

# EE241B : Advanced Digital Circuits

## Lecture 5 – Leakage, Delay Models

### Borivoje Nikolić



**The passing of Barrie Gilbert (1937-2020).** He was well known for his invention of numerous analog circuit concepts, holding over 100 patents worldwide, and for the discovery of the Translinear Principle. His name is attributed to a class of related topologies loosely referred to as the Gilbert cell, one of which is a mixer - a key frequency translation device - used in every modern wireless communication device.



# Announcements

- Homework 1 posted today, due in 2 weeks

# Outline

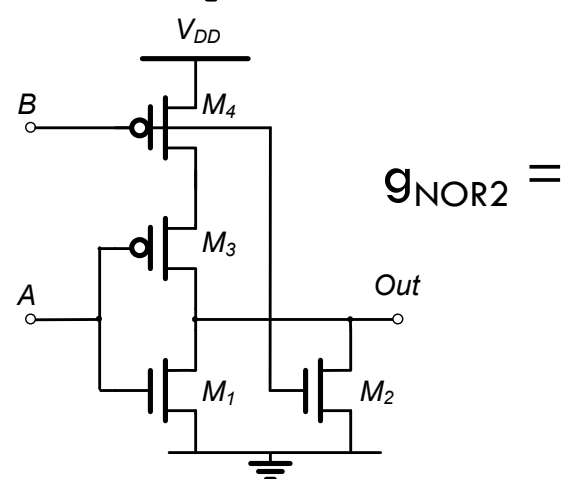
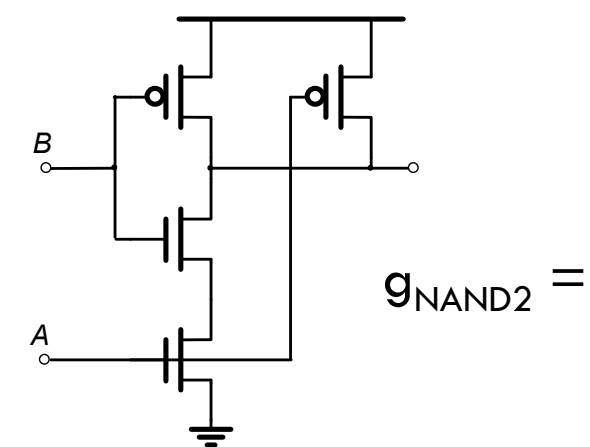
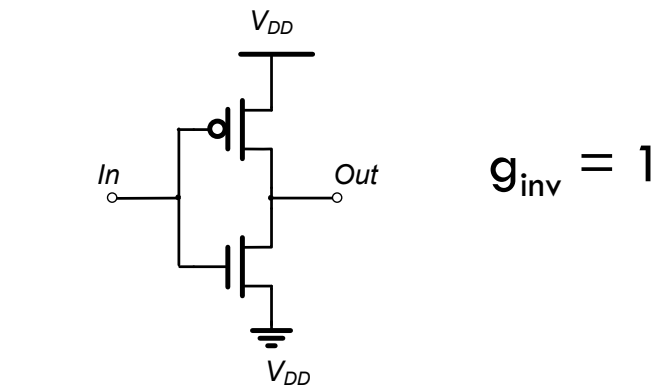
- **Module 2**
  - MOS transistor leakage
  - C-V models
  - Delay revisited

## Recap last lecture

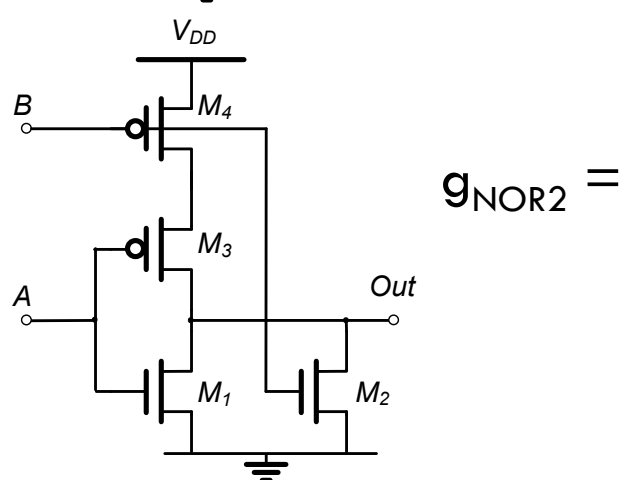
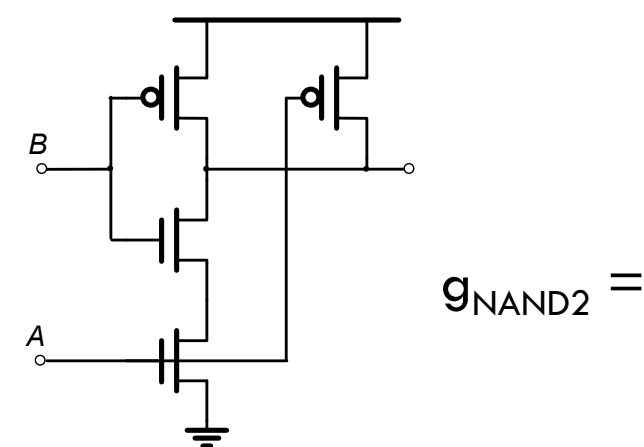
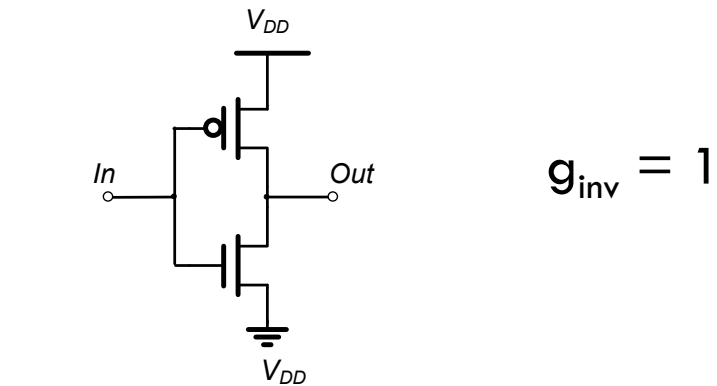
- Scaled transistors are different than the ones in textbooks
- But the same principles still apply
- Sizing:
  - P/N ratio is set by mobilities
    - Mobility enhancements are more effective on PMOS devices
    - $\sim 1:1$  in sub 16nm
  - Stack sizing set by velocity saturation
  - Stack of 2 reduces NMOS current by  $\sim 2/3$ 
    - PMOS depends on the degree of saturation, also  $\sim 2/3$  in sub 16nm

# Recap last lecture: Logical Effort

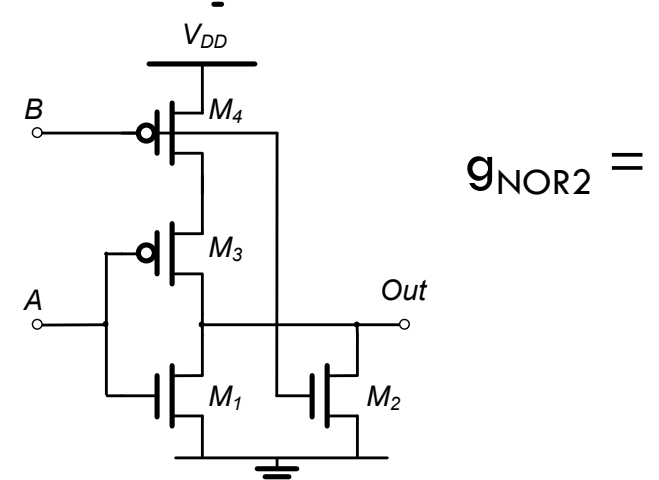
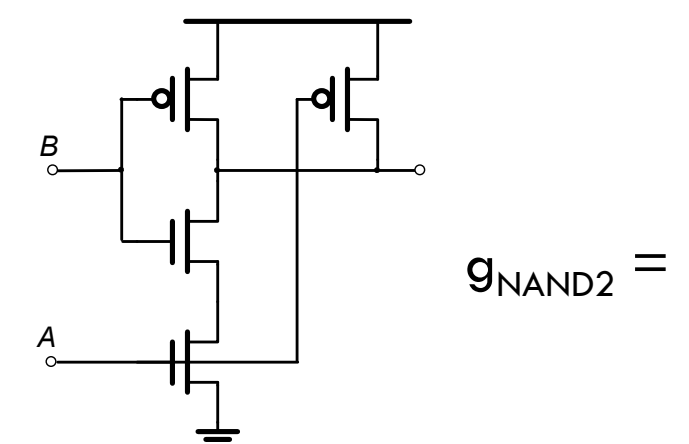
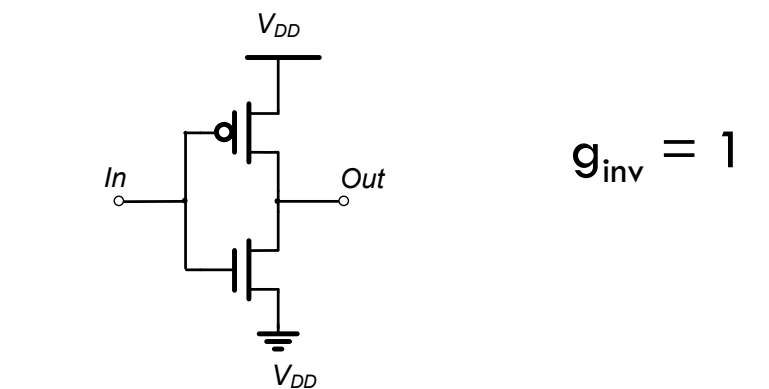
- Older CMOS ( $> 1 \mu\text{m}$ )



