Announcements

• Project emails
  - Main theme: do micro-to-macro comparison using a table

<table>
<thead>
<tr>
<th>Macro</th>
<th>Micro</th>
</tr>
</thead>
<tbody>
<tr>
<td>V_{L/min}</td>
<td>10,000 psi</td>
</tr>
<tr>
<td>2 psi</td>
<td></td>
</tr>
</tbody>
</table>

• Project Outbrief sign-up sheets will be on my door by this afternoon

• HW#6:
  - Now due this Thursday
  - Let’s go through it since there’s been no discussion section for the past two weeks
Announcements

HW#6 (cont)

- **SPICE**
  - Why use it? Why make equivalent electrical circuits for mechanical devices?
  - Answer: Noise analysis is needed to determine the minimum detectable signal for a MEMS-based sensor
  - Simulink will not do this nearly as conveniently
Lecture Outline

• Reading: Senturia Chpt. 16, 19

• Lecture Topics:
  - Non-Ideal Op Amps
    - Input Offset Voltage, $V_{OS}$
  - Determining Sensor Resolution
    - Noise
    - Noise Sources
    - Equivalent Input-Referred Noise Sources
    - Example: Gyro MDS Calculation
Back to Op Amp Non-Idealities

Actual Op Amps Are Not Ideal

• Actual op amps, of course, are not ideal; rather, they ...
  - Generate noise
  - Have finite gain, $A$
  - Have finite bandwidth, $\omega_b$
  - Have finite input resistance, $R_i$
  - Have finite input capacitance, $C_i$
  - Have finite output resistance, $R_o$
  - Have an offset voltage $V_{OS}$ between their (+) and (-) terminals
  - Have input bias currents
  - Have an offset $I_{OS}$ between the bias currents into the (+) and (-) terminals
  - Have finite slew rate
  - Have finite output swing (governed by the supply voltage used, $-L$ to $+L$)

• And what’s worse: All of the above can be temperature (or otherwise environmentally) dependent!
**Input Offset Voltage \( V_{0S} \)**

**Input Offset Voltage, \( V_{0S} \):**

\[
\begin{align*}
  v_0 &= A(v_+ - v_-) \\
  \text{Ideal case: } & v_0 = 0 \\
  \text{Reality: } & v_0 \neq 0 \text{ (usually, } v_0 = L^+ \text{ or } L^- : \text{ it rails out!)}
\end{align*}
\]

Why? Internal mismatches within the op amp → cause a dc offset. Model this with an equivalent input offset voltage \( V_{0S} \).

Typically, \( V_{0S} = 1 \text{mV} - 5 \text{mV} \)

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**Effect of \( V_{0S} \) on Op Amp Circuits**

**Example: Non-Inverting Amplifier**

\[
V_0 = V_{0S} \left(1 + \frac{R_2}{R_1}\right)
\]

\[e.g., \frac{R_2}{R_1} = 9, \ V_{0S} = 5 \text{mV} \rightarrow V_0 = 50 \text{mV}\]

(not so bad …)
Effect of $V_{OS}$ on Op Amp Circuits (cont.)

Example: Integrator

To fix this, place a resistor in shunt with the $C \rightarrow$ then:

$$v_0 = V_{OS} \left(1 + \frac{R_f}{R}\right)$$

$$v_0 = V_{OS} + \frac{1}{C} \int_{t_0}^{t} i \, dt$$

$$= V_{OS} + \frac{1}{C} \int_{0}^{t} V_{OS} \, dt$$

$$= V_{OS} \left(1 + \frac{t}{RC}\right) + v_C|_{t=0}$$

Will continue to increase until op amp saturates

Noise
Determining Sensor Resolution

Minimum Detectable Signal (MDS)

- Minimum Detectable Signal (MDS): Input signal level when the signal-to-noise ratio (SNR) is equal to unity

\[ \text{Minimum Detectable Signal} = \frac{\text{Sensed Signal}}{\text{Circuit Gain}} \]

- The sensor scale factor is governed by the sensor type
- The effect of noise is best determined via analysis of the equivalent circuit for the system

<table>
<thead>
<tr>
<th>Sensor Scale Factor</th>
<th>Sensor Noise</th>
</tr>
</thead>
<tbody>
<tr>
<td>+</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Circuit Gain</th>
<th>Circuit Output Noise</th>
</tr>
</thead>
<tbody>
<tr>
<td>+</td>
<td></td>
</tr>
</tbody>
</table>

- Output
- Includes desired output plus noise
Noise

- **Noise**: Random fluctuation of a given parameter $I(t)$
- In addition, a noise waveform has a zero average value (e.g. could be DC current)
- We can’t handle noise at instantaneous times
- But we can handle some of the averaged effects of random fluctuations by giving noise a power spectral density representation
- Thus, represent noise by its mean-square value:

$$ i^2 = (I - I_D)^2 = \lim_{T \to \infty} \frac{1}{T} \int_0^T |I - I_D|^2 \, dt $$
Noise Spectral Density

- We can plot the spectral density of this mean-square value:

\[ \frac{i^2}{\Delta f} \text{ [units}^2/\text{Hz]} \]

One-sided spectral density → used in circuits → measured by spectrum analyzers

Two-sided spectral density (1/2 the one-sided)

Often used in systems courses

\[ \bar{i}^2 = \text{integrated mean-square noise spectral density over all frequencies (area under the curve)} \]

Circuit Noise Calculations

- Deterministic: \[ v_o(j\omega) = H(j\omega)v_i(j\omega) \]

- Random:

\[ S_o(\omega) = |H(j\omega)H^*(j\omega)|S_i(\omega) = |H(j\omega)|^2S_i(\omega) \]

\[ \sqrt{S_o(\omega)} = |H(j\omega)|\sqrt{S_i(\omega)} \]

Root mean square amplitudes

How is it we can do this?
Handling Noise Deterministically

- Can do this for noise in a tiny bandwidth (e.g., 1 Hz)

\[ \frac{v_{n1}^2}{\Delta f} = S_1(f) \]

\[ v_{n1} = \sqrt{S_1(f) \cdot B} \]

Can approximate this by a sinusoidal voltage generator (especially for small B, say 1 Hz)

Why? Neither the amplitude nor the phase of a signal can change appreciably within a time period 1/B.

[This is actually the principle by which oscillators work -> oscillators are just noise going through a tiny bandwidth filter]