Lecture 13: Material Properties

Lecture Outline

- Reading: Senturia Chpt. 8
- Lecture Topics:
  - Elasticity: Nomenclature
    - Stress
    - Strain
    - Poisson Ratio
  - Material Properties
    - Young's modulus
    - Yield strength
    - Quality factor
  - On-chip Measurement of Material Properties
  - Anisotropic Material Properties
**Normal Stress (1D)**

If the force acts normal to a surface, then the stress is called a normal stress.

\[
\sigma = \frac{F}{A} \quad \text{[N/m}^2\text{] or [Pa]}
\]

 stress per unit area

- Force assumed uniform over the whole area A
- Standard mks unit

Microscopic Definition: Force per unit area acting on the surface of a differential volume element of a solid body.

Note: Force acts uniformly over each face.

**Differential volume element**

**Strain (1D)**

Strain : \( \varepsilon = \frac{L' - L}{L} = \frac{\Delta L}{L} \) (Unitless)

Sometimes a unit called "microstrain" is used, where

\[ \mu \varepsilon = \frac{\Delta L}{L} \text{ per unit in } 10^6 \]

In the elastic regime (i.e., for "small" strain)

\[ \sigma = E \varepsilon \] (Unitless)

For solids:

- MPA: Glass
- GPa: PolySi: 150 GPa

Yield stress relates to stress of elasticity.
The Poisson Ratio

Apply normal stress to a free-standing object

\( \varepsilon_x = \frac{\Delta w}{w} \)  
\( \gamma = -\nu \varepsilon_x \)

Shear Stress & Strain (2D)

Note: Assume compensating forces are applied to the vertical faces to avoid a net torque. (This by convention)

Shear Stress:
\[ \tau = \frac{F}{A} \]  

Shear Strain:
\[ \gamma = \frac{\tau}{G} \]  
\[ G = \frac{E}{2(1+\nu)} \]
2D and 3D Considerations

• Important assumption: the differential volume element is in static equilibrium → no net forces or torques (i.e., rotational movements)
  - Every σ must have an equal σ in the opposite direction on the other side of the element
  - For no net torque, the shear forces on different faces must also be matched as follows:
    \[ \tau_{xy} = \tau_{yx} \quad \tau_{xz} = \tau_{zx} \quad \tau_{yz} = \tau_{zy} \]

2D Strain

• In general, motion consists of
  - rigid-body displacement (motion of the center of mass)
  - rigid-body rotation (rotation about the center of mass)
  - Deformation relative to displacement and rotation

• Must work with displacement vectors
• Differential definition of axial strain:
  \[ \varepsilon_x = \frac{u_x(x + \Delta x) - u_x(x)}{\Delta x} = \frac{\partial u_x}{\partial x} \]
2D Shear Strain

For shear strains, must remove any rigid body rotation that accompanies deformation.

\[ \gamma_{xy} = \theta_1 + \theta_2 = \frac{\Delta u_x}{\Delta y} + \frac{\Delta u_y}{\Delta x} = \left( \frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x} \right) \]

For small amplitude deformations
\[ \xi = \phi = 0 \]

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Volume Change for a Uniaxial Stress

Stresses acting on a differential volume element. Given an x-directed uniaxial stress, \( \sigma_x \), what is the \( \Delta V \)?

\[ \Delta x \to \Delta x(1+\epsilon_x) \]
\[ \Delta y \to \Delta y(1-\epsilon_y) \]
\[ \Delta z \to \Delta z(1-\epsilon_z) \]

The resulting change in volume \( \Delta V \)

\[ \Delta V = \Delta x \Delta y \Delta z (1+\epsilon_x)(1-\epsilon_y)(1-\epsilon_z) \]

(\text{Assume small strain}) \[ \Delta V = \Delta x \Delta y \Delta z [(1+\epsilon_x)(1-\epsilon_y)(1-\epsilon_z) - 1] \]

\[ (1+\epsilon_x)^2 \approx 1+2\epsilon_x \]

For \( \epsilon < 0.5 \) (small), no \( \Delta V \)

If \( \epsilon < 0.5 \) (finite), \( \Delta V \) is finite.
Isotropic Elasticity in 3D