- Planar CMOS ( $\sim 28\text{nm}$ , bulk, FDSOI)



- FinFET (1.6nm-7nm)





## 2.E Other Velocity Saturation Models

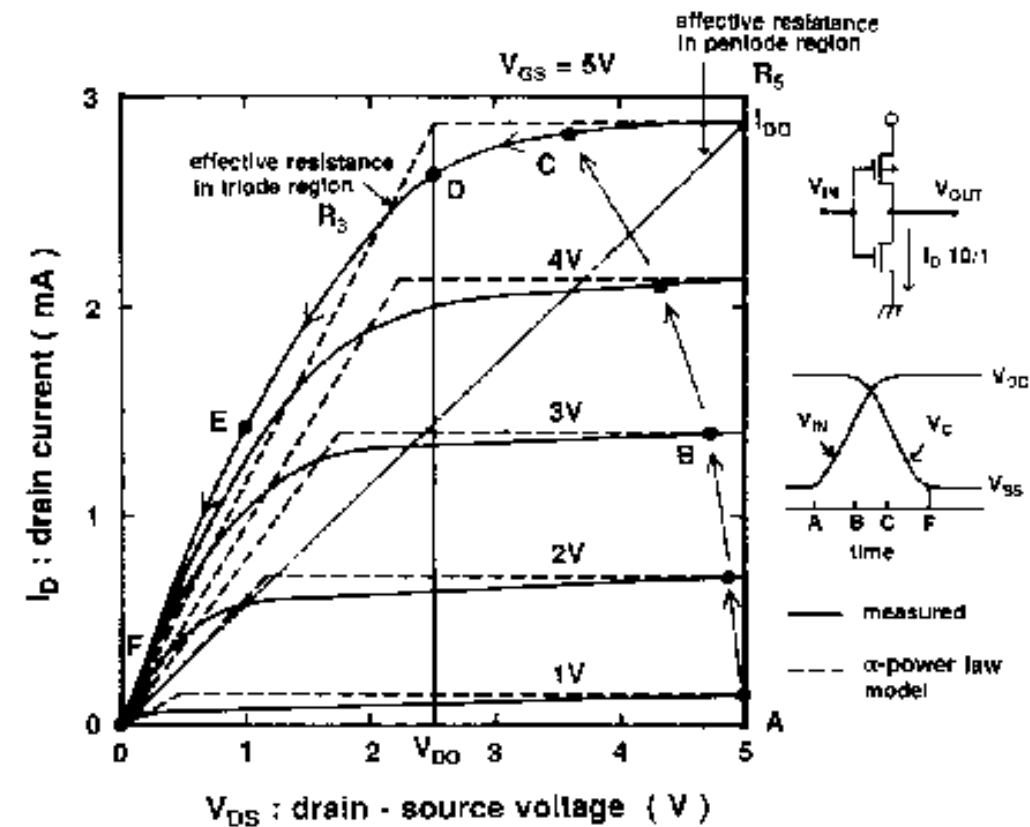
# Other Models: Alpha Power Law Model

- Simple model, sometimes useful for hand analysis

$$I_{DS} = \frac{W}{2L} \mu C_{ox} (V_{GS} - V_{Th})^\alpha$$

Parameter  $\alpha$  is between 1 and 2.

Sakurai, Newton, JSSC 4/90



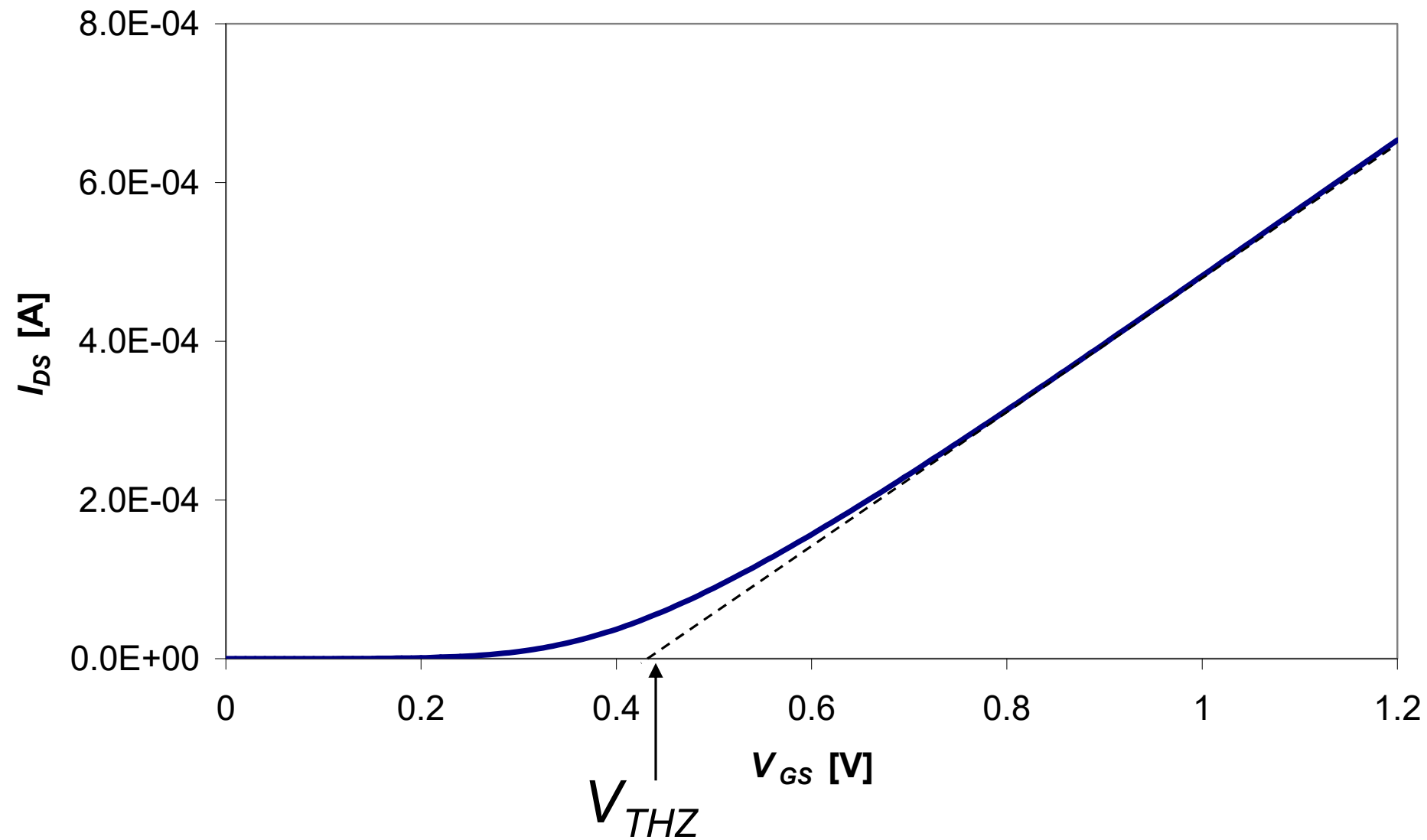
# Alpha Power Law Model

- This is not a physical model
- Simply empirical:
  - Can fit (in minimum mean squares sense) to variety of  $\alpha$ 's,  $V_{Th}$
  - Need to find one with minimum square error – fitted  $V_{Th}$  can be different from physical
  - Can also fit to  $\alpha = 1$ 
    - What is  $V_{Th}$ ?



