EE C245 – ME C218
Introduction to MEMS Design
Fall 2007

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Lecture 24: Output Sensing

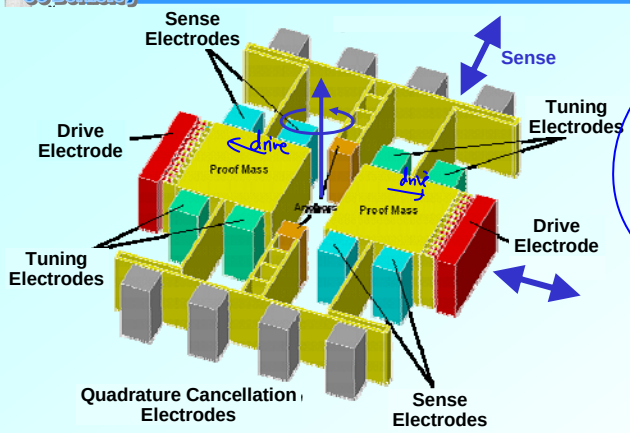
Announcements

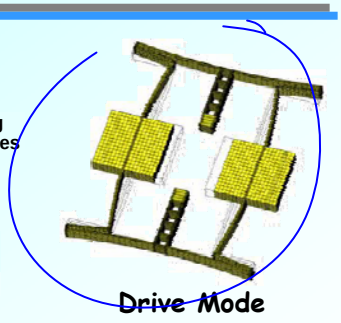
- Make-Up Lecture:
  - Monday, 11/19, from 10-11:30 a.m.
  - Room 126 Barrows Hall

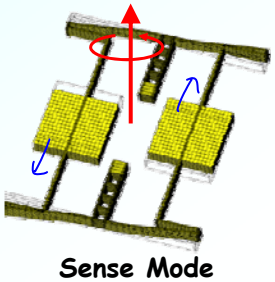
Lecture Outline

- Reading: Senturia Chpts. 5, 6, Handouts: "Electrostatic-Comb Drive of Lateral Polysilicon Resonators", "Electrostatic Comb Drive Levitation And Control Method", "Micromachined Inertial Sensors"
- Lecture Topics:
  - Gyroscopes
  - Output Modeling
  - Complete Electrical Equivalent Circuit
  - Sensing Circuits
    - Ideal Op Amps

