EE C245 – ME C218
Introduction to MEMS Design
Fall 2007

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Lecture 23: Input Modeling & Gyros

Announcements

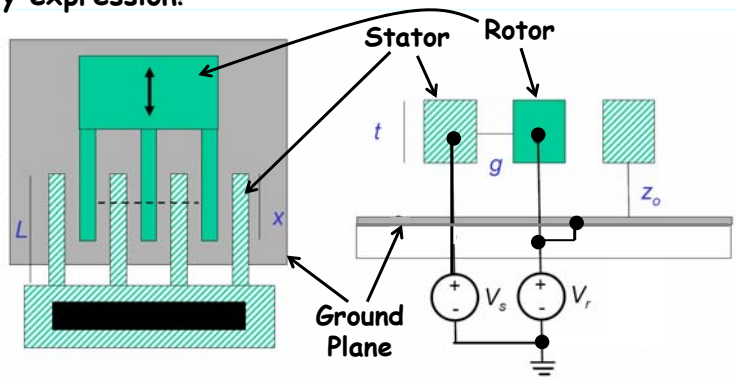
- Make-Up Lecture:
  - Friday, 11/16, from 1:00 p.m. to 2:30 p.m.
  - Room 145 McCone
- Still trying to get a room for a Monday, 11/19, make-up lecture
  - Trying for 10 a.m., but will take what I can get
- Updates to HW#6 posted
  - Basically, you can ignore the masses of the springs, since the proof masses are so large
  - Also, just identify the electrodes in part (c)
- Times for project outbriefs
  - Wednesday, Dec. 12 (morning), 9:30-12 noon
  - Sunday, Dec. 16?
  - Wednesday, Dec. 19?
  - Thursday, Dec. 20?

Lecture Outline

- Reading: Senturia Chpts. 5, 6, Handouts: "Electrostatic-Comb Drive of Lateral Polysilicon Resonators", "Electrostatic Comb Drive Levitation And Control Method", "Micromachined Inertial Sensors"
- Lecture Topics:
  - Electrostatic Comb-Drive
    - 1st Order Analysis
    - 2nd Order Analysis
    - Levitation
    - Electrical Stiffness
  - Input Circuit Modeling
  - Gyroscopes

Comb-Drive Force Equation (2nd Pass)

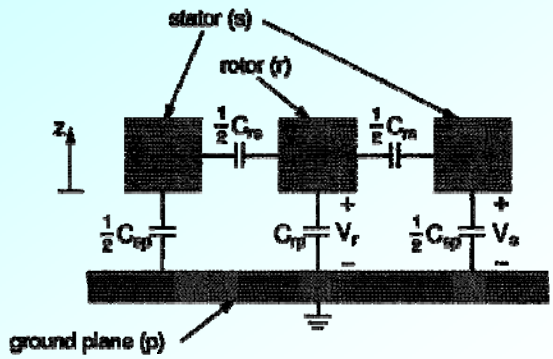
- In our 1st pass, we neglected
  - Fringing fields
  - Parallel-plate capacitance between stator and rotor
  - Capacitance to the substrate
- All of these capacitors must be included when evaluating the energy expression!



Comb-Drive Force With Ground Plane Correction

- Finger displacement changes not only the capacitance between stator and rotor, but also between these structures and the ground plane → modifies the capacitive energy

$$F_{e,x} = \frac{\partial W'}{\partial x} = \frac{1}{2}\frac{dC_{sp}}{dx}V_s^2 + \frac{1}{2}\frac{dC_{rp}}{dx}V_r^2 + \frac{1}{2}\frac{dC_{rs}}{dx}(V_s - V_r)^2$$



[Gary Fedder, Ph.D., UC Berkeley, 1994]

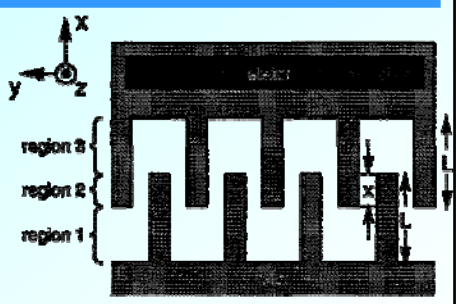
Capacitance Expressions

- Case: $V_r = V_p = 0V$ Ground Plane
- C_{sp} depends on whether or not fingers are engaged

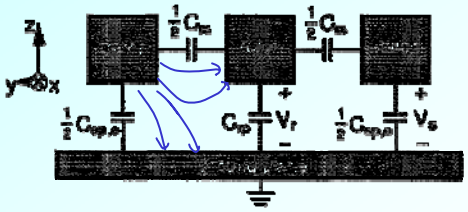
$$C_{sp} = N[C'_{sp,e}x + C'_{sp,u}(L-x)]$$

$$C_{rs} = NC'_{rs}x$$

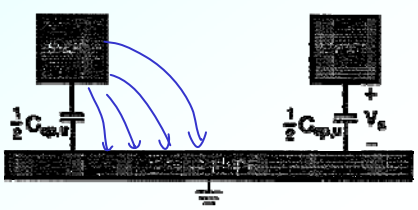
Capacitance per unit length



Region 2



Region 3



[Gary Fedder, Ph.D., UC Berkeley, 1994]