# $K(V_{GS} - V_{THZ})$ Model ( $\alpha = 1$ )

Drain current vs. gate-source voltage



# Saturation Currents

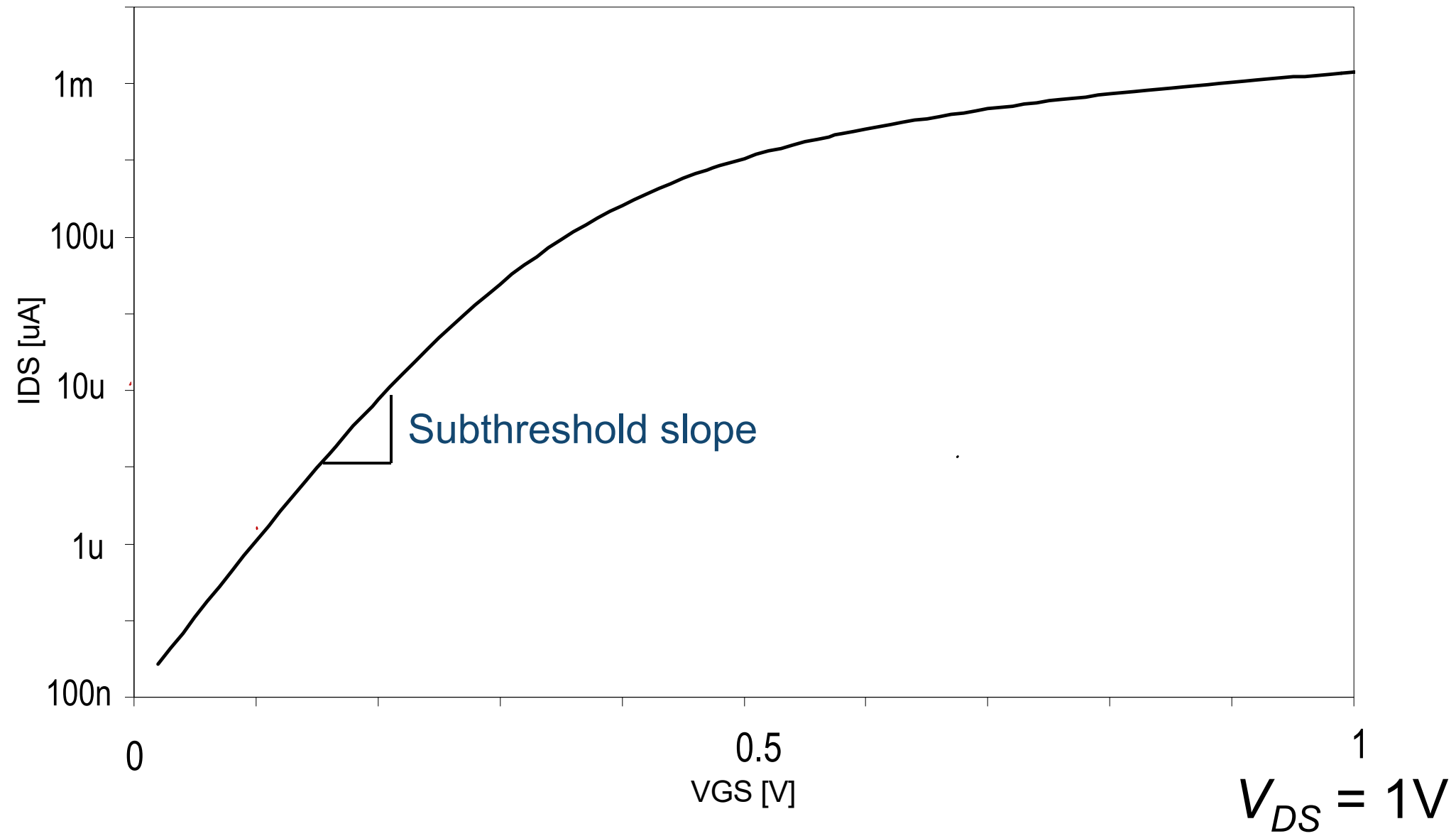
Model	Usage
$I_{DS} = K \frac{W}{L} (V_{GS} - V_{THZ})$	Delay estimates with $V_{DD} \gg V_{TH}$
$I_{DS} = \frac{W}{L} \frac{\mu C_{ox}}{2} (V_{GS} - V_{TH})^2$	Long channel devices (rare in digital)
$I_{DS} = \frac{W}{L} \frac{\mu C_{ox}}{2} (V_{GS} - V_{TH})^\alpha$	Delay estimates in a wider range of $V_{DD}$ 's
$I_{DS} = \frac{W}{L} \mu C_{ox} \left( (V_{GS} - V_{TH}) V_{Dsat} - \frac{V_{Dsat}^2}{2} \right)$	Easy to remember, does not handle stacks correctly
$I_{DS} = \frac{W}{L} \frac{\mu C_{ox}}{2} \frac{E_C L (V_{GS} - V_{TH})^2}{(V_{GS} - V_{TH}) + E_C L}$	Handles stacks correctly, sizing