MEMS-Based Tuning Fork Gyroscope

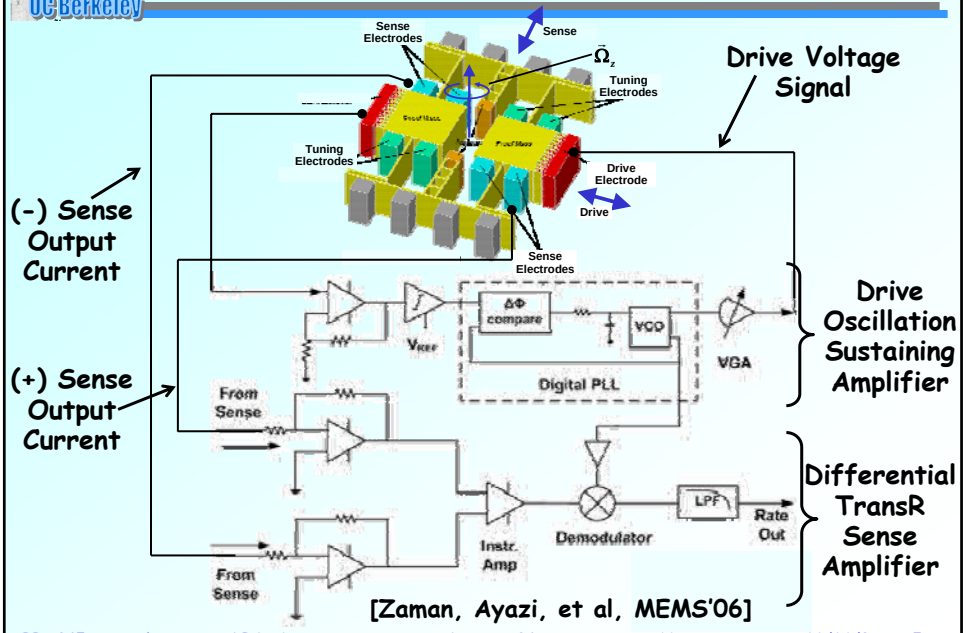






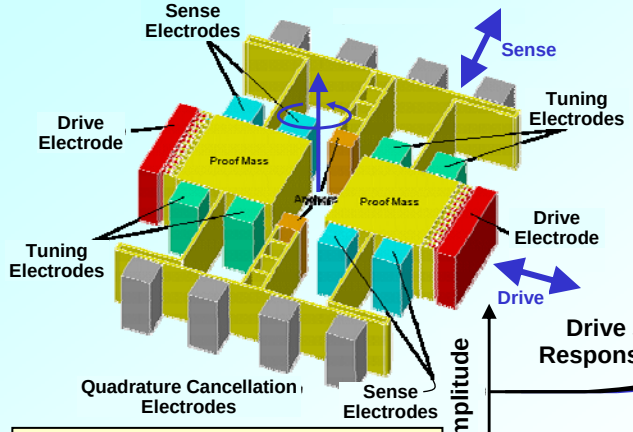
- In-plane drive and sense modes pick up z-axis rotations
- Mode-matching for maximum output sensitivity
- From [Zaman, Ayazi, et al, MEMS'06]

MEMS-Based Tuning Fork Gyroscope

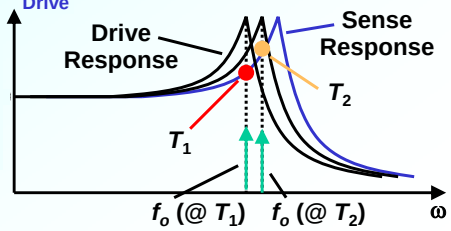


MEMS-Based Tuning Fork Gyroscope

- Drive and sense axes must be stable or at least track one another to avoid output drift



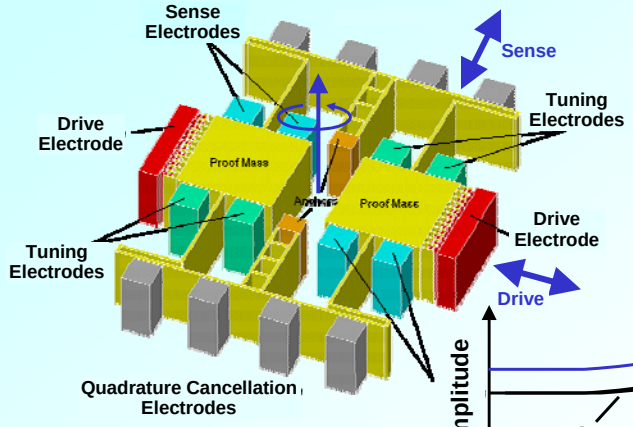
Problem: if drive frequency changes relative to sense frequency, output changes ⇨ bias drift



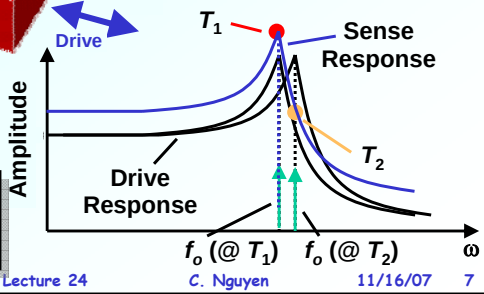
Need: small or matched drive and sense axis temperature coefficients to suppress drift

Mode Matching for Higher Resolution

- For higher resolution, can try to match drive and sense axis resonance frequencies and benefit from Q amplification



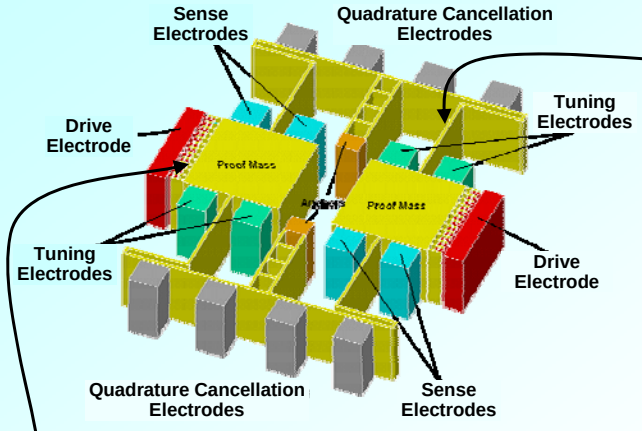
Problem: mismatch between drive and sense frequencies ⇨ even larger drift!

