Announcements

• Move midterm date to Thursday, Nov. 1 (1 week later than original tentative date)
• Website issues
Lecture Outline

- Reading: Senturia Chpts. 8, 9
- Lecture Topics:
  - Material Properties
    - Young's modulus
    - Yield strength
    - Quality factor
  - On-chip Measurement of Material Properties
  - Anisotropic Material Properties
  - Beam Bending

Quality Factor (or Q)
**Clamped-Clamped Beam μResonator**

- Resonator Beam
- Electrode
- Young’s Modulus
- Density

**Frequency:**

\[
f_o = \frac{1}{2\pi} \sqrt{k_r m_r} = 1.03 \left( \frac{E h}{\rho I_r^2} \right)
\]

**Mass:** (e.g., \( m_r = 10^{-13} \) kg)

**Quality Factor (or \( Q \))**

- Measure of the frequency selectivity of a tuned circuit
- Definition:
  \[
  Q = \frac{\text{Total Energy Per Cycle}}{\text{Energy Lost Per Cycle}} = \frac{f_o}{BW_{3dB}}
  \]
- Example: series LCR circuit
  \[
  Q = \frac{\text{Im}(Z)}{\text{Re}(Z)} = \frac{\omega_o L}{R} = \frac{1}{\omega_o CR}
  \]
- Example: parallel LCR circuit
  \[
  Q = \frac{\text{Im}(Y)}{\text{Re}(Y)} = \frac{\omega_o C}{G} = \frac{1}{\omega_o LG}
  \]
Selective Low-Loss Filters: Need Q

- In resonator-based filters: high tank Q ⇔ low insertion loss
- At right: a 0.1% bandwidth, 3-res filter @ 1 GHz (simulated)
  → heavy insertion loss for resonator Q < 10,000

Oscillator: Need for High Q

- Main Function: provide a stable output frequency
- Difficulty: superposed noise degrades frequency stability

\[
\omega_0 = \frac{2\pi}{T_0}
\]

Ideal Sinusoid: \( v_0(t) = V_0 \sin(2\pi f_0 t) \)

Real Sinusoid: \( v_0(t) = (V_0 + \alpha t) \sin(2\pi f_0 t + \theta(t)) \)
**Attaining High Q**

- **Problem**: IC's cannot achieve Q's in the thousands
  - Transistors ⇒ consume too much power to get Q
  - On-chip spiral inductors ⇒ Q's no higher than ~10
  - Off-chip inductors ⇒ Q's in the range of 100's
- **Observation**: vibrating mechanical resonances ⇒ Q > 1,000
- **Example**: quartz crystal resonators (e.g., in wristwatches)
  - Extremely high Q's ~ 10,000 or higher (Q ~ 10^6 possible)
  - Mechanically vibrates at a distinct frequency in a thickness-shear mode

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**Energy Dissipation and Resonator Q**

\[
\frac{1}{Q} = \frac{1}{Q_{\text{defects}}} + \frac{1}{Q_{\text{TED}}} + \frac{1}{Q_{\text{viscous}}} + \frac{1}{Q_{\text{support}}}
\]

- **Material Defect Losses**
- **Gas Damping**
- **Thermoelastic Damping (TED)**
- **Anchor Losses**
  - At high frequency, this is our big problem!
Thermoelastic Damping (TED)

- Occurs when heat moves from compressed parts to tensioned parts → heat flux = energy loss

\[ \zeta = \Gamma(T)\Omega(f) = \frac{1}{2Q} \]

\[ \Gamma(T) = \frac{\alpha^2TE}{4\rho C_p} \]

\[ \Omega(f_0) = 2\left( \frac{f_{TED}f}{f_{TED}^2 + f^2} \right) \]

\[ f_{TED} = \frac{\pi K}{2\rho C_p h^2} \]

\[ \zeta = \text{thermoelastic damping factor} \]
\[ \alpha = \text{thermal expansion coefficient} \]
\[ T = \text{beam temperature} \]
\[ E = \text{elastic modulus} \]
\[ \rho = \text{material density} \]
\[ C_p = \text{heat capacity at const. pressure} \]
\[ K = \text{thermal conductivity} \]
\[ f = \text{beam frequency} \]
\[ h = \text{beam thickness} \]
\[ f_{TED} = \text{characteristic TED frequency} \]

TED Characteristic Frequency

\[ f_{TED} = \frac{\pi K}{2\rho C_p h^2} \]

- Governed by
  - Resonator dimensions
  - Material properties

**TABLE 1. MATERIAL PROPERTIES**

<table>
<thead>
<tr>
<th>Property</th>
<th>Silicon</th>
<th>Quartz</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thermal expansion</td>
<td>2.09</td>
<td>13.70</td>
<td>ppm/K</td>
</tr>
<tr>
<td>Elastic modulus</td>
<td>1.79</td>
<td>0.78</td>
<td>1610^5 dynes/cm²</td>
</tr>
<tr>
<td>Material density</td>
<td>2.33</td>
<td>2.60</td>
<td>g/cm³</td>
</tr>
<tr>
<td>Heat capacity</td>
<td>0.75</td>
<td>0.75</td>
<td>J/g/K</td>
</tr>
<tr>
<td>Thermal conductivity</td>
<td>1.30</td>
<td>0.10</td>
<td>10¹⁶ dynes/cm²/K</td>
</tr>
<tr>
<td>Peak Damping</td>
<td>1.06</td>
<td>11.34</td>
<td>10⁻⁴</td>
</tr>
<tr>
<td>Q at 300K</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Critical Damping Factor, \( \zeta \)**