## 2.F Transistor Leakage

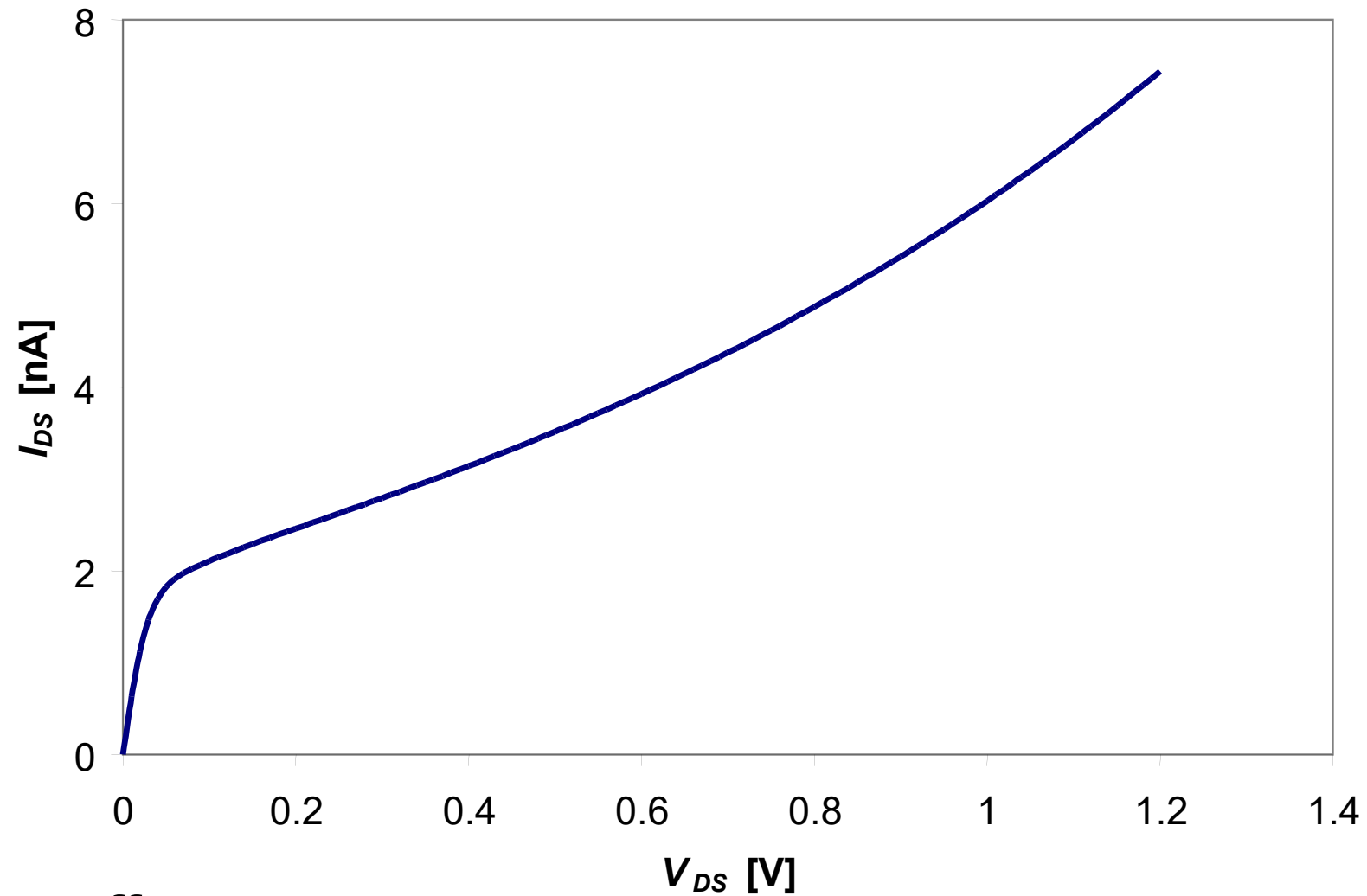
# Transistor Leakage

# Transistor Leakage



Leakage current is exponential with  $V_{GS}$

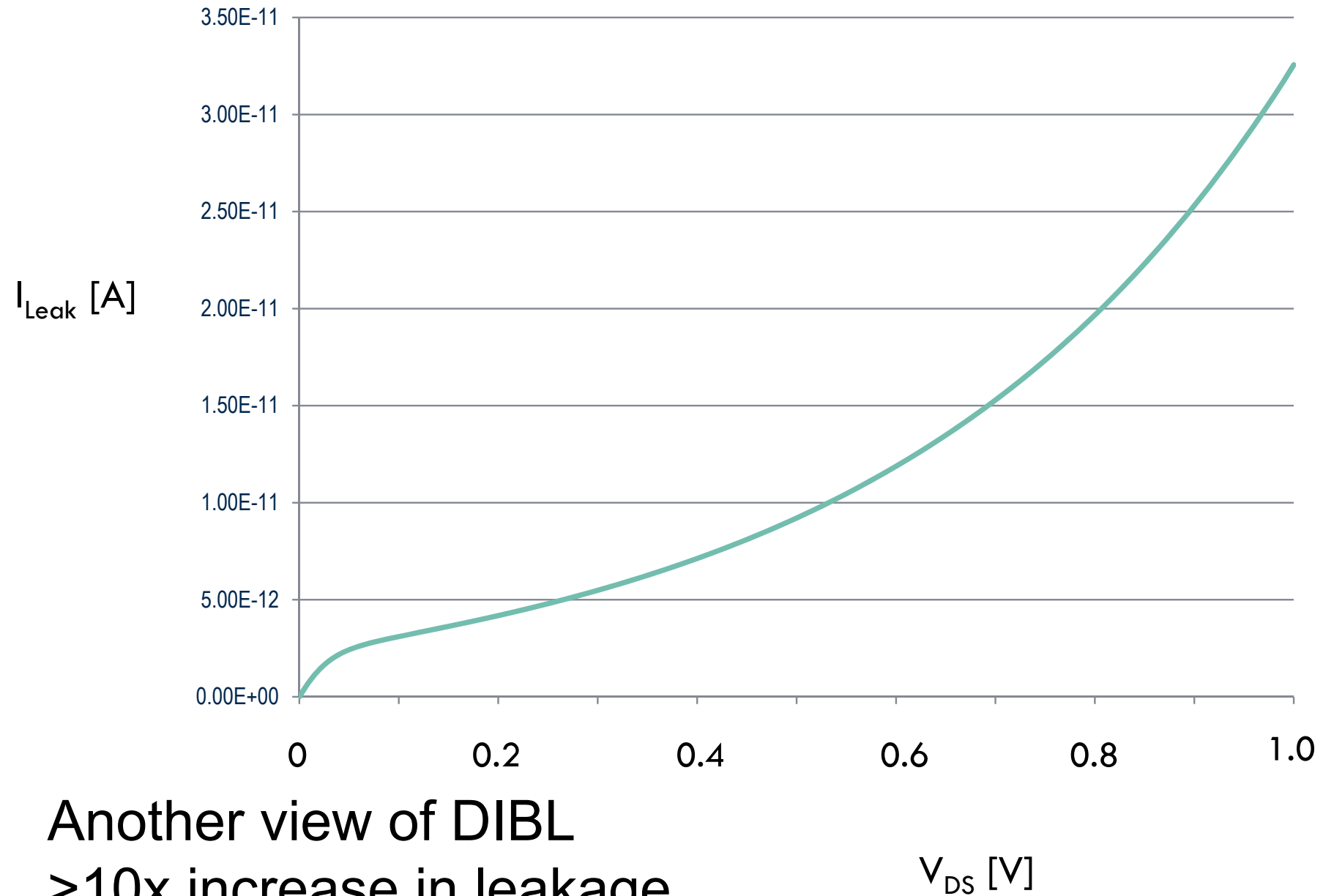
# Transistor Leakage (130nm)



Two effects:

- diffusion current (like a bipolar transistor)
- exponential increase with  $V_{DS}$  (DIBL)

# Transistor Leakage (32nm LP PTM)



Another view of DIBL  
>10x increase in leakage

# Subthreshold Current

- Subthreshold behavior can be modeled physically

$$I_{ds,subth} = \mu_{eff} C_{ox} \frac{W}{L} (m-1) \left( \frac{kT}{q} \right)^2 e^{\frac{V_{GS}-V_{Th}}{m kT/q}} \left( 1 - e^{-\frac{V_{ds}}{kT/q}} \right)$$

$$m = 1 + \frac{C_{dm}}{C_{ox}} \quad (m \sim 1.1-1.4)$$

Or (approx):

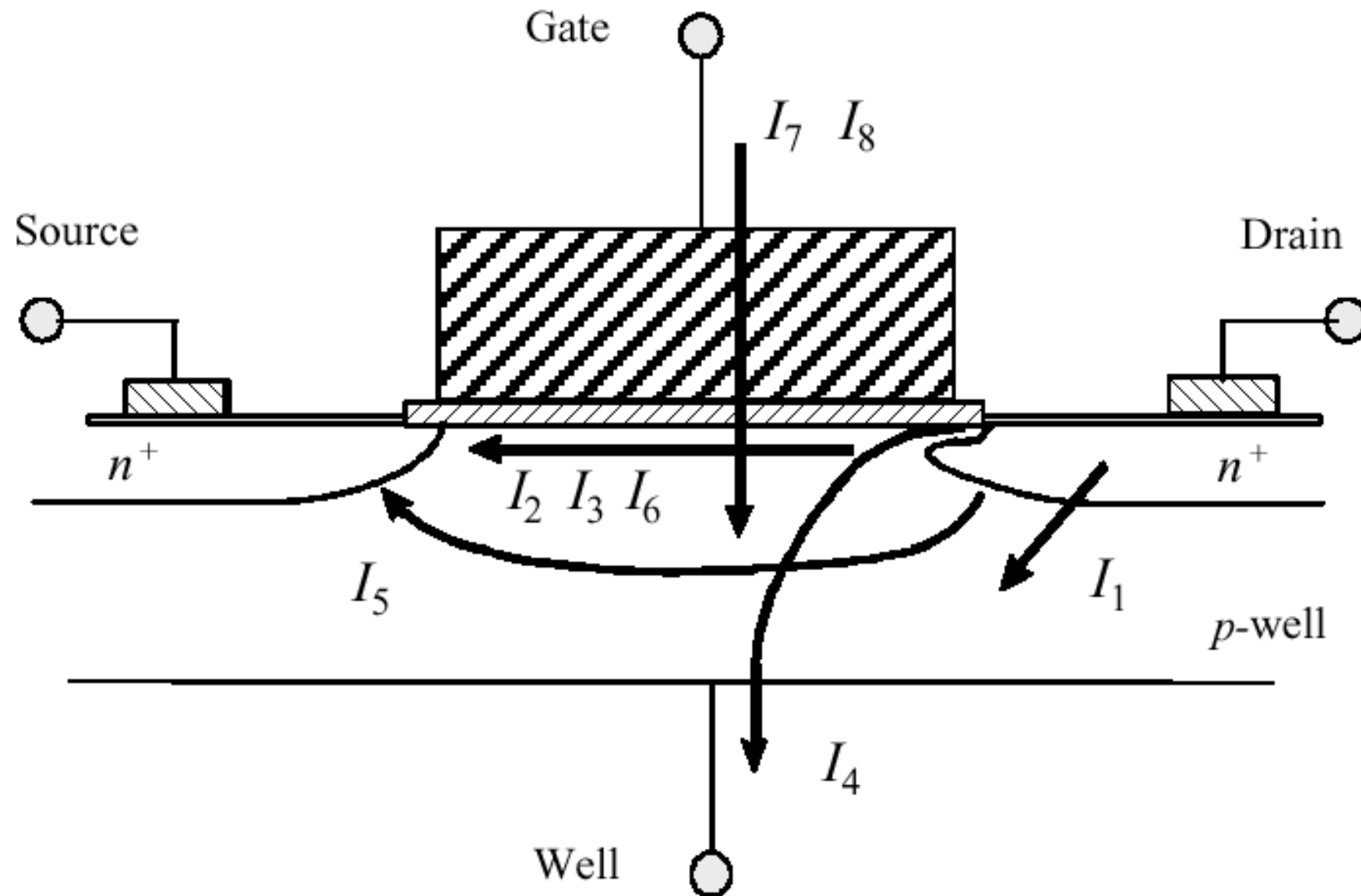
$$I_{ds,subth} = I_0 \frac{W}{W_0} 10^{\frac{(V_{gs}-V_{Th})+\gamma V_{ds}}{S}}$$