Comb-Drive Force With Ground Plane Correction

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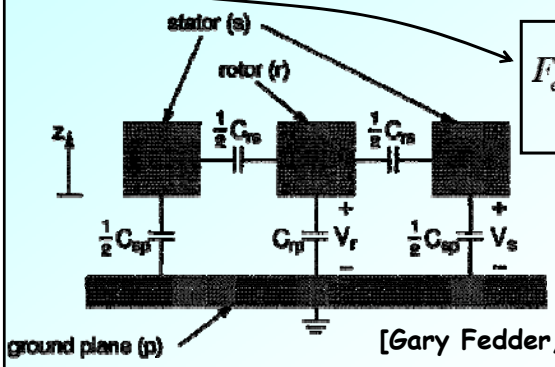
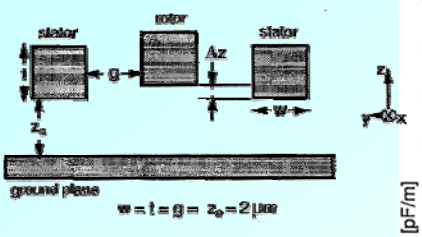
$$F_{e,x} = \frac{N}{2}(C'_{rs} + C'_{sp,e} - C'_{sp,u})V_s^2$$



[Gary Fedder, Ph.D., UC Berkeley, 1994]

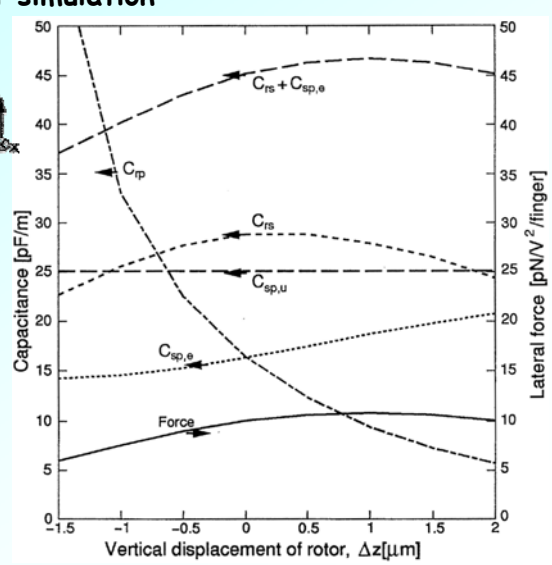
Simulate to Get Capacitors → Force

- Below: 2D finite element simulation

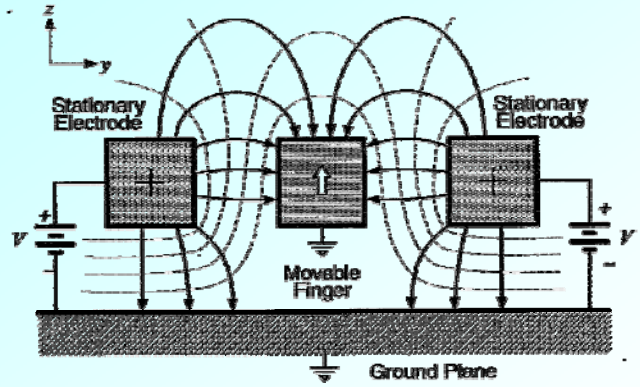


$$F_{e,x} = \frac{N}{2}(C'_{rs} + \underbrace{C'_{sp,e} - C'_{sp,u}})V_s^2$$

20-40% reduction of $F_{e,x}$



Vertical Force (Levitation)

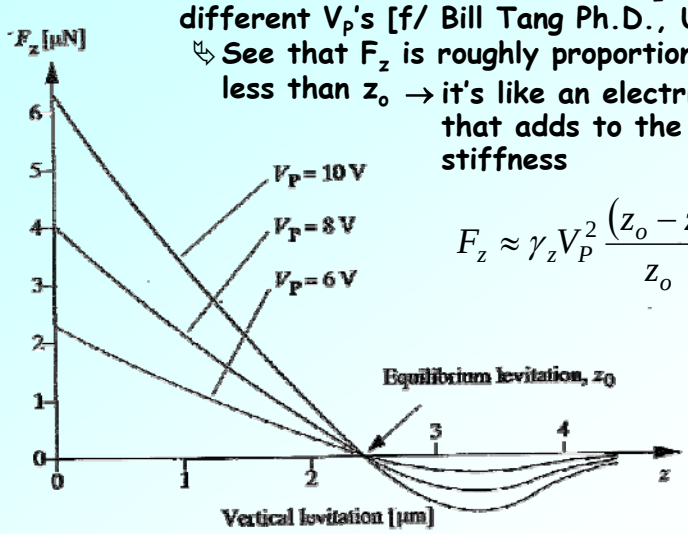


$$F_{e,z} = \frac{\partial W'}{\partial z} = \frac{1}{2}\frac{dC_{sp}}{dz}V_s^2 + \frac{1}{2}\frac{dC_{rp}}{dz}V_r^2 + \frac{1}{2}\frac{dC_{rs}}{dz}(V_s - V_r)^2$$

- For V_r = 0V (as shown): $F_{e,z} = \frac{1}{2}Nx\left[\frac{d\left(C'_{sp,e} + C'_{rs}\right)}{dz}\right]V_s^2$

