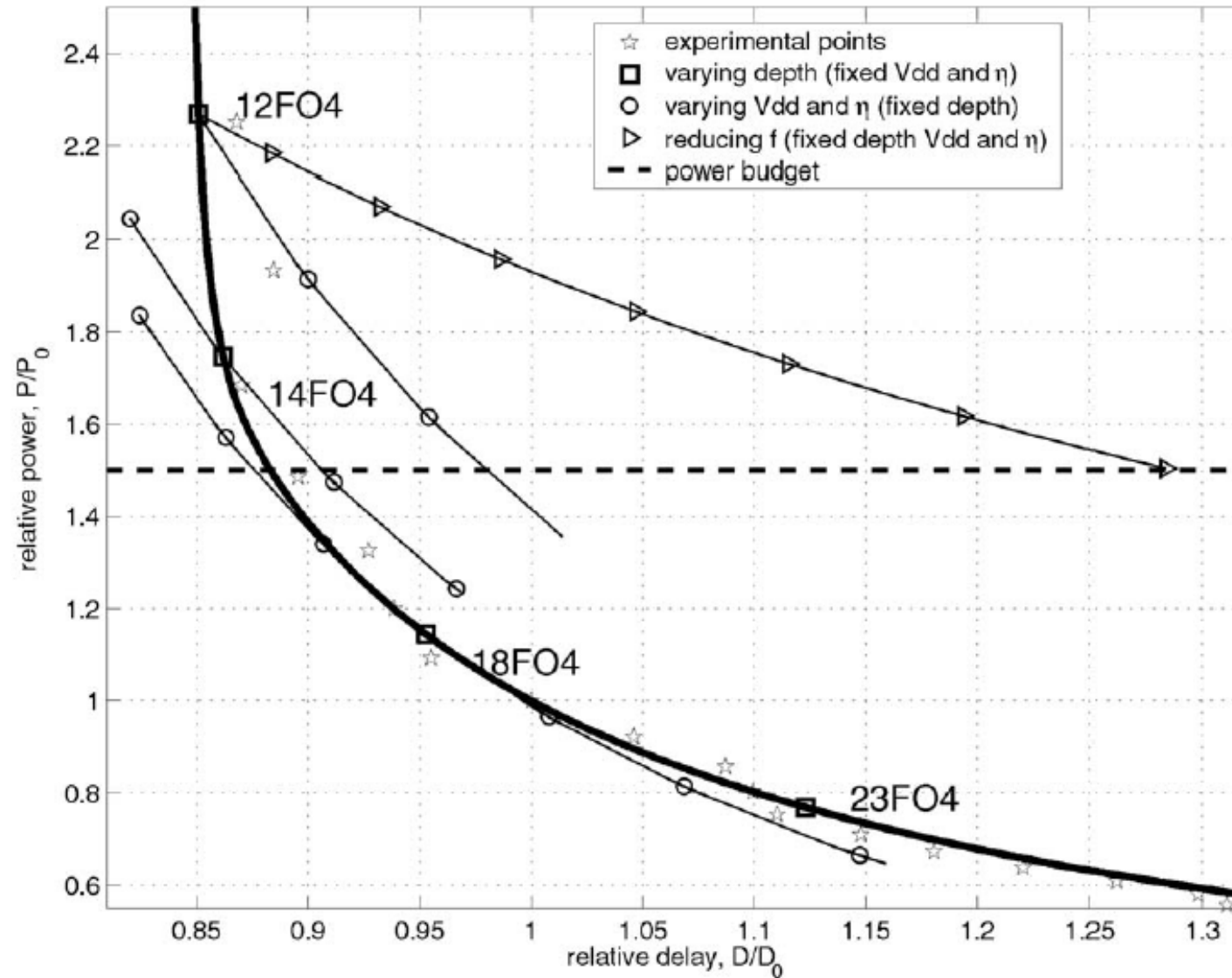
$$F_c = (E/E_0)(D/D_0)^\eta \quad 0 \leq \eta < +\infty,$$