$$S = 2.3m \frac{kT}{q}$$

Taur, Ning, Modern VLSI Devices



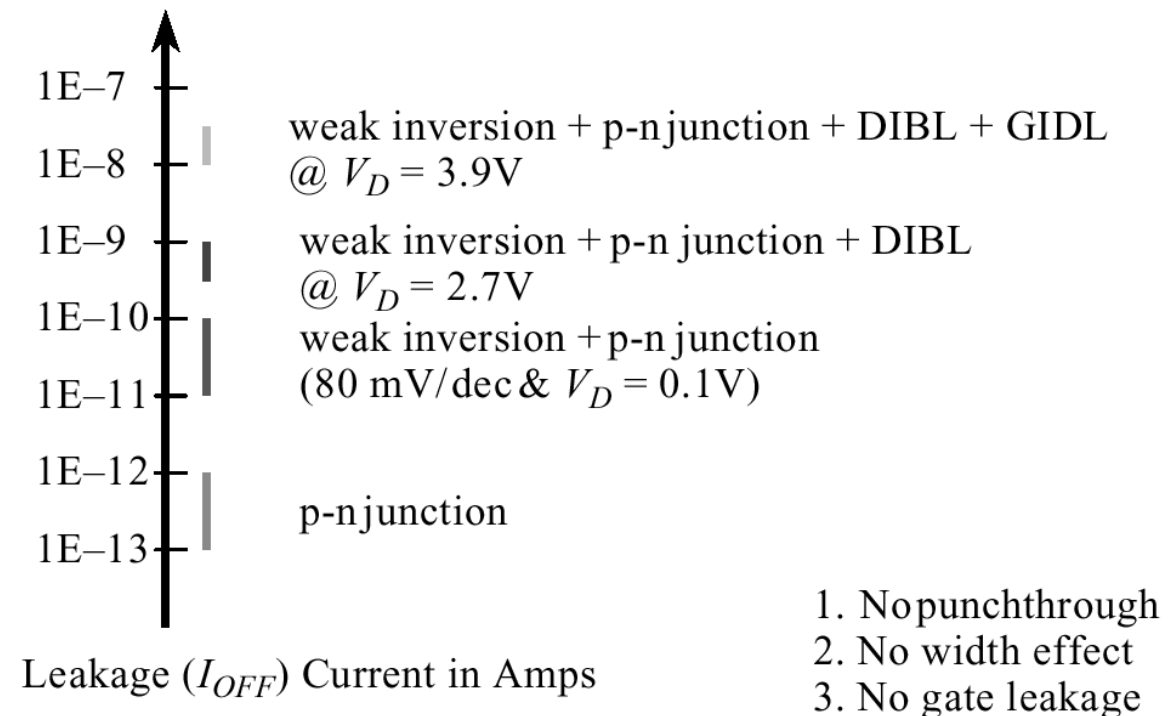
# Leakage Components



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# Leakage Components (250nm)

1. pn junction reverse bias current
2. Weak inversion
3. Drain-induced barrier lowering (DIBL)
4. Gate-induced drain leakage (GIDL)
5. Punchthrough
6. Narrow width effect
7. Gate oxide tunneling
8. Hot carrier injection



# Leakage Components

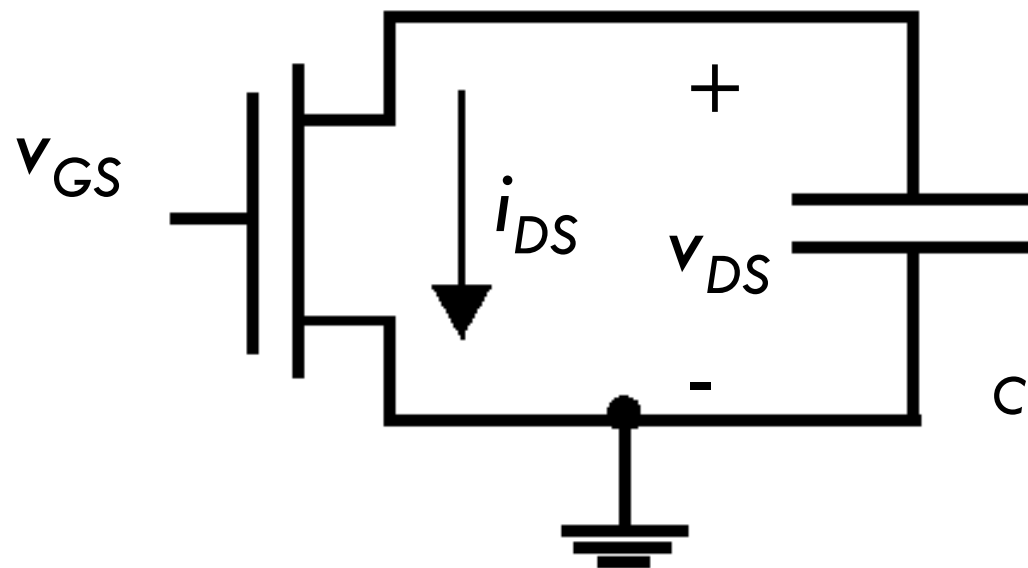
- **Drain-induced barrier lowering (DIBL)**
  - Voltage at the drain lowers the source potential barrier
  - Lowers  $V_{Th}$ , no change on  $S$
- **Gate-induced drain leakage (GIDL)**
  - High field between gate and drain increases injection of carriers into substrate -> leakage  
(band-to-band leakage)



## 2.H Transistor C-V

# MOS Transistor as a Switch

## Discharging a capacitor



- Prefer using equivalent resistances
- Find  $t_{pHL}$
- Find equivalent  $C$ ,  $R$

- Can solve:

$$i_{DS} = i_{DS}(v_{DS})$$

$$i_{DS} = C(v_{DS}) \frac{dv_{DS}}{dt}$$

$$t_{pHL} = \int \frac{C(v_{DS}) dv_{DS}}{i_{DS}(v_{GS}, v_{DS})}$$

# MOS Capacitances

- Gate capacitance
  - Non-linear channel capacitance
  - Linear overlap, fringing capacitances
  - Miller effect on overlap, fringing capacitance
- Non-linear drain diffusion capacitance
  - PN junction
- Wiring capacitances
  - Linear

# Gate and Drain Capacitances

Gate capacitance

Drain capacitance

# Gate Capacitances



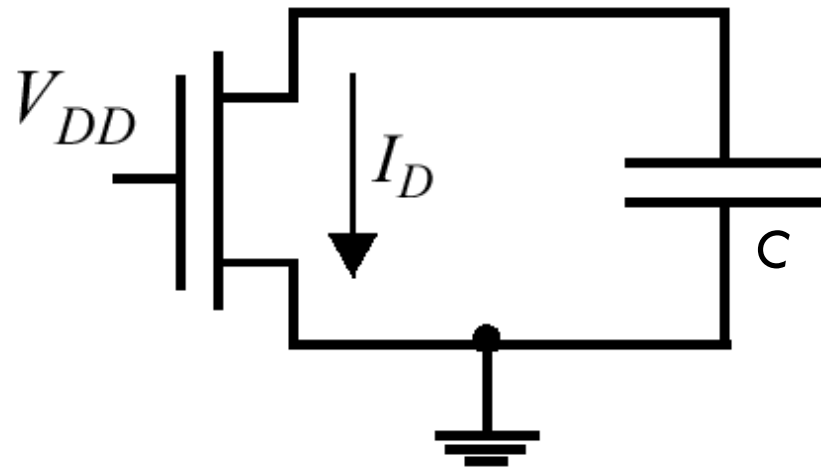
# Gate Capacitances

- Gate capacitance is non-linear
  - First order approximation with  $C_{ox}WL$  ( $C_{ox}L = 2\text{fF}/\mu\text{m}$ )
- Need to find the actual equivalent capacitance by simulating it
- Since this is a linear approximation of non-linear function, it is valid only over the certain range
  - Different capacitances for HL, LH transitions and power computation
- Drain capacitance non-linearity compensates
  - But this changes with fanout