Simulated Levitation Force

- Below: simulated vertical force F_z vs. z at different V_P's [f/ Bill Tang Ph.D., UCB, 1990]
  - See that F_z is roughly proportional to –z for z less than z_o → it's like an electrical stiffness that adds to the mechanical stiffness



$$F_z \approx \gamma_z V_P^2 \frac{(z_o - z)}{z_o} = k_e(z_o - z)$$

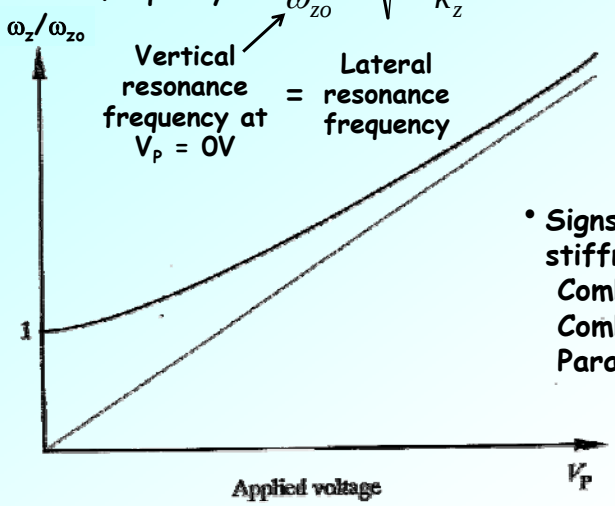
Electrical Stiffness

Vertical Resonance Frequency

Vertical resonance frequency

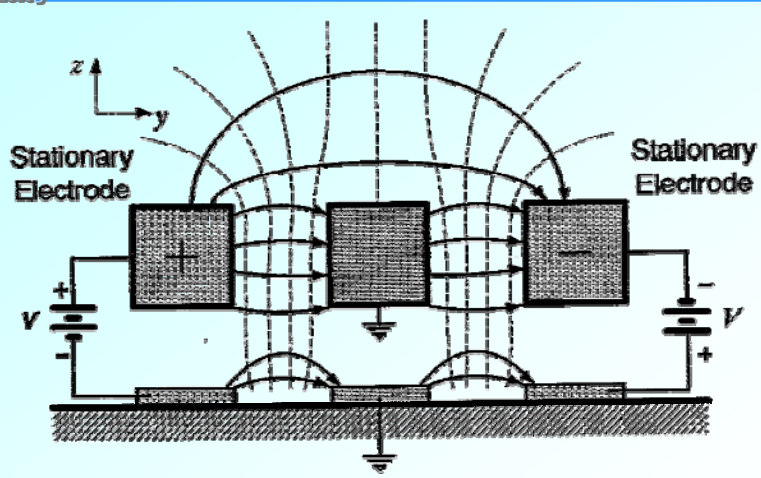
$$\frac{\omega_z}{\omega_{zo}} = \sqrt{\frac{k_z + k_e}{k_z}} \quad \text{where} \quad k_e = \left(\frac{\gamma_z}{z_o}\right)V^2$$

Vertical resonance frequency at V_P = 0V = Lateral resonance frequency



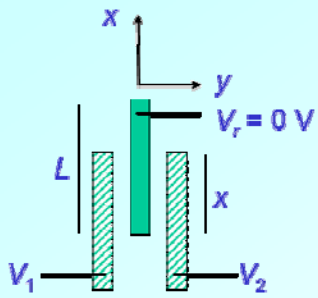
- Signs of electrical stiffnesses in MEMS:
  - Comb (x-axis) → k_e = 0
  - Comb (z-axis) → k_e > 0
  - Parallel Plate → k_e < 0

Suppressing Levitation



- Pattern ground plane polysilicon into differentially excited electrodes to minimize field lines terminating on top of comb
- Penalty: x-axis force is reduced

Force of Comb-Drive vs. Parallel-Plate



gap = g = 1 μm,
thickness = t = 2 μm
finger length = L = 100 μm
overlap length x = 75 μm

- Comb drive (x-direction)
  - $V_1 = V_2 = V_S = 1V$

$$F_{e,x} = \frac{\varepsilon_o t}{g} V_s^2$$

- Differential Parallel-Plate (y-direction)
  - $V_1 = 0V,\ V_2 = 1V$

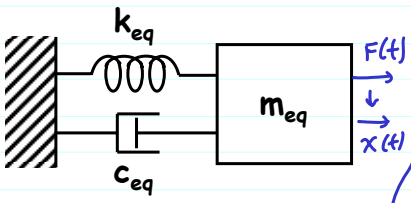
$$F_{e,y} = \frac{1}{2}\frac{\varepsilon_o t x}{g^2} V_2^2$$

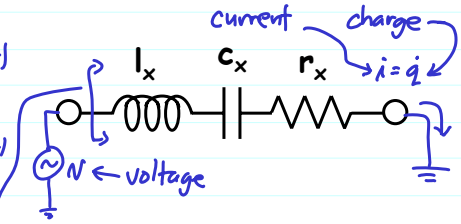
$$\frac{F_{e,y}}{F_{e,x}} = \frac{\frac{\varepsilon_o t x}{g^2} V_2^2}{\frac{2\varepsilon_o t}{g} V_s^2} = \frac{1}{2}\frac{x}{g}$$