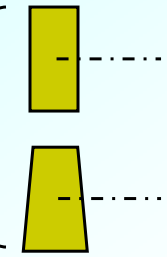


Need: small or matched drive and sense axis temperature coefficients to make this work

Issue: Zero Rate Bias Error

- Imbalances in the system can lead to zero rate bias error





Mass imbalance ⇨ off-axis motion of the proof mass

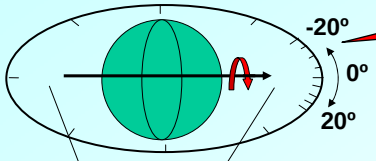
Quadrature output signal that can be confused with the Coriolis acceleration

Drive imbalance ⇨ off-axis motion of the proof mass

Output signal in phase with the Coriolis acceleration

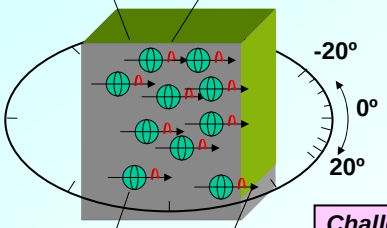
Nuclear Magnetic Res. Gyroscope

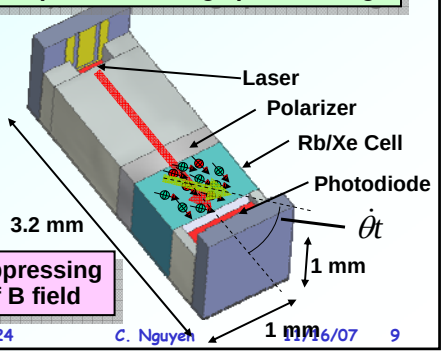
- The ultimate in miniaturized spinning gyroscopes?
  - ⇘ from CSAC, we may now have the technology to do this



Better if this is a noble gas nucleus (rather than e-), since nuclei are heavier ⇨ less susceptible to B field

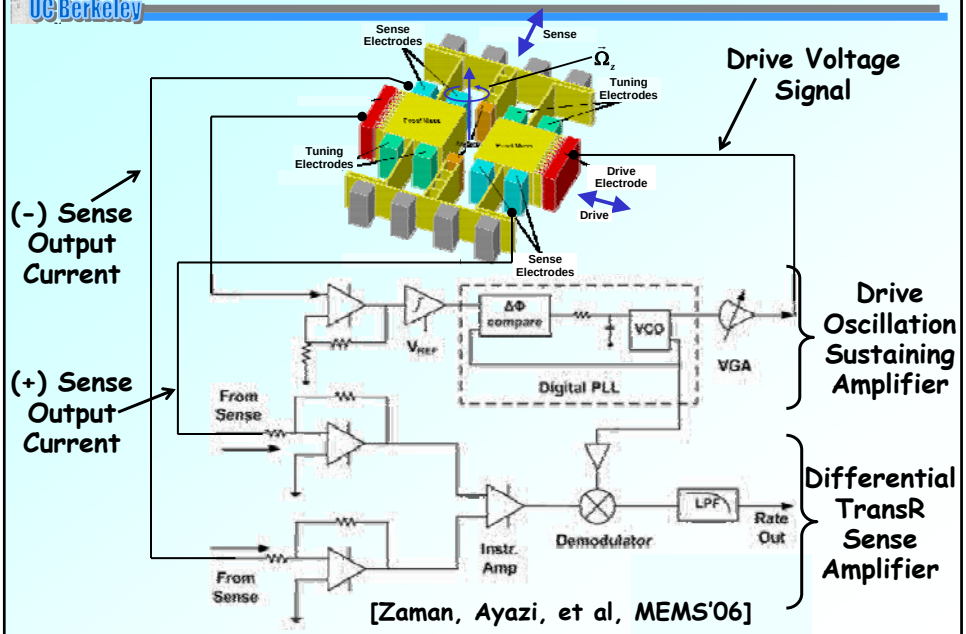
Soln: Spin polarize Xe^{129} nuclei by first polarizing e- of Rb^{87} (a la CSAC), then allowing spin exchange