$$\eta = - \left. \frac{D \partial E}{E \partial D} \right|_v$$

**Slope of the optimal E-D curve at the chosen design point**



# Optimum Across Hierarchy Layers



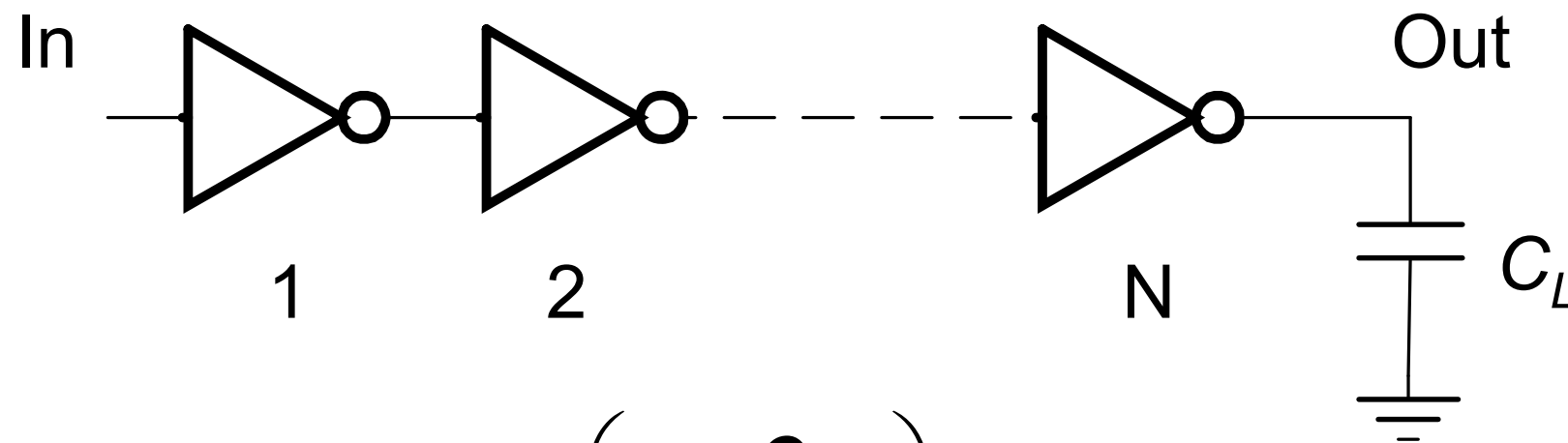
**Optimal logic depth in pipelined processors is ~18FO4**

Relatively flat in the 16-22FO4 range



## 5.D Circuit-Level Tradeoffs

# Alpha-Power Based Delay Model



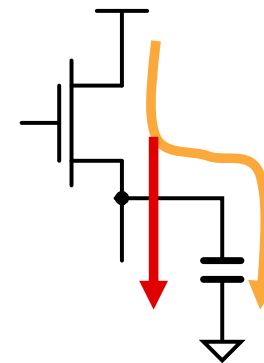
$$t_{pi} = \frac{K_d V_{DD}}{(V_{DD} - V_{Th})^\alpha} \left( 1 + \frac{C_{L,i}}{C_{in,i}} \right)$$

$$D = \sum t_{pi} = \sum \frac{K_d V_{DD}}{(V_{DD} - V_{Th})^\alpha} \left( 1 + \frac{W_{L,i}}{W_{in,i}} \right)$$