Parallel-plate generates a much larger force; but at the cost of linearity

Input Modeling

Electromechanical Analogies





$F(t) = F\cos(\omega t) \rightarrow x(t) = X\cos\omega t$

Equation of Motion:

$$m_{eq}\ddot{x} + c_{eq}\dot{x} + k_{eq}x = F(t)$$

⇒ using phasor concepts:

$$F = j\omega m_{eq}\dot{x} + \frac{k_{eq}}{j\omega}\dot{x} + c_{eq}\dot{x}$$

⇒ by analogy:

$$\boxed{\begin{array}{lll} F \rightarrow v & m_{eq} \rightarrow l_x & c_{eq} \rightarrow r_x \\ \dot{x} \rightarrow i & k_{eq} \rightarrow \frac{1}{c_x} & \end{array}}$$

[Parameter Relationships in the Current Analogy]

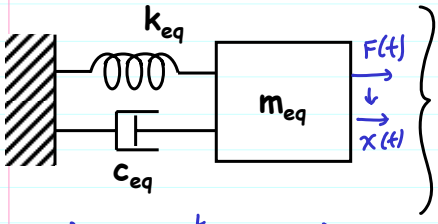
$v(t) = V\cos\omega t \rightarrow i(t) = I\cos\omega t$

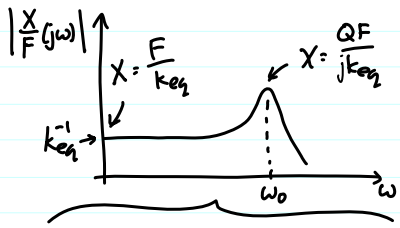
Impedance looking in:

$$\frac{v}{i} = j\omega l_x + \frac{1}{j\omega c_x} + r_x$$

$$v = j\omega l_x i + \frac{(1/c_x)}{j\omega} i + r_x i$$

Bandpass Biquad Transfer Function





$$F = j\omega m_{eq}\dot{x} + \frac{k_{eq}}{j\omega}\dot{x} + c_{eq}\dot{x}$$

⇒ converting to full phasor form:

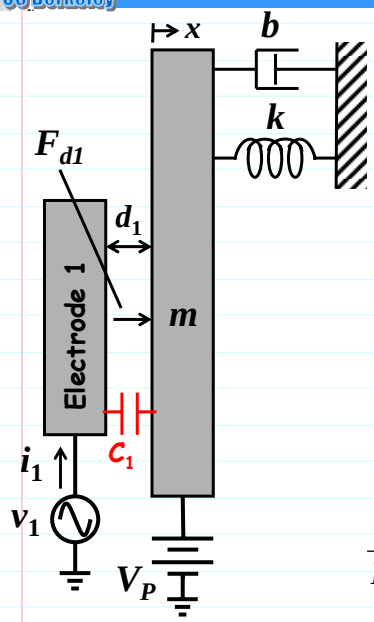
$$F = (j\omega)(j\omega X)m_{eq} + \frac{k_{eq}}{\cancel{j\omega}}(\cancel{j\omega}X) + c_{eq}(j\omega X)$$

$$\boxed{\frac{X}{F}(j\omega) = \frac{k_{eq}^{-1}}{1 - \left(\frac{\omega}{\omega_0}\right)^2 + j\frac{\omega}{Q\omega_0}}}$$

$$\frac{X}{F}(j\omega) = \frac{1}{k_{eq}}\left[-\omega^2\frac{m_{eq}}{k_{eq}} + 1 + j\frac{c_{eq}\omega}{k_{eq}}\right]^{-1} = \frac{1}{k_{eq}}\left[-\left(\frac{\omega}{\omega_0}\right)^2 + 1 + j\frac{\omega}{Q\omega_0}\right]^{-1}$$

$$\left[\frac{k_{eq}}{m_{eq}} = \omega_0^2,\ Q = \frac{m_{eq}\omega_0}{c_{eq}} = \frac{k_{eq}}{\omega_0 c_{eq}} \rightarrow \frac{k_{eq}}{c_{eq}} = Q\omega_0\right]$$

Force-to-Velocity Relationship



- The relationship between input voltage v_1 and force F_{d1}:

$$F_{d1} \approx -V_P \frac{\partial C_1}{\partial x} v_1$$

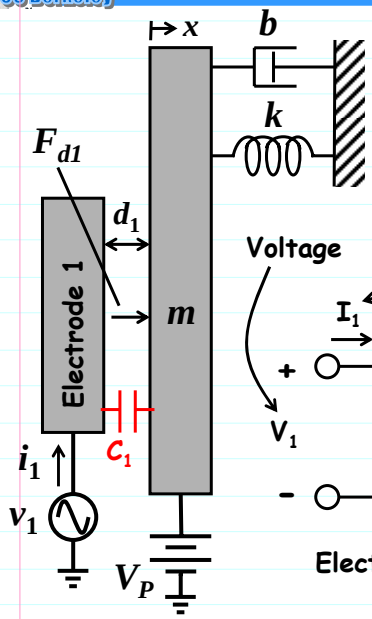
- When displacement x is the mechanical output variable:

$$\frac{X(s)}{F_{d1}(s)} = \frac{1}{k} \frac{\omega_o^2}{s^2 + (\omega_o/Q)s + \omega_o^2}$$