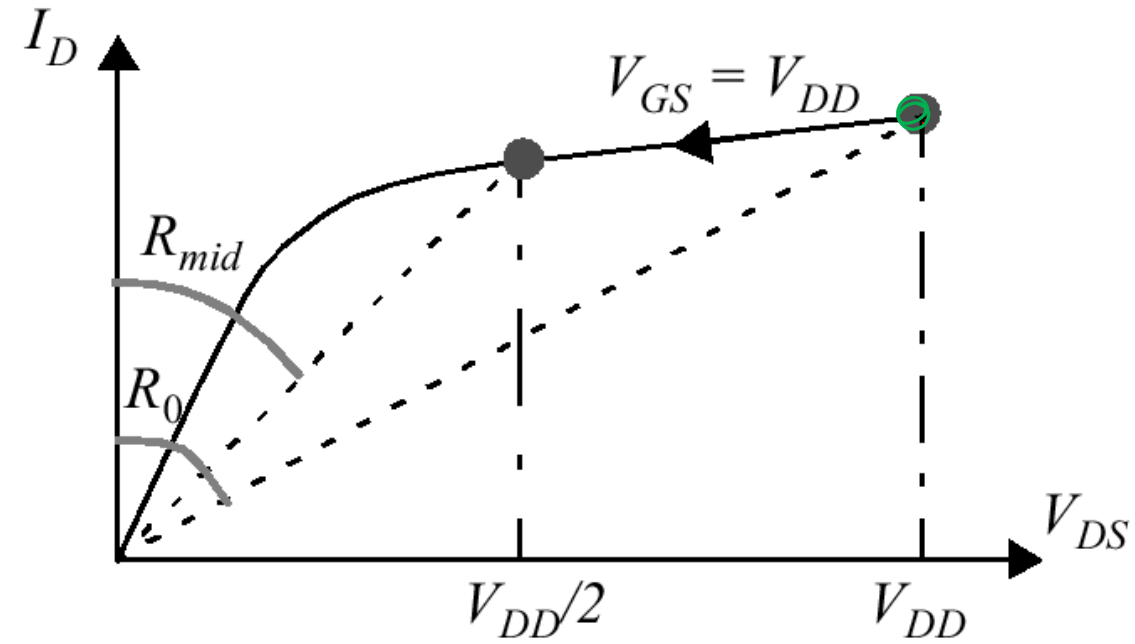


## 2.1 Delay Revisited

# MOS Transistor as a Switch (EECS251A)



Traversed path



$$R_{eq} = \text{average}_{t=t_1 \dots t_2} (R_{on}(t)) = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} R_{on}(t) dt = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} \frac{V_{DS}(t)}{I_D(t)} dt$$

$$\approx \frac{1}{2} (R_{on}(t_1) + R_{on}(t_2))$$

# MOS Transistor as a Switch (EE241A)

Solving the integral:

$$R_{eq} = \frac{1}{-V_{DD}/2} \int_{V_{DD}}^{V_{DD}/2} \frac{V}{I_{DSAT}(1 + \lambda V)} dV \approx \frac{3}{4} \frac{V_{DD}}{I_{DSAT}} \left( 1 - \frac{7}{9} \lambda V_{DD} \right)$$

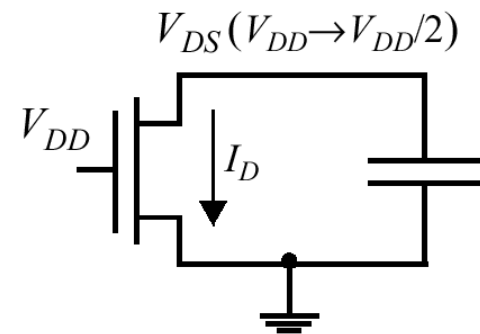
with appropriately calculated  $I_{dsat}$

Averaging resistances:

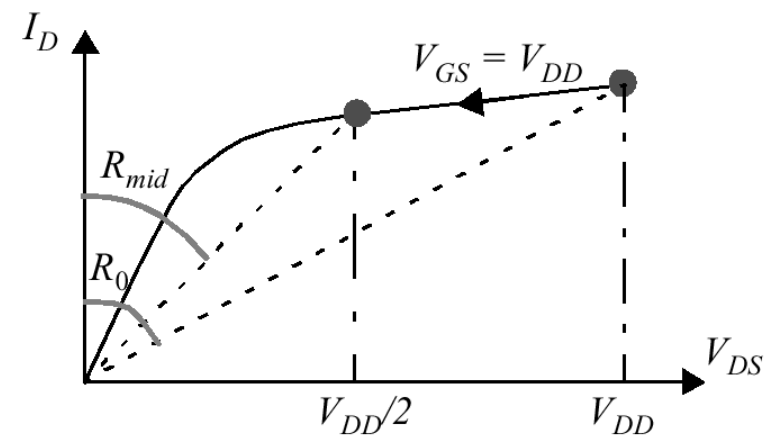
$$R_{eq} = \frac{1}{2} \left( \frac{V_{DD}}{I_{DSAT}(1 + \lambda V_{DD})} + \frac{V_{DD}/2}{I_{DSAT}(1 + \lambda V_{DD}/2)} \right) \approx \frac{3}{4} \frac{V_{DD}}{I_{DSAT}} \left( 1 - \frac{5}{6} \lambda V_{DD} \right)$$

# CMOS Performance

Propagation delay:  $t_{pHL} = (\ln 2)R_{eqn}C_L$      $t_{pLH} = (\ln 2)R_{eqp}C_L$



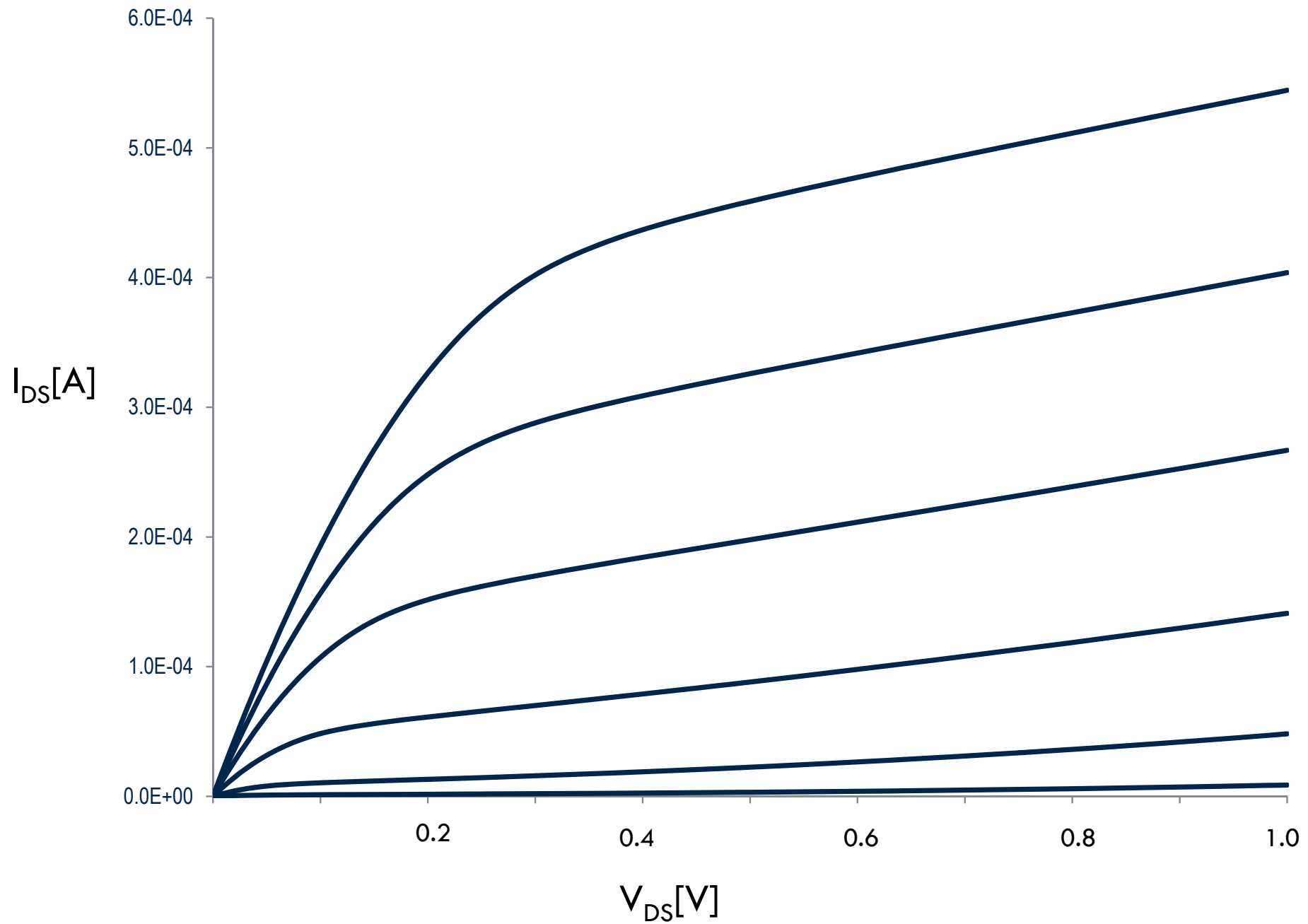
(a) schematic



(b) trajectory traversed on ID-VDS curve.