# Energy Models

## ◆ Switching

$$E_{Sw} = \alpha_{0 \rightarrow 1} (C_{L,i} + C_{int,i}) V_{DD}^2$$



## ◆ Leakage

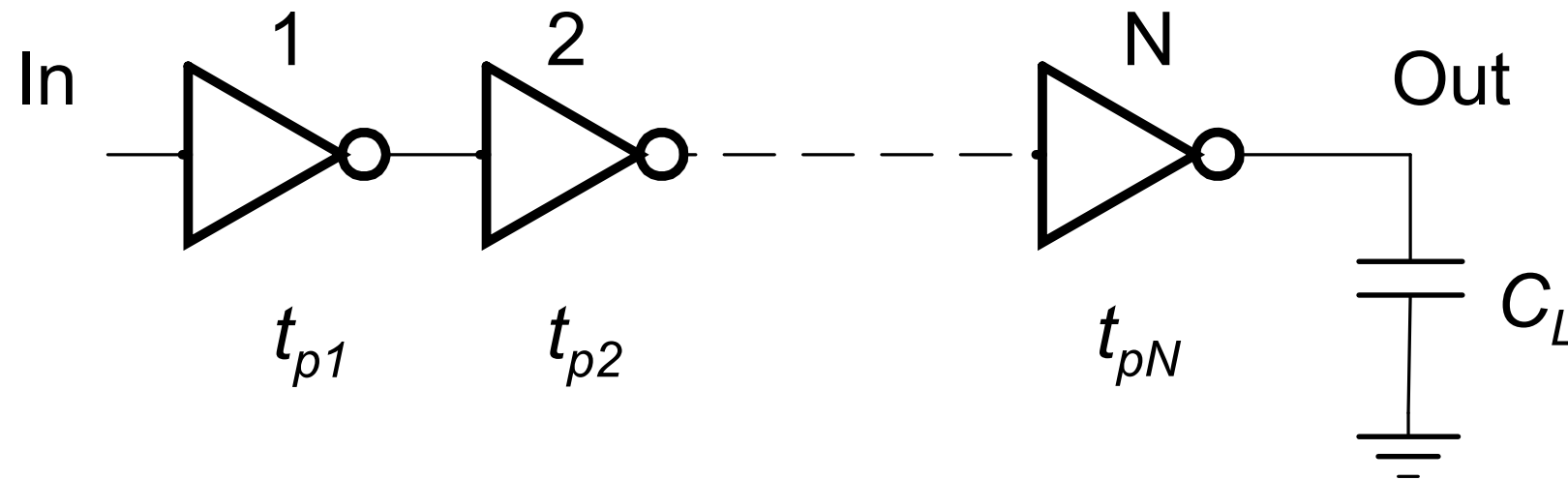
$$E_{Lk} = W_{ln} I_0 e^{\frac{-(V_{Th} - \gamma V_{DD})}{nV_t}} V_{DD} D$$

# Sizing, Supply, Threshold Optimization

- Transistor sizing can yield large power savings with small delay penalties
  - Gate sizing
  - Beta-ratio adjustments
  - (Stack resizing)
- Supply voltage affects both active and leakage energy
- Threshold voltage affects primarily the leakage

$$\beta = W_p/W_n$$

# Apply to Sizing of an Inverter Chain



*Unconstrained energy: find min  $D = \sum t_{pi}$*

$$C_{gin,j} = \sqrt{C_{gin,j-1} C_{gin,j+1}}$$

$$W_j = \sqrt{W_{j-1} W_{j+1}}$$

*Constrained energy: find min  $D$ , under  $E < E_{max}$*

*Where  $E = \sum e_i$*

# Constrained Optimization

- Find  $\min(D)$  subject to  $E = E_{max}$ 
  - *Constrained function minimization*
- E.g. Lagrange multipliers

Or dual:

$$\Lambda(x) = D(x) + \lambda(E(x) - E_{max}) \quad \mathbb{K}(x) = E(x) + \lambda(D - D_{max})$$

