Announcements

• Make-Up Lecture:
  ✈ Friday, 11/16, from 1:00 p.m. to 2:30 p.m.
  ✈ Room 145 McCone

• Still trying to get a room for a Monday, 11/19, make-up lecture
  ✈ Trying for 10 a.m., but will take what I can get

• Updates to HW#6 posted
  ✈ Basically, you can ignore the masses of the springs, since the proof masses are so large
  ✈ Also, just identify the electrodes in part (c)

• Times for project outbriefs
  ✈ Wednesday, Dec. 12 (morning), 9:30-12 noon
  ✈ Sunday, Dec. 16?
  ✈ Wednesday, Dec. 19?
  ✈ Thursday, Dec. 20?
Lecture Outline

- Reading: Senturia Chpts. 5, 6, Handouts: "Electrostatic-Comb Drive of Lateral Polysilicon Resonators", "Electrostatic Comb Drive Levitation And Control Method", "Micromachined Inertial Sensors"

- Lecture Topics:
  - Electrostatic Comb-Drive
    - 1st Order Analysis
    - 2nd Order Analysis
    - Levitation
    - Electrical Stiffness
  - Input Circuit Modeling
  - Gyroscopes

Comb-Drive Force Equation (2nd Pass)

- In our 1st pass, we neglected:
  - Fringing fields
  - Parallel-plate capacitance between stator and rotor
  - Capacitance to the substrate

- All of these capacitors must be included when evaluating the energy expression!
Finger displacement changes not only the capacitance between stator and rotor, but also between these structures and the ground plane → modifies the capacitive energy

$$F_{e,x} = \frac{dW}{dx} = \frac{1}{2} \frac{dC_{sp}}{dx} V_s^2 + \frac{1}{2} \frac{dC_{rp}}{dx} V_r^2 + \frac{1}{2} \frac{dC_{rs}}{dx} (V_s - V_r)^2$$

Gary Fedder, Ph.D., UC Berkeley, 1994

- Case: $V_r = V_p = 0V$
- $C_{sp}$ depends on whether or not fingers are engaged

$$C_{sp} = N[C_{sp,x} + C_{sp,zz} (L - x)]$$

$$C_{rs} = NC_{rs,xx}$$

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Finger displacement changes not only the capacitance between stator and rotor, but also between these structures and the ground plane → modifies the capacitive energy

\[ F_{e,x} = \frac{\partial W'}{\partial x} = \frac{1}{2} \frac{dC_{sp}}{dx} V_s^2 + \frac{1}{2} \frac{dC_{rp}}{dx} V_r^2 + \frac{1}{2} \frac{dC_{rp}}{dx} (V_s - V_r)^2 \]

Simulate to Get Capacitors → Force

* Below: 2D finite element simulation

\[ F_{e,x} = \frac{N}{2} (C_{rs} + C_{sp,e} - C_{sp,r}) V_s^2 \]

20-40% reduction of \( F_{e,x} \)
**Vertical Force (Levitation)**

\[
F_{e,z} = \frac{\partial W'}{\partial z} = \frac{1}{2} \frac{dC_{sp}}{dz} V_s^2 + \frac{1}{2} \frac{dC_{rp}}{dz} V_r^2 + \frac{1}{2} \frac{dC_{rs}}{dz} (V_s - V_r)^2
\]

* For \( V_r = 0 \text{V} \) (as shown): \( F_{e,z} = \frac{1}{2} N_e \left[ \frac{d(C'_{sp,e} + C'_{rs})}{dz} \right] V_s^2 \)

**Simulated Levitation Force**

* Below: simulated vertical force \( F_z \) vs. \( z \) at different \( V_p \)'s [f/ Bill Tang Ph.D., UCB, 1990]

See that \( F_z \) is roughly proportional to \( -z \) for \( z \) less than \( z_o \) → it’s like an electrical stiffness that adds to the mechanical stiffness

\[
F_z \approx \gamma_z V_p^2 \frac{(z_o - z)}{z_o} = k_e(z_o - z)
\]