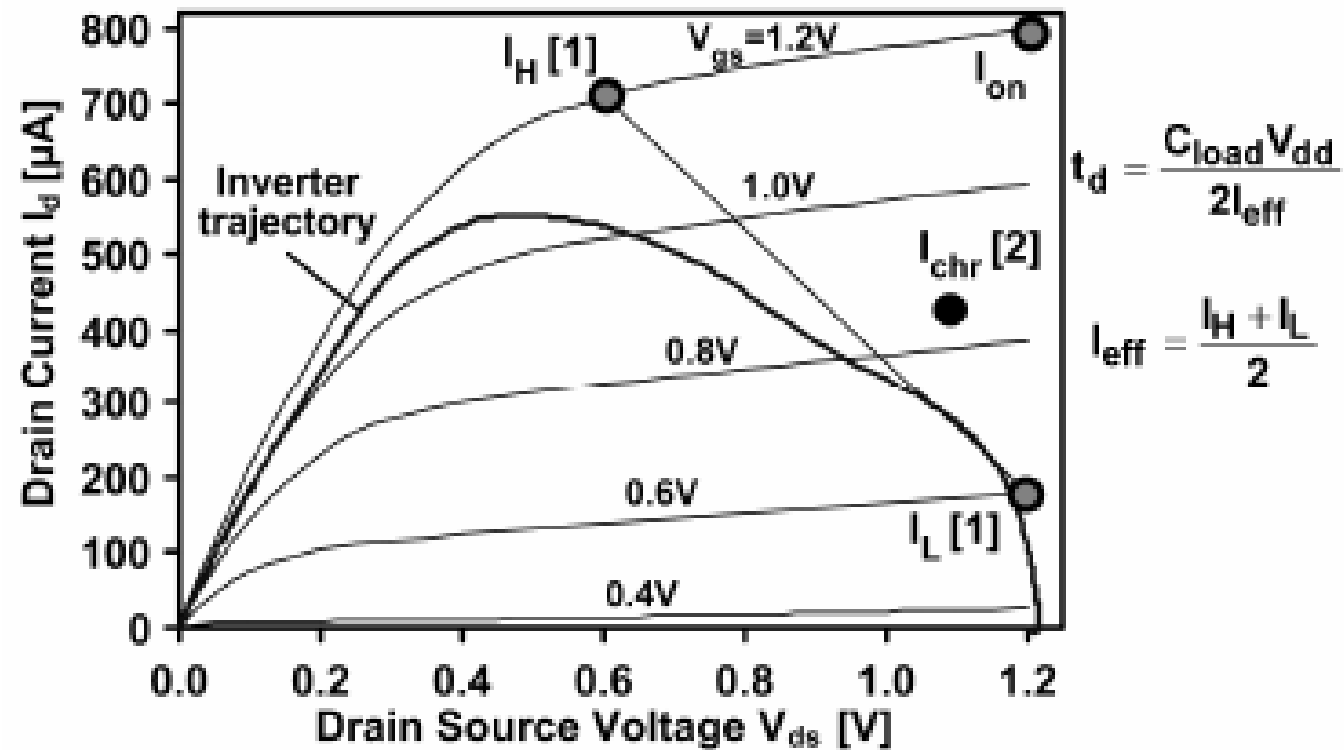
$\ln 2 = 0.7$

# Switching Trajectory



# Effective Current

- $I_{on}(V_{DD})$  is never reached
- Define  $I_{eff} = (I_H + I_L)/2$
- $I_L = I_{DS}(V_{GS}=V_{DD}/2, V_{DS}=V_{DD}); I_H = I_{DS}(V_{GS}=V_{DD}, V_{DS}=V_{DD}/2),$