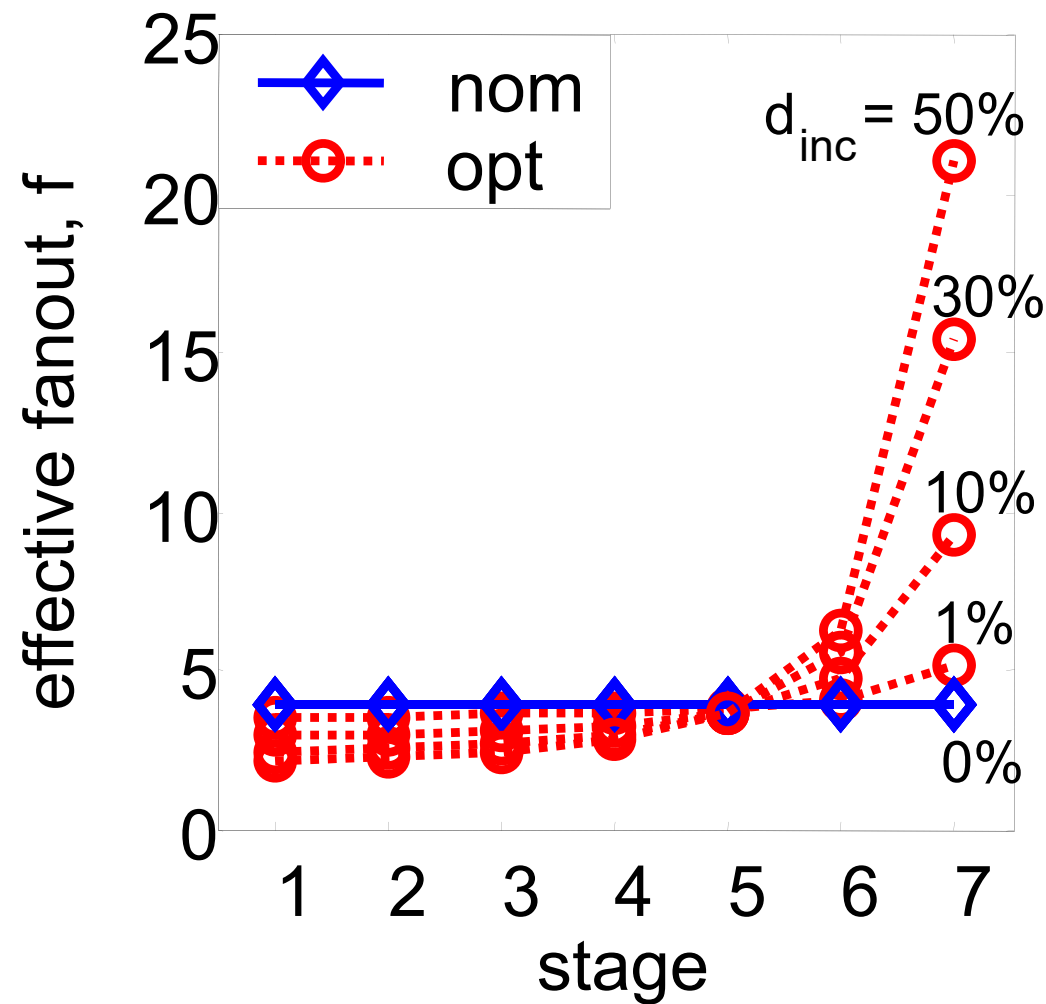
$$\frac{\partial \Lambda}{\partial x} = 0$$

- Can solve analytically for  $x = W_i, V_{DD}, V_{Th}$



# Inverter Chain: Sizing Optimization

# Inverter Chain: Sizing Optimization



$$W_j = \sqrt{\frac{W_{j-1}W_{j+1}}{1 + \lambda W_{j-1}}}$$

[Ma, Franzon, *IEEE JSSC*, 9/94]

$$\lambda = -\frac{2KV_{DD}^2}{\tau_{nom} S_W}$$

$$S_W \propto \frac{e_j}{f_j - f_{j-1}}$$

$e_j$  – energy per stage  
 $f_j$  – fanout per stage

Stojanovic, ICCAD'02

- **Variable taper achieves minimum energy**
- **Reduce number of stages at large  $d_{inc}$**