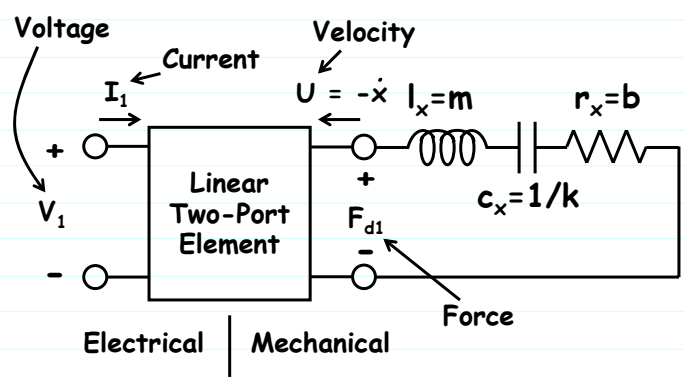
- When velocity υ is the mechanical output variable:

$$\frac{\upsilon(s)}{F_{d1}(s)} = \frac{sX(s)}{F_{d1}(s)} = \frac{1}{k} \frac{\omega_o^2 s}{s^2 + (\omega_o/Q)s + \omega_o^2}$$

Force-to-Velocity Equiv. Ckt.

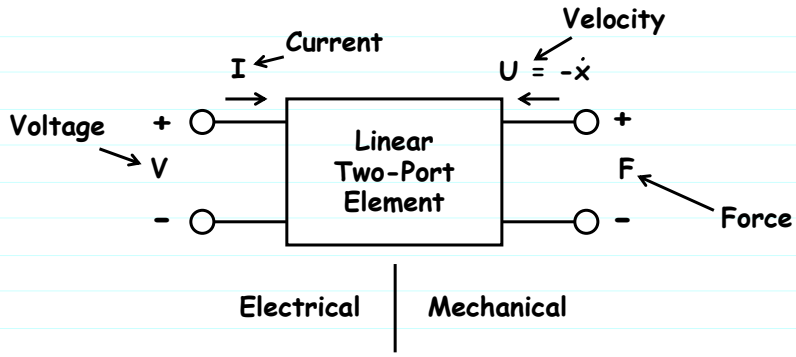


- Combine the previous lumped LCR mechanical equivalent circuit with a circuit modeling the capacitive transducer → circuit model for voltage-to-velocity

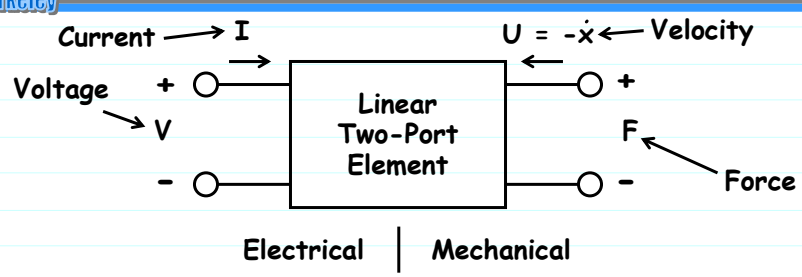


Equiv. Circuit for a Linear Transducer

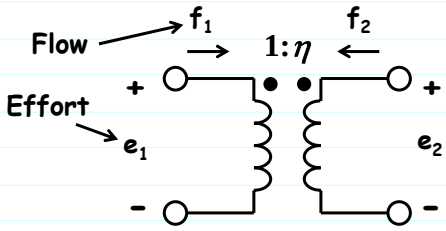
- A transducer …
  - converts energy from one domain (e.g., electrical) to another (e.g., mechanical)
  - has at least two ports
  - is not generally linear, but is virtually linear when operated with small signals (i.e., small displacements)



Equiv. Circuit for a Linear Transducer



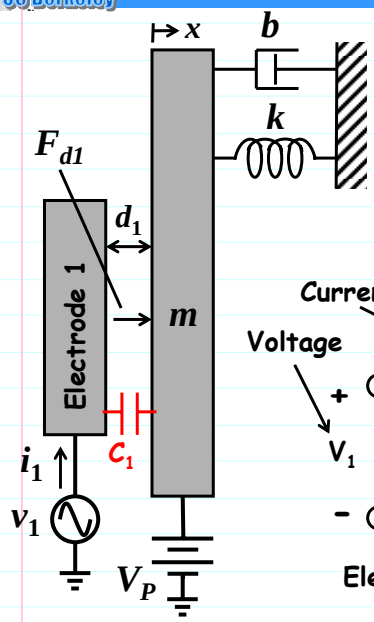
- For physical consistency, use a transformer equivalent circuit to model the energy conversion from the electrical domain to mechanical domain



$$\begin{bmatrix} e_2 \\ f_2 \end{bmatrix} = \begin{bmatrix} \eta & 0 \\ 0 & -\frac{1}{\eta} \end{bmatrix} \begin{bmatrix} e_1 \\ f_1 \end{bmatrix}$$