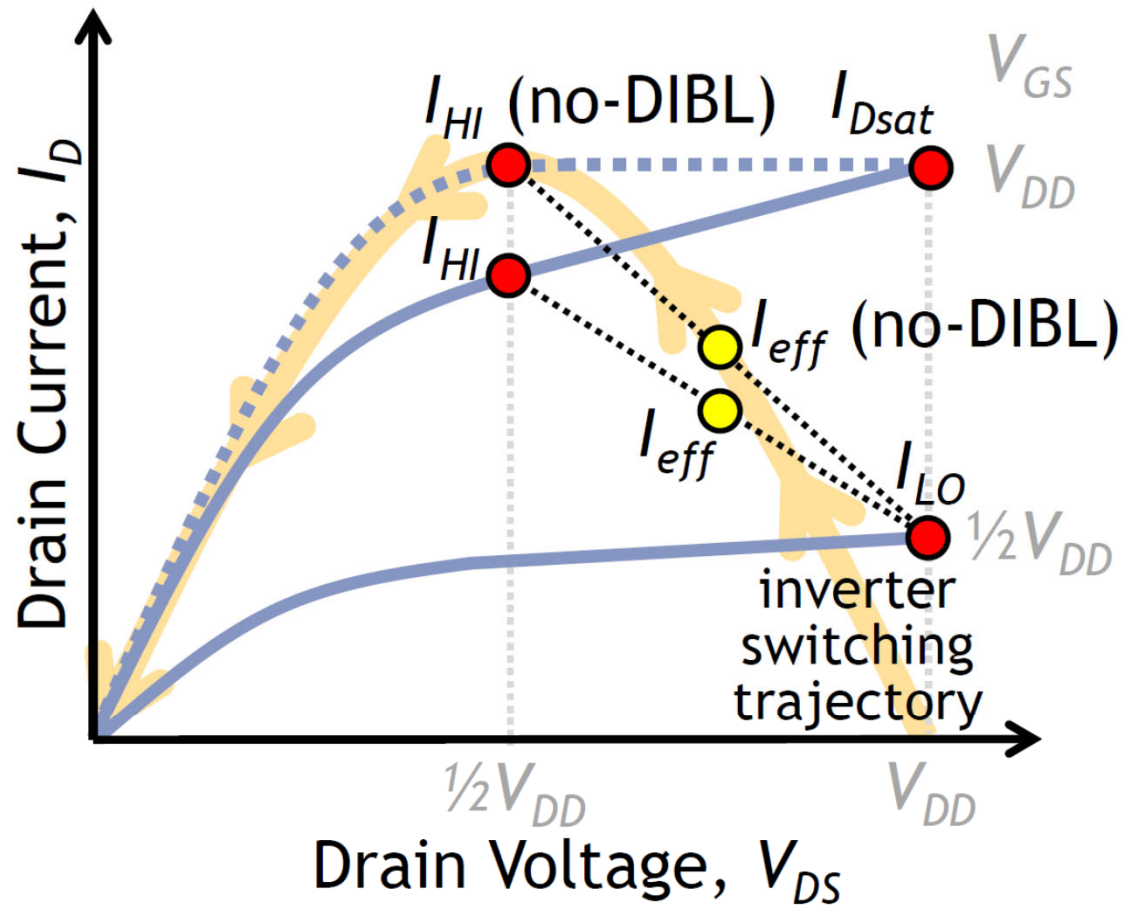


Na, IEDM'2002

Von Arnim, IEDM'2007

# DIBL Matters

- A. Loke, VLSI'16
  - FinFET, FDSOI – less DIBL



$$I_{eff} = \frac{I_{LO} + I_{HI}}{2}$$

$I_{LO}$  @  $V_{GS} = 1/2 V_{DD}$ ,  $V_{DS} = V_{DD}$

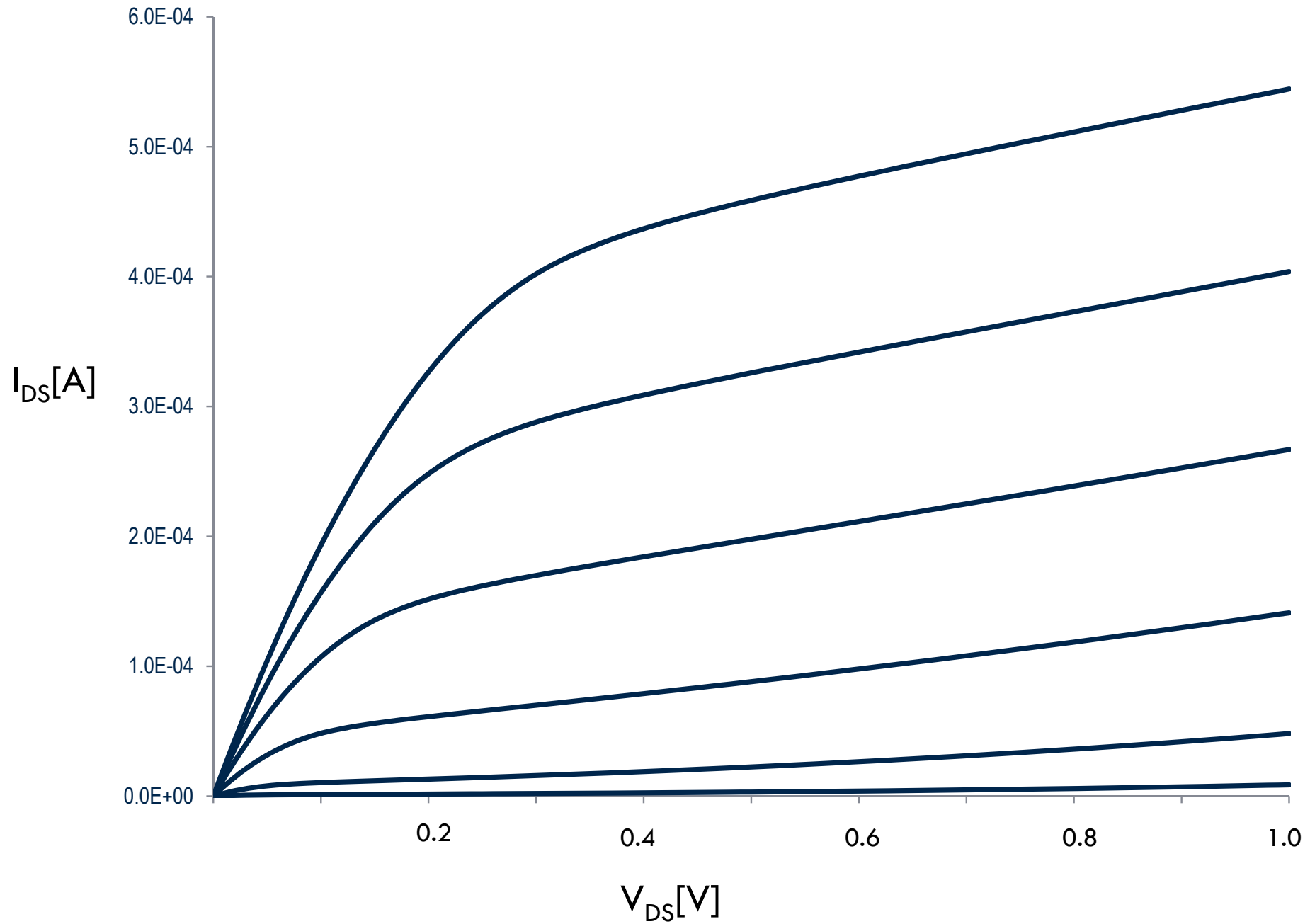
$I_{HI}$  @  $V_{GS} = V_{DD}$ ,  $V_{DS} = 1/2 V_{DD}$

$I_{eff}$  is better than  $I_{Dsat}$  for estimating inverter  $CV/I$  switching delay

Less DIBL → higher  $I_{eff}$  &  $r_{out}$  for same  $I_{Dsat}$

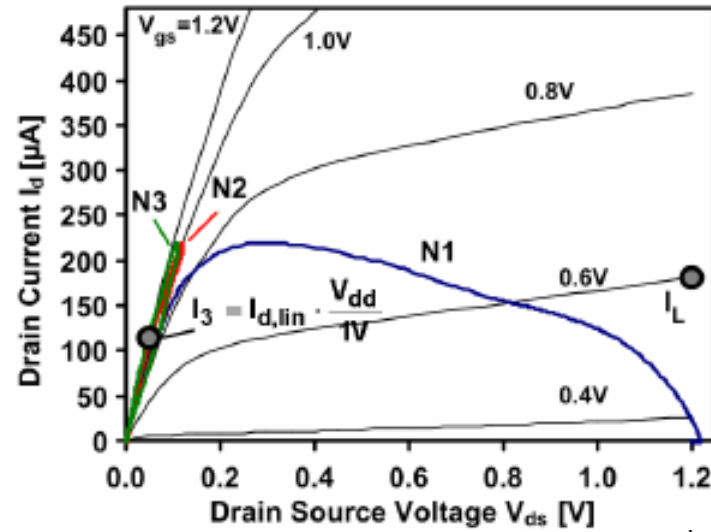


# Transistor Stacks

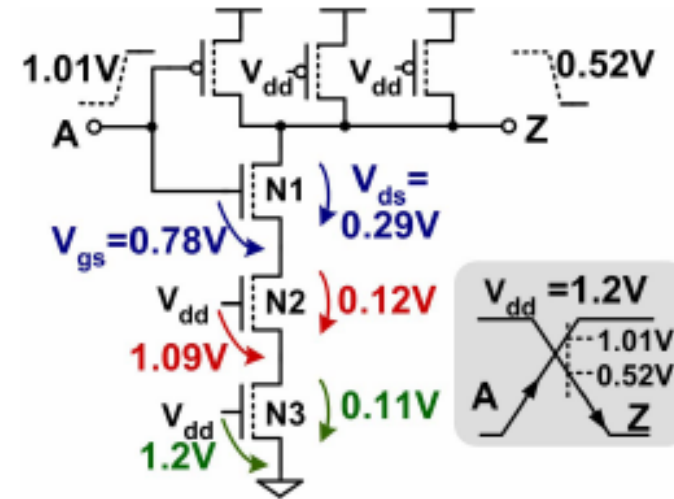


# Effective Current in Stacks

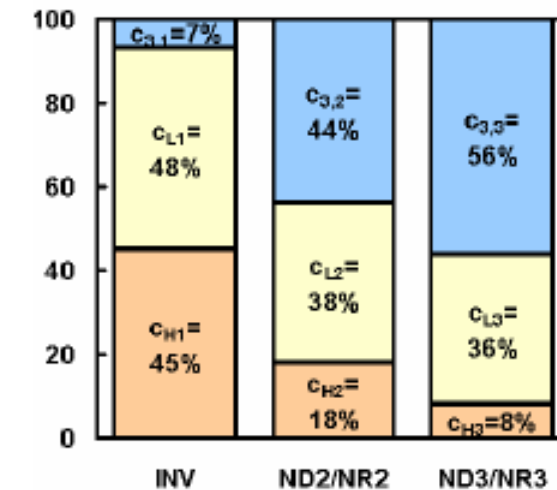
➤ Add linear current,  $I_3$



$V_{gs}$	$V_t$
$V_{dd}$	$V_{dd}/2$
$V_{dd}/2$	$V_{dd}$
$V_{dd}$	$0.05 \cdot V_{dd}$



**Model:**  $I_{stack,i} = c_{H,i} I_H + c_{L,i} I_L + c_{3,i} I_3 \Rightarrow$   
**Inverter:**  $I_{stack1} = 0.45 \cdot I_H + 0.48 \cdot I_L + 0.07 \cdot I_3$   
**NAND2/NOR2:**  $I_{stack2} = 0.18 \cdot I_H + 0.38 \cdot I_L + 0.44 \cdot I_3$   
**NAND3/NOR3:**  $I_{stack3} = 0.08 \cdot I_H + 0.36 \cdot I_L + 0.56 \cdot I_3$



Von Arnim, IEDM'2007



## Next Lecture

- Standard Cells