# Sensitivity to Sizing and Supply

- Gate sizing ( $W_i$ )

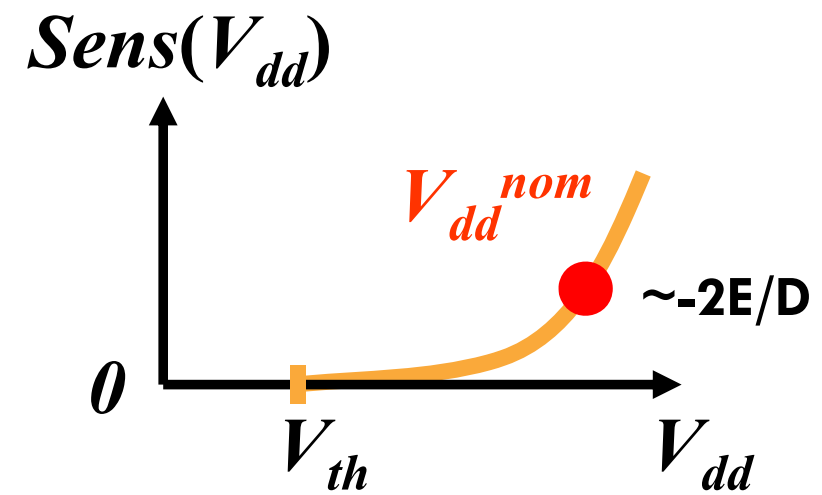
$$-\frac{\partial E_{sw} / \partial W_j}{\partial D / \partial W_j} = \frac{e_j}{\tau_{nom} (f_j - f_{j-1})}$$

$\infty$  for equal  $f_{eff}$   
( $D_{min}$ )

- Supply voltage ( $V_{dd}$ )

$$-\frac{\partial E_{sw} / \partial V_{DD}}{\partial D / \partial V_{DD}} = \frac{E_{sw}}{D} 2 \frac{1 - x_v}{\alpha - 1 + x_v}$$

$$x_v = (V_{Th} + \Delta V_{Th}) / V_{dd}$$



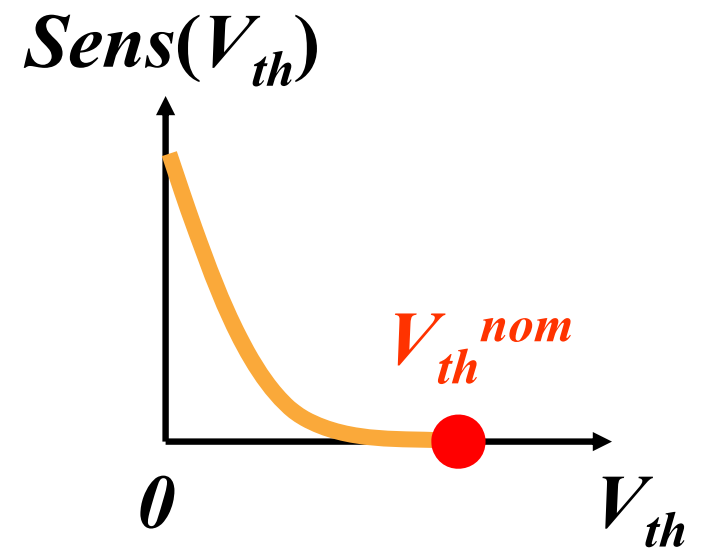
## Sensitivity to $V_{th}$

- Threshold voltage ( $V_{th}$ )

$$-\frac{\frac{\partial E}{\partial \Delta V_{Th}}}{\frac{\partial D}{\partial \Delta V_{th}}} = P_{Lk} \left( \frac{V_{DD} - V_{Th} - \Delta V_{Th}}{\alpha n V_t} - 1 \right)$$

**Low initial leakage**

**⇒ speedup comes for “free”**



## Next Lecture

- **Low-power design**
  - Lowering supply voltage