Electrical Stiffness
Vertical Resonance Frequency

Vertical resonance frequency

\[ \frac{\omega_z}{\omega_{zo}} = \sqrt{\frac{k_z + k_e}{k_z}} \]

where

\[ k_e = \left( \frac{Y_z}{z_0} \right) V^2 \]

- Signs of electrical stiffnesses in MEMS:
  - Comb (x-axis) → \( k_e = 0 \)
  - Comb (z-axis) → \( k_e > 0 \)
  - Parallel Plate → \( k_e < 0 \)

Suppressing Levitation

- Pattern ground plane polysilicon into differentially excited electrodes to minimize field lines terminating on top of comb
- Penalty: x-axis force is reduced
Force of Comb-Drive vs. Parallel-Plate

• Comb drive (x-direction)
  \( V_1 = V_2 = V_S = 1V \)
  \[ F_{e,x} = \frac{\varepsilon_0 t V_s^2}{g} \]

• Differential Parallel-Plate (y-direction)
  \( V_1 = 0V, V_2 = 1V \)
  \[ F_{e,y} = \frac{1}{2} \frac{\varepsilon_0 t x V_2^2}{g^2} \]
  Parallel-plate generates a much larger force; but at the cost of linearity

Input Modeling
**Electromechanical Analogies**

Equation of Motion:

\[ m_{eq} \ddot{x} + c_{eq} \dot{x} + k_{eq} x = F(t) \]

\[ \Rightarrow \text{using phaser concepts:} \]

\[ F = j \omega m_{eq} \dot{x} + \frac{k_{eq}}{j \omega} \dot{x} + c_{eq} \dot{x} \]

\[ \Rightarrow \text{by analogy:} \]

\[ F \rightarrow N, \quad m_{eq} \rightarrow k_{eq}, \quad c_{eq} \rightarrow r_{eq} \]

\[ \dot{x} \rightarrow \dot{v}, \quad k_{eq} \rightarrow \frac{1}{C_{eq}} \]

**Bandpass Biquad Transfer Function**

\[ F = j \omega m_{eq} \dot{x} + \frac{k_{eq}}{j \omega} \dot{x} + c_{eq} \dot{x} \]

\[ \Rightarrow \text{Converting to full phasor form:} \]

\[ F = (j \omega) \left( j \omega \right) m_{eq} \dot{x} + \frac{k_{eq}}{j \omega} \left( j \omega \right) x + c_{eq} \left( j \omega \right) x \]

\[ X = \frac{E}{j \omega k_{eq}} \]

\[ \frac{X}{F} (j \omega) = \frac{k_{eq} \left( \frac{k_{eq}}{j \omega \omega_{0}} + 1 + \frac{j \omega}{\omega_{0}} \right)}{1 - \left( \frac{\omega}{\omega_{0}} \right)^{2} + \frac{j \omega}{\omega_{0}}} \]

\[ \left[ k_{eq} = \omega_{0}^{2}, \quad Q = \frac{\omega_{0}}{C_{eq}} = \frac{k_{eq}}{C_{eq}} = \frac{k_{eq}}{Q_{eq}} = \frac{k_{eq}}{C_{eq}} \right] \]
**Force-to-Velocity Relationship**

- The relationship between input voltage $v_1$ and force $F_{d1}$:

  \[ F_{d1} \approx -V_p \frac{\partial C_1}{\partial x} v_1 \]

- When displacement $x$ is the mechanical output variable:

  \[ X(s) = \frac{1}{s} \frac{\omega_o^2}{k s^2 + (\omega_o/Q)s + \omega_o^2} \]

  \[ F_{d1}(s) = \frac{1}{k s^2 + (\omega_o/Q)s + \omega_o^2} \]

- When velocity $\upsilon$ is the mechanical output variable:

  \[ \upsilon(s) = sX(s) = \frac{1}{s} \frac{\omega_o^2 s}{k s^2 + (\omega_o/Q)s + \omega_o^2} \]

**Force-to-Velocity Equiv. Ckt.**

- Combine the previous lumped LCR mechanical equivalent circuit with a circuit modeling the capacitive transducer → circuit model for voltage-to-velocity
A transducer ...
- converts energy from one domain (e.g., electrical) to another (e.g., mechanical)
- has at least two ports
- is not generally linear, but is virtually linear when operated with small signals (i.e., small displacements)