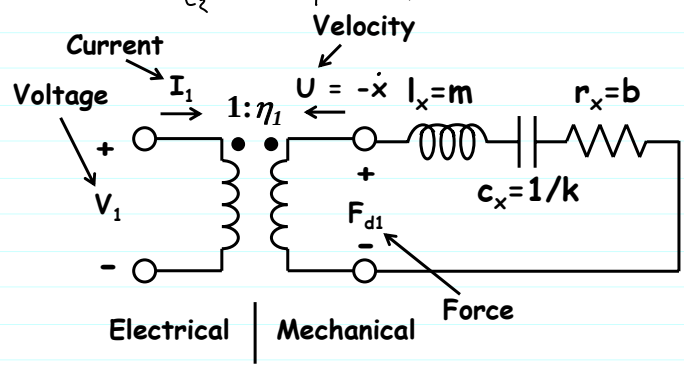
Describing Matrix

Electromechanical Equivalent Circuit



- $e_2 = F_{d1}$, $e_1 = v_1$, just need η_1:
- From the matrix: $e_2 = \eta e_1$

$$\underbrace{F_{d1}}_{e_2} \approx \underbrace{-V_P \frac{\partial C_1}{\partial x}}_{\eta} \underbrace{v_1}_{e_1} \longrightarrow \boxed{\eta_1 = \left| V_P \frac{\partial C_1}{\partial x} \right|}$$

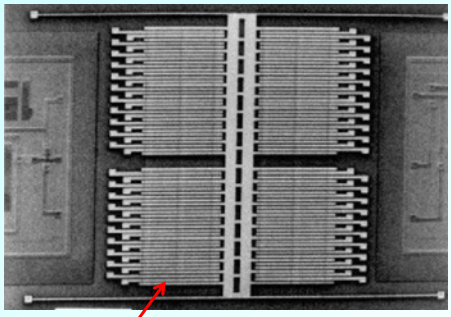


Output Modeling

Types of Capacitive Outputs

Low Frequency

- Example: accelerometer
- Acceleration inputs are generally at low frequency, e.g., near DC

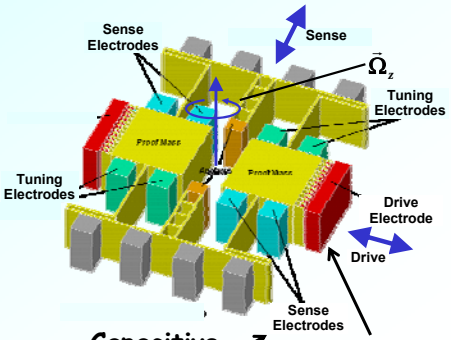


Differential Capacitive Pickoff

ADXL-50

High Frequency

- Example: gyroscope
- Rotation-derived sense forces are generally sinusoidal, e.g., resonance



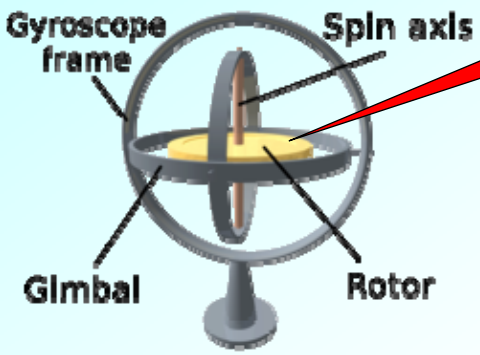
Capacitive Pickoff

Capacitive Drive

Gyroscopes

Classic Spinning Gyroscope

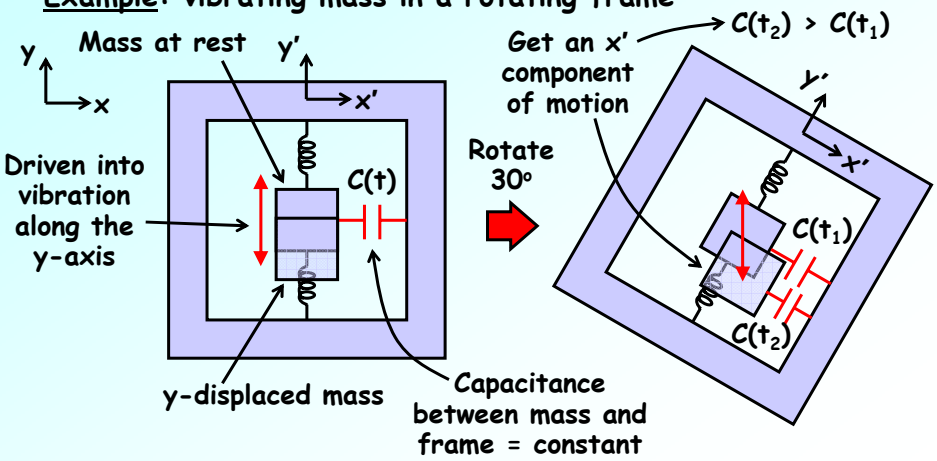
- A gyroscope measures rotation rate, which then gives orientation → very important, of course, for navigation
- Principle of operation based on conservation of momentum
- Example: classic spinning gyroscope



Rotor will preserve its angular momentum (i.e., will maintain its axis of spin) despite rotation of its gimbled chassis

Vibratory Gyroscopes

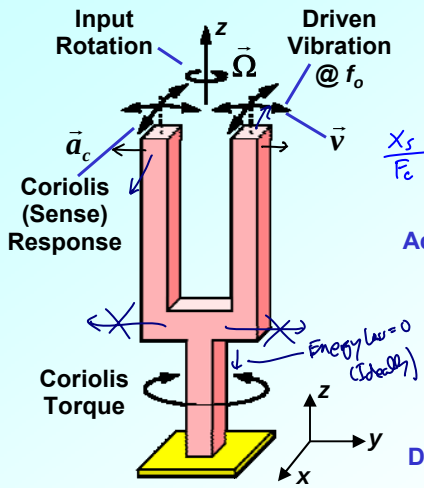
- Generate momentum by vibrating structures
- Again, conservation of momentum leads to mechanisms for measuring rotation rate and orientation
- Example: vibrating mass in a rotating frame