Challenge: suppressing the effects of B field

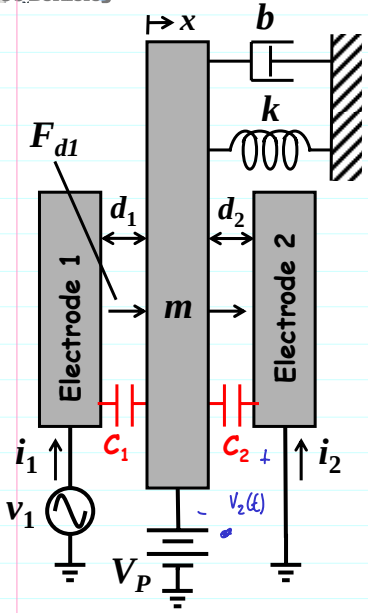
MEMS-Based Tuning Fork Gyroscope



[Zaman, Ayazi, et al, MEMS'06]

Output Modeling

Output Current Into Ground



- When the mass moves with time-dependent displacement x(t), the electrode-to-mass capacitors $C_1(x,t)$ and $C_2(x,t)$ vary with time
- This generates an output current:

$$\left[q = CV\right] \Rightarrow i = \frac{dq}{dt} = C\frac{\partial V}{\partial t} + V\frac{\partial C}{\partial t}$$

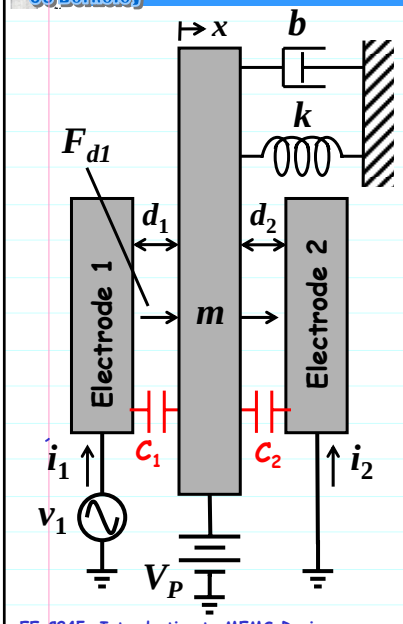
$$i_2(t) = C_2(x,t)\frac{dV_2(t)}{dt} + V_2(t)\frac{dC_2(x,t)}{dt}$$

$$\left[V_2(t) = -V_P\right] \Rightarrow i_2 = -V_P\frac{dC_2}{dt} = -V_P\frac{\partial C_2}{\partial x}\frac{\partial x}{\partial t}$$

In phasor form: $I_2(j\omega) = -V_P\frac{\partial C_2}{\partial x}(j\omega x)$

$$I_2(j\omega) = -j\omega V_P\frac{\partial C_2}{\partial x}x$$

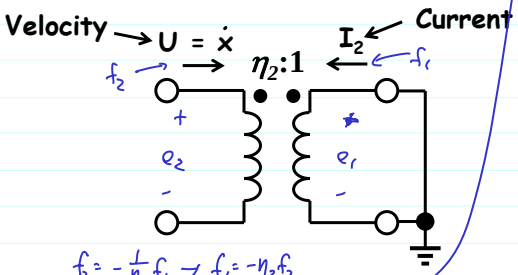
Output Current Into Ground



$I_2(j\omega) = -j\omega V_P \frac{\partial C_2}{\partial x} X = -V_P \frac{\partial C}{\partial x} U$ ← velocity

90° phase shift (+) (+) → $I_2(j\omega) = (-)$ when $x = (+)$ ✓

- Again, model with a transformer:

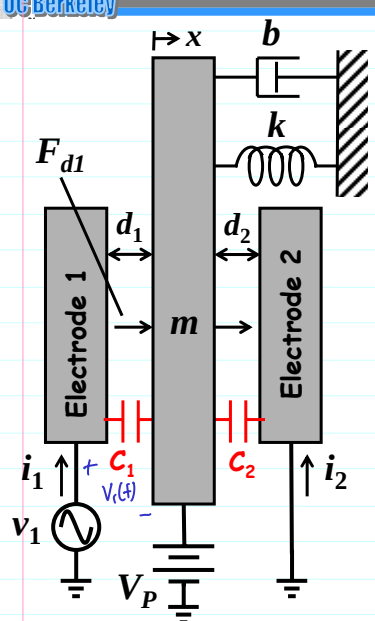


$f_2 = -\frac{1}{\eta_2} f_1 \rightarrow f_1 = -\eta_2 f_2$

$[f_1 = I_2, f_2 = U] \Rightarrow I_2 = -\eta_2 U$

$\therefore \boxed{\eta_2 = V_P \frac{\partial C_2}{\partial x}}$

Input Current Expression



Get $I_1(j\omega)$:

$i_1(t) = C_1(x,t)\frac{dV_1(t)}{dt} + V_1(t)\frac{dC_1(x,t)}{dt}$

$[V_1(t) = v_1 - V_P] \Rightarrow i_1 = C_1 \frac{dv_1}{dt} + [v_1 - V_P]\frac{\partial C_1}{\partial x}\frac{\partial x}{\partial t}$

ac S.S. DC

$\therefore I_1(j\omega) = C_1(j\omega V_1) + V_1 \frac{\partial C_1}{\partial x}(j\omega X) - V_P \frac{\partial C_1}{\partial x}(j\omega X)$

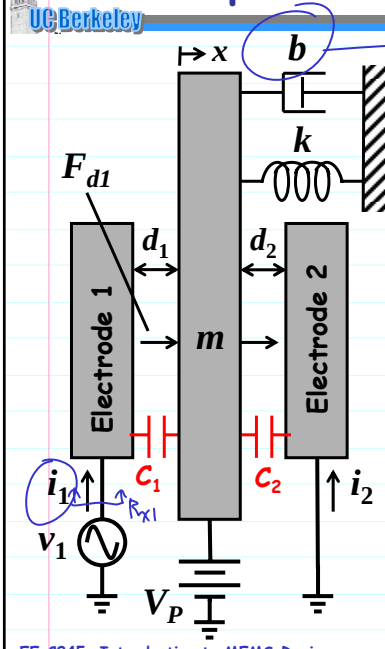
$= j\omega C_1 V_1 + j\omega V_1 \frac{\partial C_1}{\partial x} X - j\omega V_P \frac{\partial C_1}{\partial x} X$

$[v_1 << V_P] \Rightarrow I_1(j\omega) = \underbrace{j\omega C_1 V_1}_{\text{Feedthrough Current Thru the Static } C_1} - \underbrace{j\omega V_P \frac{\partial C_1}{\partial x} X}_{\text{Motional Current}}$ (due to mass movement)

@DC: $x = \frac{F_{d1}}{k} = -\frac{1}{k} V_P \frac{\partial C_1}{\partial x} v_1$ (−)

@resonance: $x = -\frac{Q F_{d1}}{jk} = -\frac{Q}{jk} V_P \frac{\partial C_1}{\partial x} v_1$ 90° phase shift

Input Current Expression (cont)



Thus: @ resonance

$$I_1(j\omega) = j\omega_0 C_1 V_1 + j\omega_0 \overbrace{\left(V_P \frac{\partial C_1}{\partial x}\right)^2 \frac{Q}{jk}}^{\eta_1} V_1$$

interference $= \underbrace{j\omega_0 C_1 V_1}_{\text{90° phase-shifted from } V_1} + \underbrace{\omega_0 \frac{Q}{k} \eta_1^2 V_1}_{\text{In phase w/ } V_1}$ ← care

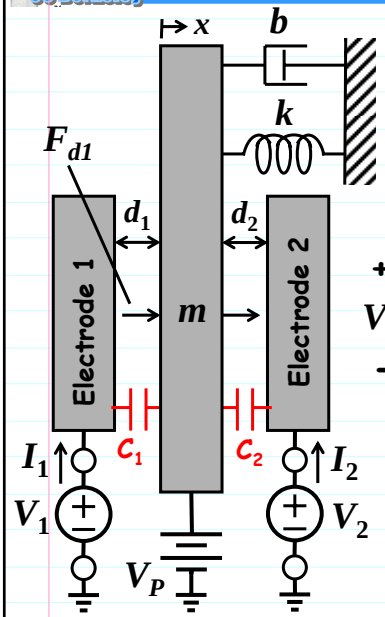
This is a capacitor in shunt w/ the input.

