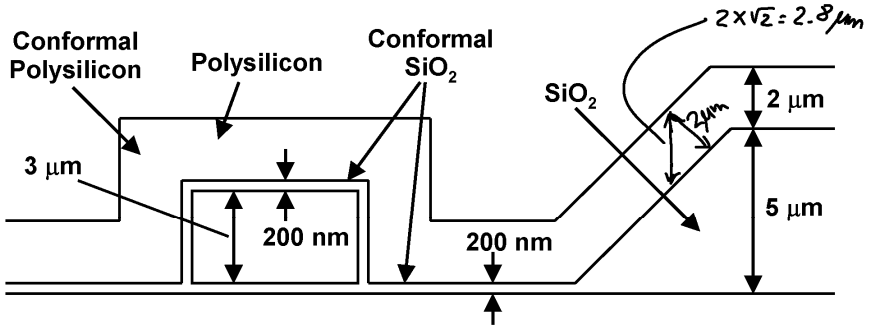
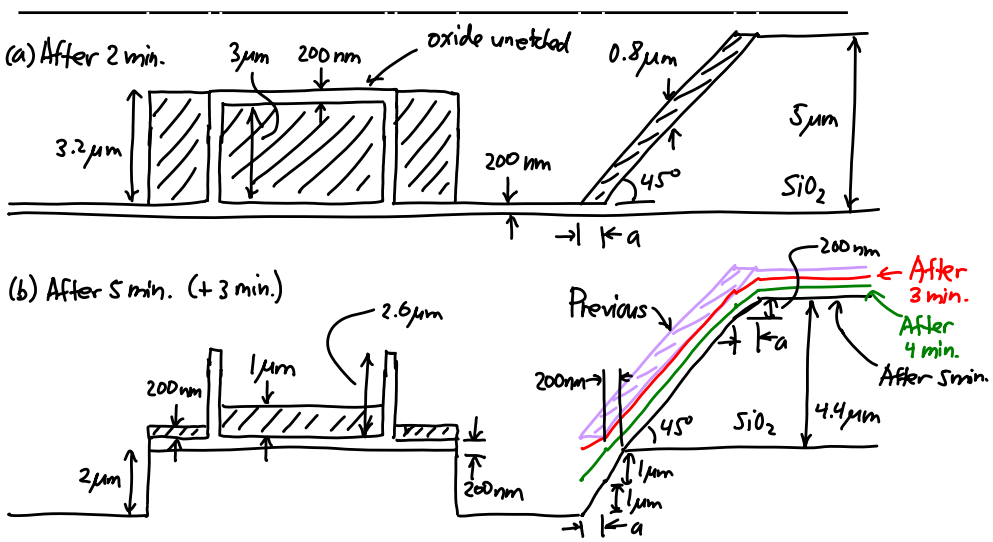
Midterm Solutions

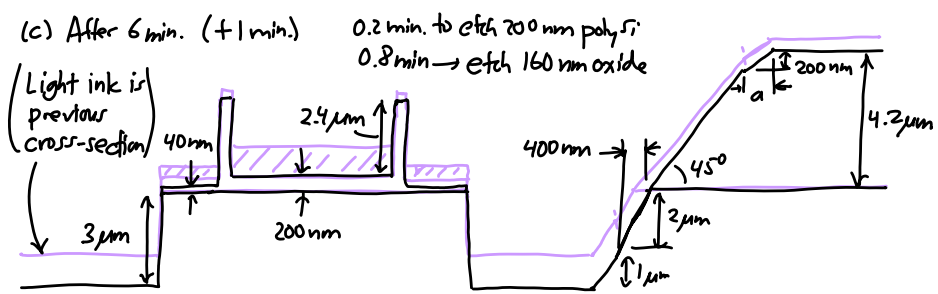
Problem 1. **Total 40 points**

The cross-section below is to be etched via reactive ion etching (RIE). For this problem, assume that the RIE etch is 100% anisotropic and that it etches polysilicon at the rate of 1 μm/min and has a silicon-to-oxide selectivity of 5:1. Draw cross-sections of the structure after etching for (a) 2 min.; (b) 5 min.; and (c) 6 min.

Oxide etches @ 200 nm/min.