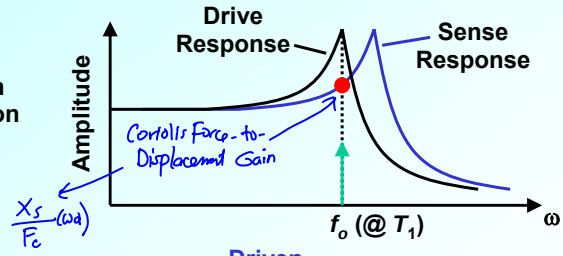
Basic Vibratory Gyroscope Operation

Principle of Operation

- Tuning Fork Gyroscope:



Drive/Sense Response Spectra:



Coriolis Acceleration — Driven Velocity — Rotation Rate

$$\vec{a}_c = 2\vec{v} \times \vec{\Omega}$$

Coriolis Force — Beam Mass

$$\vec{x} = \frac{\vec{F}_c}{k} = \frac{m\vec{a}_c}{k} = \frac{\vec{a}_c}{\omega_r^2}$$

Coriolis Displacement — Beam Stiffness — Sense Frequency

Basic Vibratory Gyroscope Operation

Principle of Operation

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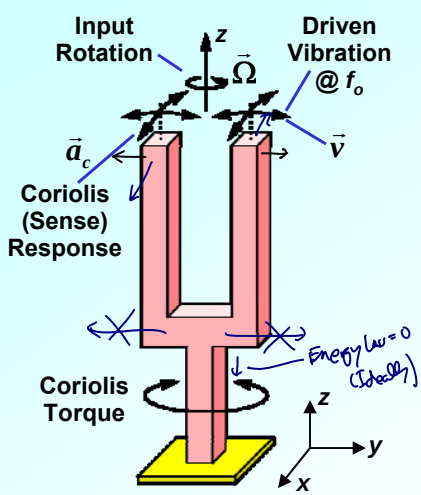
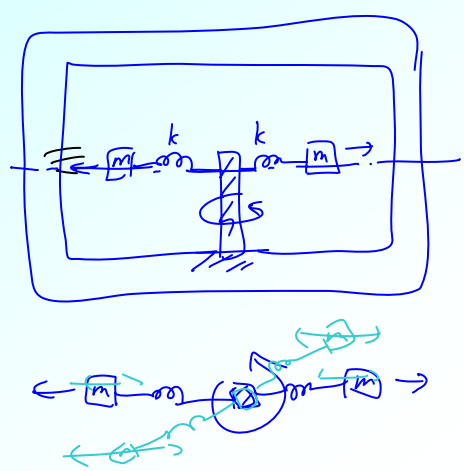




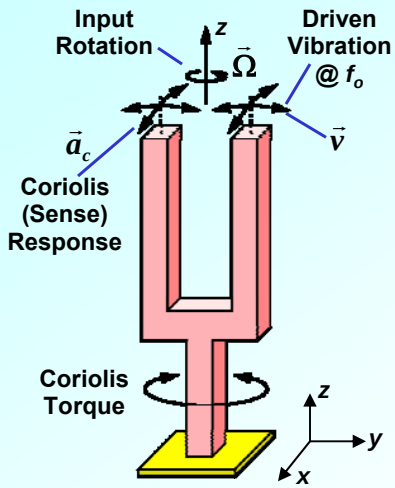
Vibratory Gyroscope Performance

Principle of Operation

- Tuning Fork Gyroscope:

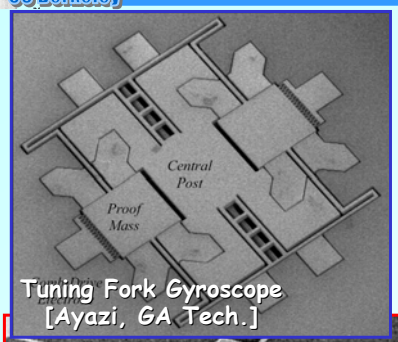
$$\vec{x} = \frac{\vec{F}_c}{k} = \frac{m\vec{a}_c}{k} = \frac{\vec{a}_c}{\omega_r^2} \qquad \vec{a}_c = 2\vec{v} \times \vec{\Omega}$$

Beam Mass; Beam Stiffness; Sense Frequency; Driven Velocity

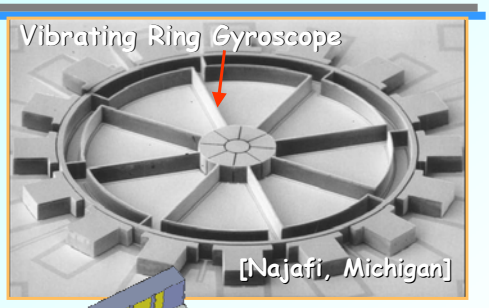


- To maximize the output signal x, need:
  - Large sense-axis mass
  - Small sense-axis stiffness
  - (Above together mean low resonance frequency)
  - Large drive amplitude for large driven velocity (so use comb-drive) → $v = \omega_o X$ want X = large!
  - If can match drive freq. to sense freq., then can amplify output by Q times ←