This is an effective resistance seen looking into Electrode 1

Motional Resistance:

$$R_{x1} = \frac{V_1}{I_1} = \frac{k}{\omega_0 Q \eta_1^2} = \frac{m\omega_0}{Q\eta_1^2} = \boxed{\frac{b}{\eta_1^2} = R_{x1}}$$

Complete Electrical-Port Equiv. Circuit

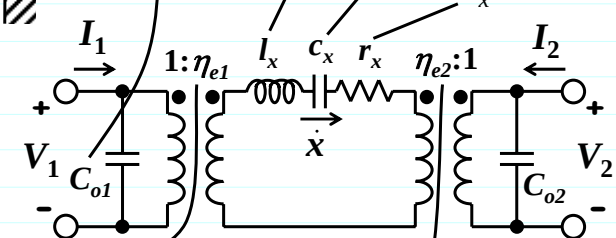


Static electrode-to-mass overlap capacitance

$l_x = m$

$c_x = \frac{1}{k}$

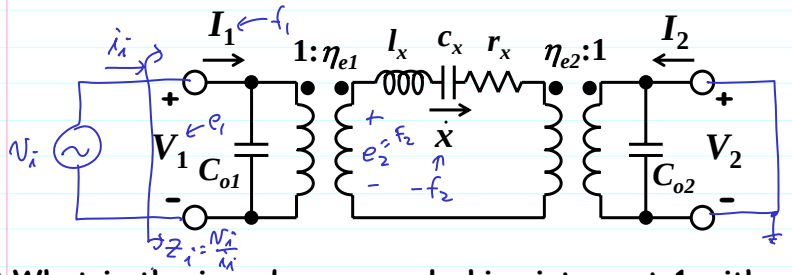
$r_x = b$



$$\eta_{e1} = V_P \frac{\partial C_1}{\partial x} = V_P \frac{C_{o1}}{d_1}$$

$$\eta_{e2} = V_P \frac{\partial C_2}{\partial x} = V_P \frac{C_{o2}}{d_2}$$

Input Impedance Into Port 1



- What is the impedance seen looking into port 1 with port 2 shorted to ground?

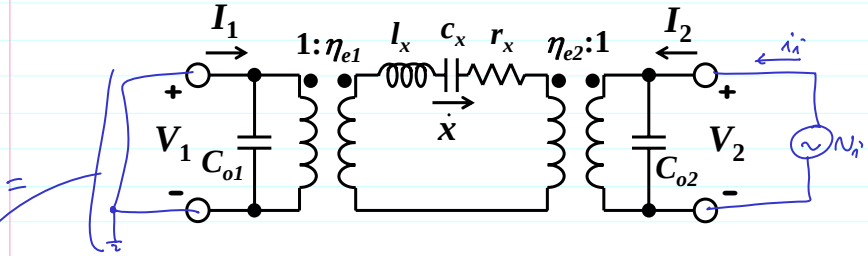
From our transformer model: $\begin{bmatrix} e_2 \\ f_2 \end{bmatrix} = \begin{bmatrix} \eta & 0 \\ 0 & -\frac{1}{\eta} \end{bmatrix} \begin{bmatrix} e_1 \\ f_1 \end{bmatrix} \Rightarrow e_2 = \eta e_1 \rightarrow e_1 = \frac{e_2}{\eta}$, $f_2 = -\frac{1}{\eta} f_1 \rightarrow f_1 = -\eta f_2$

$\frac{e_1}{f_1} = \frac{e_2}{\eta}\left(\frac{1}{-\eta f_2}\right) = -\frac{1}{\eta^2}\left(\frac{e_2}{f_2}\right) \rightarrow \frac{v_{i1}}{i_{i1}} = Z_{i1} = \frac{1}{\eta_{e1}}\frac{F_2}{(-\dot{x}_2)} = \frac{1}{\eta_{e1}^2} Z_x$