- Isotropic = same in all directions
- The complete stress-strain relations for an isotropic elastic solid in 3D: (i.e., a generalized Hooke’s Law)

\[
\begin{align*}
\epsilon_x &= \frac{1}{E} \left[ \sigma_x - \nu (\sigma_y + \sigma_z) \right] \\
\gamma_{xy} &= \frac{1}{G} \tau_{xy} \\
\epsilon_y &= \frac{1}{E} \left[ \sigma_y - \nu (\sigma_z + \sigma_x) \right] \\
\gamma_{yz} &= \frac{1}{G} \tau_{yz} \\
\epsilon_z &= \frac{1}{E} \left[ \sigma_z - \nu (\sigma_x + \sigma_y) \right] \\
\gamma_{zx} &= \frac{1}{G} \tau_{zx}
\end{align*}
\]

Basically, add in off-axis strains from normal stresses in other directions

Important Case: Plane Stress

- Common case: very thin film coating a thin, relatively rigid substrate (e.g., a silicon wafer)

- At regions more than 3 thicknesses from edges, the top surface is stress-free → \( \sigma_z = 0 \)
- Get two components of in-plane stress:

\[
\begin{align*}
\epsilon_x &= (1/E)[\sigma_x - \nu(\sigma_y + 0)] \\
\epsilon_y &= (1/E)[\sigma_y - \nu(\sigma_x + 0)]
\end{align*}
\]
**Important Case: Plane Stress (cont.)**

- Symmetry in the xy-plane → $\sigma_x = \sigma_y = \sigma$
- Thus, the in-plane strain components are: $\varepsilon_x = \varepsilon_y = \varepsilon$

  where

  $$\varepsilon_x = (1/E)[\sigma - \nu \sigma] = \frac{\sigma}{[E/(1-\nu)]} = \frac{\sigma}{E'}$$

  and where

  Biaxial Modulus $E' = \frac{E}{1-\nu}$

---

**Edge Region of a Tensile ($\sigma>0$) Film**

Net non-zero in-plane force (that we just analyzed)

At free edge, in-plane force must be zero

Film must be bent back, here

There's no Poisson contraction, so the film is slightly thicker, here

Shear stresses $F \neq 0$

F = 0

Extra peel force

Discontinuity of stress at the attached corner → stress concentration

Peel forces that can peel the film off the surface
Linear Thermal Expansion

- As temperature increases, most solids expand in volume
- Definition: linear thermal expansion coefficient

\[
\Delta \alpha_T = \frac{d\varepsilon}{dT} \quad \text{[Kelvin}^{-1}]\]

Remarks:
- \(\alpha_T\) values tend to be in the 10\(^{-6}\) to 10\(^{-7}\) range
- Can capture the 10\(^{-6}\) by using dimensions of \(\mu\)strain/K, where 10\(^{-6}\) K\(^{-1}\) = 1 \(\mu\)strain/K
- In 3D, get volume thermal expansion coefficient \(\frac{\Delta V}{V} = 3\alpha_T\Delta T\)

For moderate temperature excursions, \(\alpha_T\) can be treated as a constant of the material, but in actuality, it is a function of temperature

\[\alpha_T \text{ As a Function of Temperature}\]

Cubic symmetry implies that \(\alpha\) is independent of direction

[Madou, Fundamentals of Microfabrication, CRC Press, 1998]
Thin-Film Thermal Stress

Assume film is deposited stress-free at a temperature $T_d$, then the whole thing is cooled to room temperature $T_r$.

Substrate much thicker than thin film $\rightarrow$ substrate dictates the amount of contraction for both it and the thin film.

\[ \varepsilon_s = -\alpha_{Ts} \Delta T, \quad \text{where} \quad \Delta T = T_d - T_r \]

Silicon Substrate ($\alpha_{Ts} = 2.8 \times 10^{-6} \text{ K}^{-1}$)

Thin Film

Linear Thermal Expansion

But the film is attached to the substrate, so the actual strain in the film:

\[ \varepsilon_{f,\text{attached}} = \varepsilon_s = -\alpha_{Ts} \Delta T \]

Thermal Mismatch Strain:

\[ \varepsilon_{f,\text{mismatch}} = (\alpha_{Ts} - \alpha_{Tf}) \Delta T \]

Note that this is a mismatch strain.