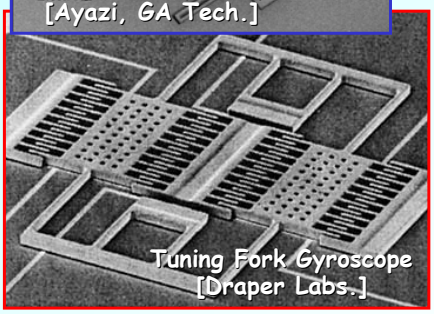
MEMS-Based Gyroscopes



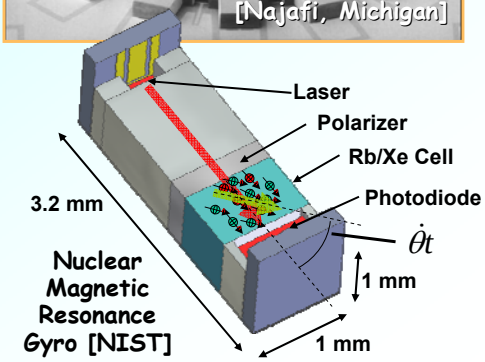
Tuning Fork Gyroscope [Ayazi, GA Tech.]



[Najafi, Michigan]

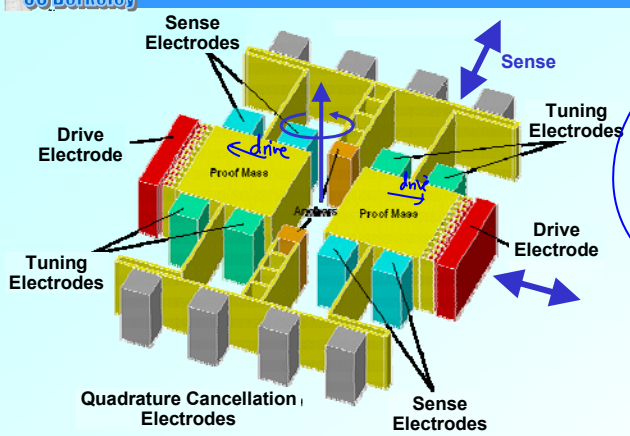


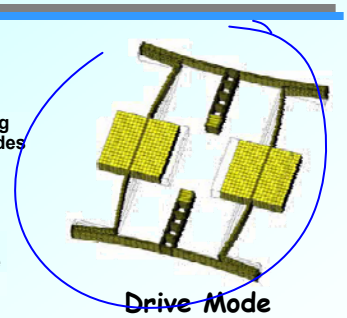
Tuning Fork Gyroscope [Draper Labs.]

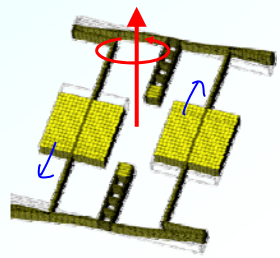


Nuclear Magnetic Resonance Gyro [NIST]

MEMS-Based Tuning Fork Gyroscope

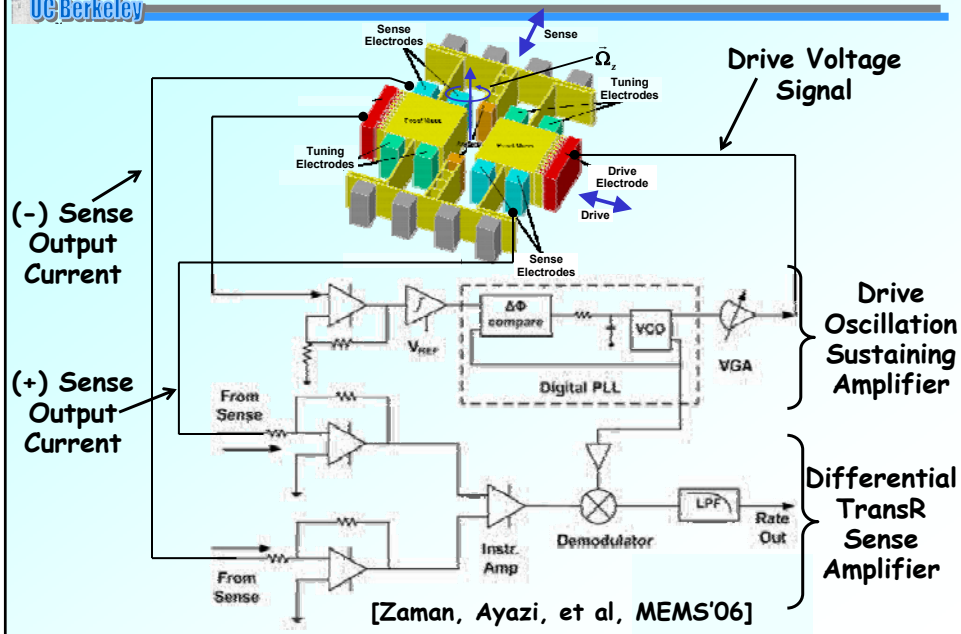






- In-plane drive and sense modes pick up z-axis rotations
- Mode-matching for maximum output sensitivity
- From [Zaman, Ayazi, et al, MEMS'06]

MEMS-Based Tuning Fork Gyroscope



[Zaman, Ayazi, et al, MEMS'06]