$Z_{i1} = \frac{1}{\eta_{e1}^2}\left(j\omega l_x + \frac{1}{j\omega c_x} + r_x\right) = j\omega\left(\frac{l_x}{\eta_{e1}^2}\right) + \frac{1}{j\omega(\eta_{e1}^2 c_x)} + \frac{r_x}{\eta_{e1}^2}$

L_{x1} C_{x1} $R_{x1} = \frac{b}{\eta_{e1}^2}$

Input Impedance Into Port 2

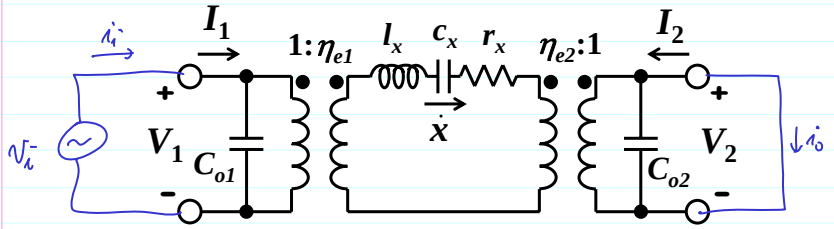


- What is the impedance seen looking into port 2 with port 1 shorted to ground?

$\frac{v_{i2}}{i_{i2}} = Z_{i2} = \frac{1}{\eta_{e2}^2}\left(j\omega l_x + \frac{1}{j\omega c_x} + r_x\right) = j\omega\left(\frac{l_x}{\eta_{e2}^2}\right) + \frac{1}{j\omega(\eta_{e2}^2 c_x)} + \frac{r_x}{\eta_{e2}^2}$

L_{x2} C_{x2} R_{x2}

Port 1 to 2 TransG Across the Circuit



- What is the transconductance from port 1 to port 2 with port 2 shorted to ground?

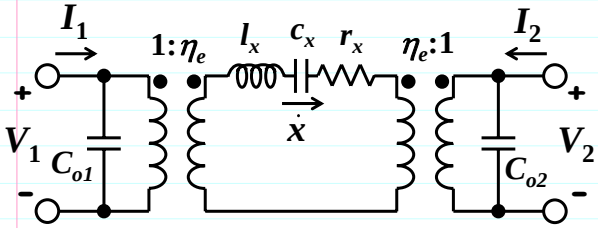
$\frac{i_o}{v_{i1}}(j\omega)$

$\dot{x} = \frac{1}{\eta_{e1}} i_{i1}$, $i_o = \eta_{e2}\dot{x}$ $\Rightarrow$ $i_o = \frac{\eta_{e2}}{\eta_{e1}} i_{i1} = \frac{\eta_{e2}}{\eta_{e1}}\left(\frac{v_{i1}}{Z_{i1}}\right) = \frac{\eta_{e2}}{\eta_{e1}} v_{i1}\left[\eta_{e1}^2 \frac{1}{j\omega l_x + \frac{1}{j\omega c_x} + r_x}\right]$

$\frac{i_o}{v_{i1}}(j\omega) = \frac{\eta_{e1}\eta_{e2}}{j\omega l_x + \frac{1}{j\omega c_x} + r_x} = \left[j\omega L_{x12} + \frac{1}{j\omega C_{x12}} + R_{x12}\right]^{-1}$

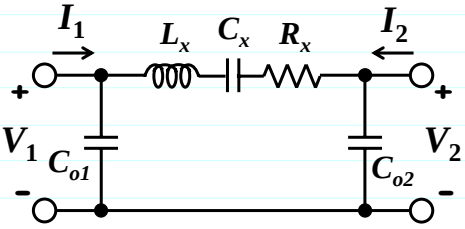
$L_{x12} = \frac{l_x}{\eta_{e1}\eta_{e2}}$, $C_{x12} = \eta_{e1}\eta_{e2} C_x$, $R_{x12} = \frac{r_x}{\eta_{e1}\eta_{e2}}$

Condensed Equiv. Circuit (Symmetrical)



Holds for the symmetrical case, where port 1 and port 2 are identical

If $\eta_{e1} = \eta_{e2}$, then …



where

$$L_x = \frac{m}{\eta_e^2}$$

$$C_x = \frac{\eta_e^2}{k}$$

$$R_x = \frac{b}{\eta_e^2}$$