- [from Roszhart, Hilton Head 1990]
**Q vs. Temperature**

**Quartz Crystal**

- $Q \approx 5,000,000$ at 30K
- $Q \approx 300,000,000$ at 4K

**Aluminum Vibrating Resonator**

- $Q \approx 500,000$ at 30K
- $Q \approx 1,250,000$ at 4K

Even aluminum achieves exceptional $Q$'s at cryogenic temperatures.

Mechanism for $Q$ increase with decreasing temperature thought to be linked to less hysteretic motion of material defects ⇒ less energy loss per cycle.

(from Braginsky, Systems With Small Dissipation)

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**Polysilicon Wine-Glass Disk Resonator**

Compound Mode (2,1)

- $f_o = 61.37$ MHz
- $Q = 145,780$

**Resonator Data**

- $R = 32 \mu m$, $h = 3 \mu m$
- $d = 80$ nm, $V_p = 3$ V

(Y.-W. Lin, Nguyen, JSSC Dec. 04)
1.51-GHz, \( Q=11,555 \) Nanocrystalline Diamond Disk \( \mu \)Mechanical Resonator

- Impedance-mismatched stem for reduced anchor dissipation
- Operated in the 2\(^{nd}\) radial-contour mode
- \( Q \sim 11,555 \) (vacuum): \( Q \sim 10,100 \) (air)
- Below: 20 \( \mu \)m diameter disk

**Design/Performance:**

- \( R=10 \mu m, \ t=2.2 \mu m, \ d=800\AA, \ V_P=7V \)
- \( f_c=1.51 \) GHz (2\(^{nd}\) mode), \( Q=11,555 \)

\[ f_0 = 1.51 \text{ GHz}, \quad Q = 11,555 \text{ (vac)}, \quad Q = 10,100 \text{ (air)} \]

**Disk Resonator Loss Mechanisms**

- **Gas Damping**
  - No motion along the nodal axis, but motion along the finite width of the stem
- **Substrate Loss Thru Anchors**
- **Hysteretic Motion of Defect**
  - Dwarfed by Substrate Loss
  - (Not Dominant in Vacuum)
- **Substrate Hysteretic Motion of Defect**
- **Electronic Carrier Drift Loss**
  - (Dwarfed By Substrate Loss)
  - (Dwomates)
  - No motion along the nodal axis, but motion along the finite width of the stem
  - \( \lambda/4 \) helps reduce loss, but not perfect
MEMS Material Property Test Structures

Stress Measurement Via Wafer Curvature

- Compressively stressed film → bends a wafer into a convex shape
- Tensile stressed film → bends a wafer into a concave shape
- Can optically measure the deflection of the wafer before and after the film is deposited
- Determine the radius of curvature $R$, then apply:

$$\sigma = \frac{E'h^2}{6Rt}$$

- $\sigma$ = film stress [Pa]
- $E' = E/(1-\nu) =$ biaxial elastic modulus [Pa]
- $h =$ substrate thickness [m]
- $t =$ film thickness
- $R =$ substrate radius of curvature [m]
MEMS Stress Test Structure

- **Simple Approach:** use a clamped-clamped beam
  - Compressive stress causes buckling
  - Arrays with increasing length are used to determine the critical buckling load, where
    \[ \sigma_{\text{critical}} = -\frac{\pi^2 E h^2}{3 L^2} \]
    - \( E \) = Young's modulus [Pa]
    - \( I = \frac{1}{12} W h^3 \) = moment of inertia
    - \( L, W, h \) indicated in the figure
  - **Limitation:** Only compressive stress is measurable

More Effective Stress Diagnostic

- **Single structure measures both compressive and tensile stress**
- **Expansion** or **contraction** of test beam \(\rightarrow\) deflection of pointer
- **Vernier movement** indicates type and magnitude of stress
  - Expansion \(\rightarrow\) Compression
  - Contraction \(\rightarrow\) Tensile
**Q Measurement Using Resonators**

- Compound Mode (2,1)
- Wine Glass Disk Resonator
  - \( R = 32 \mu m \)
- Anchor
- Output
- Support Beams

**Resonator Data**
- \( R = 32 \mu m, h = 3 \mu m \)
- \( d = 80 \text{ nm}, V_p = 3 \text{ V} \)

**Folded-Beam Comb-Drive Resonator**

- **Issue w/ Wine-Glass Resonator**: non-standard fab process
- **Solution**: use a folded-beam comb-drive resonator

- \( f_0 = 342.5 \text{ kHz} \)
- \( Q = 41,000 \)
Comb-Drive Resonator in Action