It can only be developed by an in-plane biaxial stress:

\[ \sigma_{f,\text{mismatch}} = \frac{E}{(1-\nu)} \varepsilon_{f,\text{mismatch}} \]

Example: Thin film is polyimide $\rightarrow \alpha_{Tf} = 70 \times 10^{-6} \text{ K}^{-1}$, $E = 4.6 \text{ GPa}$

Degraded at 230°C, then cooled to RT = 25°C $\rightarrow \Delta T = 225 \text{ K}$

\[ \varepsilon_{f,\text{mismatch}} = (70 \times 10^{-6})(225) = 1.5 \times 10^{-2} \]

\[ \sigma_{f,\text{mismatch}} = \frac{4.6}{(1-0.3)}(1.5 \times 10^{-2}) = 60.5 \text{ MPa} \]

Since it is (t) tensile, an anti-parallel stress or compressive
MEMS Material Properties

Material Properties for MEMS

<table>
<thead>
<tr>
<th>Material</th>
<th>Density, $\rho$</th>
<th>Modulus, $E_s$</th>
<th>$E/\rho$</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Kg/m$^3$</td>
<td>GPa</td>
<td>GN/kg-m</td>
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Units: $(m/s)^2$

$\sqrt{E/\rho}$ is acoustic velocity

[Mark Spearing, MIT]
**Young's Modulus Versus Density**

Lines of constant acoustic velocity

[Ashby, Mechanics of Materials, Pergamon, 1992]

### Yield Strength

- **Definition:** the stress at which a material experiences significant plastic deformation (defined at 0.2% offset pt.)
- **Below the yield point:** material deforms elastically → returns to its original shape when the applied stress is removed
- **Beyond the yield point:** some fraction of the deformation is permanent and non-reversible

- **Yield Strength:** defined at 0.2% offset pt.
- **Elastic Limit:** stress at which permanent deformation begins
- **Proportionality Limit:** point at which curve goes nonlinear
- **True Elastic Limit:** lowest stress at which dislocations move
Yield Strength (cont.)

- Below: typical stress vs. strain curves for brittle (e.g., Si) and ductile (e.g., steel) materials

![Stress and Strain Diagram]

Young's Modulus and Useful Strength

<table>
<thead>
<tr>
<th>Material</th>
<th>Modulus, $E$, GPa</th>
<th>Useful Strength*, $\sigma_f$, MPa</th>
<th>$\frac{\sigma_f}{E} \times 10^3$</th>
<th>$\frac{\sigma_f^2}{E}$ MJ/m$^3$</th>
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Young’s Modulus Versus Strength

[Ashby, Mechanics of Materials, Pergamon, 1992]

Quality Factor (or Q)
Quality Factor (or $Q$)

• Measure of the frequency selectivity of a tuned circuit

**Definition:**
\[
Q = \frac{\text{Total Energy Per Cycle}}{\text{Energy Lost Per Cycle}} = \frac{f_0}{BW_{3\text{dB}}}
\]

• Example: series LCR circuit

\[
Q = \frac{\text{Im}(Z)}{\text{Re}(Z)} = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR}
\]

• Example: parallel LCR circuit

\[
Q = \frac{\text{Im}(Y)}{\text{Re}(Y)} = \frac{\omega_0 C}{G} = \frac{1}{\omega_0 LG}
\]

Selective Low-Loss Filters: Need $Q$

• In resonator-based filters: high tank $Q \Leftrightarrow$ low insertion loss

• At right: a 0.1% bandwidth, 3-res filter @ 1 GHz (simulated) 
  \[\Rightarrow\] heavy insertion loss for resonator $Q < 10,000$
Oscillator: Need for High Q

- **Main Function**: provide a stable output frequency
- **Difficulty**: superposed noise degrades frequency stability

\[ v_o(t) = V_0 \sin(2\pi f_0 t) \]

Ideal Sinusoid

\[ v_o(t) = (V_0 + \xi(t)) \sin(2\pi f_0 t + \theta(t)) \]

Real Sinusoid

**Frequency-Selective Tank**

**Sustaining Amplifier**

Higher Q

Tighter Spectrum

Zero-Crossing Point