For physical consistency, use a transformer equivalent circuit to model the energy conversion from the electrical domain to mechanical domain.

\[
\begin{bmatrix}
\eta & 0 \\
0 & \frac{1}{\eta}
\end{bmatrix}
\begin{bmatrix}
e_2 \\
f_2
\end{bmatrix}
= 
\begin{bmatrix}
\eta & 0 \\
0 & \frac{1}{\eta}
\end{bmatrix}
\begin{bmatrix}
e_1 \\
f_1
\end{bmatrix}
\]
Electromechanical Equivalent Circuit

- $e_2 = F_{d1}, e_1 = v_1$, just need $\eta_1$:
- From the matrix: $e_2 = \eta e_1$

$$F_{d1} \approx -V_P \frac{\partial C_1}{\partial x} v_1 \rightarrow \eta_1 = \left| V_P \frac{\partial C_1}{\partial x} \right|$$

Output Modeling
Types of Capacitive Outputs

Low Frequency
- Example: accelerometer
- Acceleration inputs are generally at low frequency, e.g., near DC

High Frequency
- Example: gyroscope
- Rotation-derived sense forces are generally sinusoidal, e.g., resonance

Gyrosopes
Classic Spinning Gyroscope

- A gyroscope measures rotation rate, which then gives orientation → very important, of course, for navigation
- Principle of operation based on conservation of momentum
- Example: classic spinning gyroscope

Vibratory Gyroscopes

- Generate momentum by vibrating structures
- Again, conservation of momentum leads to mechanisms for measuring rotation rate and orientation
- Example: vibrating mass in a rotating frame
Basic Vibratory Gyroscope Operation

Principle of Operation

• Tuning Fork Gyroscope:

- Input Rotation

- Driven Vibration @ $f_0$

- Coriolis (Sense) Response

- Coriolis Torque

Drive/Sense Response Spectra:

- Drive Response

- Sense Response

- Coriolis Force-to-Dispacement Gain

- Coriolis Acceleration $\ddot{a}_c = 2\dot{v} \times \Omega$

- Beam Mass

- Beam Stiffness

- Sense Frequency $\omega_c$

Basic Vibratory Gyroscope Operation

Principle of Operation

• Tuning Fork Gyroscope:

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Basic Vibratory Gyroscope Operation

Principle of Operation

• Tuning Fork Gyroscope:

- Input Rotation

- Driven Vibration @ $f_0$

- Coriolis (Sense) Response

- Coriolis Torque
Vibratory Gyroscope Performance

Principle of Operation

- Tuning Fork Gyroscope:
  \[ \ddot{x} = \frac{F_c}{k} = \frac{m\ddot{a}_x}{k} = \frac{\dddot{a}_x}{\omega^2}, \quad \dddot{a}_x = 2\Omega \times \dot{\Omega} \]

- To maximize the output signal \( x \), need:
  - Large sense-axis mass
  - Small sense-axis stiffness
  - (Above together mean low resonance frequency)
  - Large drive amplitude for large driven velocity (so use comb-drive)
  - If can match drive freq. to sense freq., then can amplify output by \( Q \) times

MEMS-Based Gyroscopes

- Vibrating Ring Gyroscope
  [Najafi, Michigan]
  - Laser
  - Polarizer
  - Rb/Xe Cell
  - Photodiode
  - 3.2 mm

- Tuning Fork Gyroscope
  [Ayazi, GA Tech.]
  [Draper Labs.]
**MEMS-Based Tuning Fork Gyroscope**

* In-plane drive and sense modes pick up z-axis rotations
* Mode-matching for maximum output sensitivity
* From [Zaman, Ayazi, et al, MEMS’06]

**MEMS-Based Tuning Fork Gyroscope**

(-) Sense Output Current

(+) Sense Output Current

Drive Voltage Signal

Drive Oscillation Sustaining Amplifier

Differential TransR Sense Amplifier

[Zaman, Ayazi, et al, MEMS’06]