- Below: fully integrated micromechanical resonator oscillator using a MEMS-last integration approach

Folded-Beam Comb-Drive Resonator

- Issue w/ Wine-Glass Resonator: non-standard fab process
- Solution: use a folded-beam comb-drive resonator
Measurement of Young’s Modulus

Use micromechanical resonators

- Resonance frequency depends on $E$
- For a folded-beam resonator:

$$f_o = \frac{4Eh(W/L)^3}{Meq} \left[\frac{1}{2}ight]$$

$h$ = thickness

Young’s modulus

$E$ = Young’s modulus

Equivalent mass

$M_{eq}$

- Extract $E$ from measured frequency $f_o$
- Measure $f_o$ for several resonators with varying dimensions
- Use multiple data points to remove uncertainty in some parameters

Anisotropic Materials
Elastic Constants in Crystalline Materials

- Get different elastic constants in different crystallographic directions → 81 of them in all
  - Cubic symmetries make 60 of these terms zero, leaving 21 of them remaining that need be accounted for
- Thus, describe stress-strain relations using a 6x6 matrix

\[
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\sigma_z \\
\tau_{yz} \\
\tau_{zx} \\
\tau_{xy}
\end{bmatrix} =
\begin{bmatrix}
C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\
C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\
C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & C_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & C_{55} & 0 \\
0 & 0 & 0 & 0 & 0 & C_{66}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\varepsilon_z \\
\gamma_{yz} \\
\gamma_{zx} \\
\gamma_{xy}
\end{bmatrix}
\]

Stresses Stiffness Coefficients Strains

Stiffness Coefficients of Silicon

- Due to symmetry, only a few of the 21 coefficients are non-zero
- With cubic symmetry, silicon has only 3 independent components, and its stiffness matrix can be written as:

\[
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\sigma_z \\
\tau_{yz} \\
\tau_{zx} \\
\tau_{xy}
\end{bmatrix} =
\begin{bmatrix}
C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\
C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\
C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & C_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & C_{55} & 0 \\
0 & 0 & 0 & 0 & 0 & C_{66}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\varepsilon_z \\
\gamma_{yz} \\
\gamma_{zx} \\
\gamma_{xy}
\end{bmatrix}
\]

where \[
\begin{align*}
C_{11} &= 165.7 \text{ GPa} \\
C_{12} &= 63.9 \text{ GPa} \\
C_{44} &= 79.6 \text{ GPa}
\end{align*}
\]
Young's Modulus in the (001) Plane

Poisson Ratio in (001) Plane
Anisotropic Design Implications

• Young's modulus and Poisson ratio variations in anisotropic materials can pose problems in the design of certain structures

• E.g., disk or ring resonators, which rely on isotropic properties in the radial directions
  - Okay to ignore variation in RF resonators, although some Q hit is probably being taken

• E.g., ring vibratory rate gyroscopes
  - Mode matching is required, where frequencies along different axes of a ring must be the same
  - Not okay to ignore anisotropic variations, here

Bending of Beams
Beams: The Springs of Most MEMS

- Springs and suspensions very common in MEMS
  - Coils are popular in the macro-world; but not easy to make in the micro-world
  - Beams: simpler to fabricate and analyze; become “stronger” on the micro-scale → use beams for MEMS

Comb-Driven Folded Beam Actuator

Bending a Cantilever Beam

- Objective: Find relation between tip deflection \( y(x=L) \) and applied load \( F \)
- Assumptions:
  1. Tip deflection is small compared with beam length
  2. Plane sections (normal to beam's axis) remain plane and normal during bending, i.e., “pure bending”
  3. Shear stresses are negligible
**Reaction Forces and Moments**

- **Point Load**: \( F \)
- **Moment due to \( F \)**: \( M_1 = F L \)
- **Moment due to \( F \)**: \( M_2 = F (L - x) \)

**Sign Conventions for Moments & Shear Forces**

- **(+)** moment leads to deformation with a (+) radius of curvature (i.e., upwards)
- **(-)** moment leads to deformation with a (-) radius of curvature (i.e., downwards)

- **(+)** shear forces produce clockwise rotation
- **(-)** shear forces produce counter-clockwise rotation
Beam Segment in Pure Bending

Small section of a beam bent in response to a transverse load.

Consider a segment bounded by the dashed lines defined by $\theta$:

At $\theta = 0$: (i.e., at the neutral axis)

$$\text{segment length} = dx = R d\theta \quad (1)$$

At any $\theta$:

$$\text{segment length} = dx = (R + \theta) d\theta \quad (2)$$

Combining (1) and (2):

$$dx = dx - \frac{\theta}{R} dx$$

Thus, the axial strain $\varepsilon_x$:

$$\varepsilon_x = \frac{\Delta l}{l} = -\frac{\theta}{R} = \varepsilon_x = -\frac{\theta}{R}$$

Note: (1) direction of $\theta$ is downward.

Points above the neutral axis go into tension.

Points below the neutral axis go into compression.

Neutral Axis $= \frac{1}{2}$ length of beam, unchanged by bending.