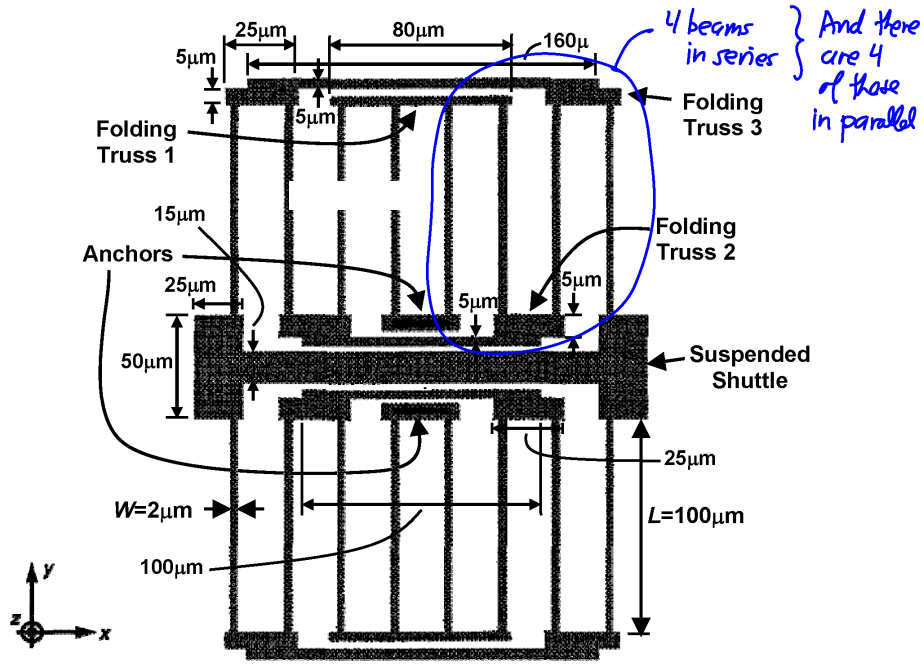






Problem 2. **Total 38 points**

The figure below presents the top view of a shuttle mass suspended 3μm above a substrate by a triple-folded beam suspension and achieved via a surface micromachining process with a 2μm-thick structural layer and using a 3μm-thick oxide as a sacrificial layer that etches in hydrofluoric acid at the rate of 1μm/min. Data on the structural material used in this problem is given in the box below the figure. Also, assume that all folding trusses are rigid in all directions, including the vertical (i.e., z) direction.

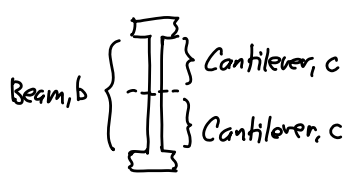


Structural Material Properties:
Young's Modulus, E = 150 GPa; Density, ρ = 2,300 kg/m^3; Poisson ratio, ν = 0.226
DI Water Contact Angle for Structural and Substrate Materials: 85°
Water-Air Interface Surface Tension: 72.75×10^{-3} N/m

(a) Write an expression for the static spring constant in the x-direction at a location on the shuttle and calculate its numerical value (with units).

(b) Assume for this part that the structure is coated by a very thin hydrophobic film on all surfaces, except for the Folding Truss 3 surfaces, which were not coated due to an error in mask layout. If the unreleased structure were dipped for 6 minutes in hydrofluoric acid, then rinsed in DI water, will the Folding Truss 3's be stuck to the substrate? Show your work for credit.

(a) To determine the total stiffness, it's easiest to break the structure into smaller pieces and combine them in series or parallel.



Each beam (b) consists of two cantilevers in series. Thus, for each beam:

$$k_b = k_c \| k_c = \frac{k_c}{2}$$

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As indicated in the figure, there are 4 groups of series connected beams in parallel. For each series connected group of 4:

$$k_{4group} = k_b \| k_b \| k_b \| k_b = \frac{k_b}{4} = \frac{k_c}{8}$$

Thus, the total x-direction stiffness is:

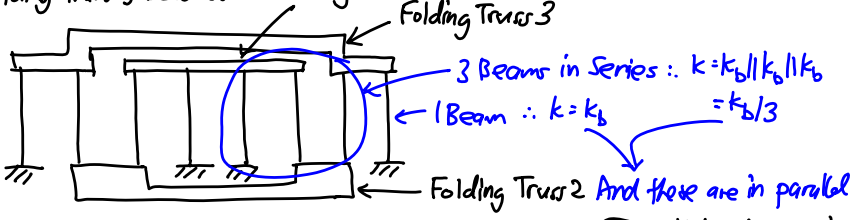
$$k_x = 4k_{4group} = 4\left(\frac{k_c}{8}\right) \Rightarrow k_x = \frac{k_c}{2} \Rightarrow \boxed{k_x = Eh\left(\frac{W}{L}\right)^3}$$

[For a cantilever of length (L/2): $k_c = \frac{24EI_z}{L^3} = \frac{24E}{L^3}\left(\frac{1}{12}W^3h\right) = 2Eh\left(\frac{W}{L}\right)^3$]

Plug in numbers:

$$k_x = (150G)(2\mu)\left(\frac{2\mu}{100\mu}\right)^3 = 2.4\,N/m$$

(b) A 6min. dip in HF will release all suspension beams and folding trusses. But it will not release the shuttle. Thus, the shuttle becomes a rigid (i.e., fixed) end condition. The effective suspension for each Folding Truss 3 becomes:



Thus: $\frac{k_{t3}}{2} = k_b + \frac{k_b}{3} = \frac{4}{3}k_b = \frac{2}{3}k_c \Rightarrow \boxed{k_{t3} = \frac{4}{3}k_c}$

Note: Vertical k_c same as lateral one.

Total Stiffness @ Folding Truss 3

Now, equate capillary force to spring restoring force and solve for z displacement. If the solution is real (i.e., not imaginary), then Folding Truss 3 does not stick; otherwise, it sticks.

$$F_{stiction} = \frac{2A_{t3}\gamma_{la}\cos\theta_c}{(g-z)} = k_{t3}z = F_{spring}$$

$A_{t3} \triangleq$ underside area of folding truss 3

$\gamma_{la} \triangleq$ water-air surface tension

$\theta_c \triangleq$ contact angle

Rearranging:

$$z^2 - gz + \frac{2A_{t3}\gamma_{la}\cos\theta_c}{k_{t3}} = 0$$

$\Rightarrow$ quadratic solution: $z = \frac{g}{2} \pm \sqrt{\frac{g^2}{4} - \frac{8A_{t3}\gamma_{la}\cos\theta_c}{4k_{t3}}}$

Plug in numbers: $g = 3\mu$, $\gamma_{la} = 72.75\times10^{-3}$, $\theta_c = 85°$

$$A_{t3} = (160\mu)(5\mu) + 2(25\mu)(5\mu) = 800\mu^2 + 250\mu^2 = 1050\mu^2$$

$$k_{t3} = \frac{4}{3}k_c = \frac{4}{3}(2)(2.4) = 6.4\,N/m$$

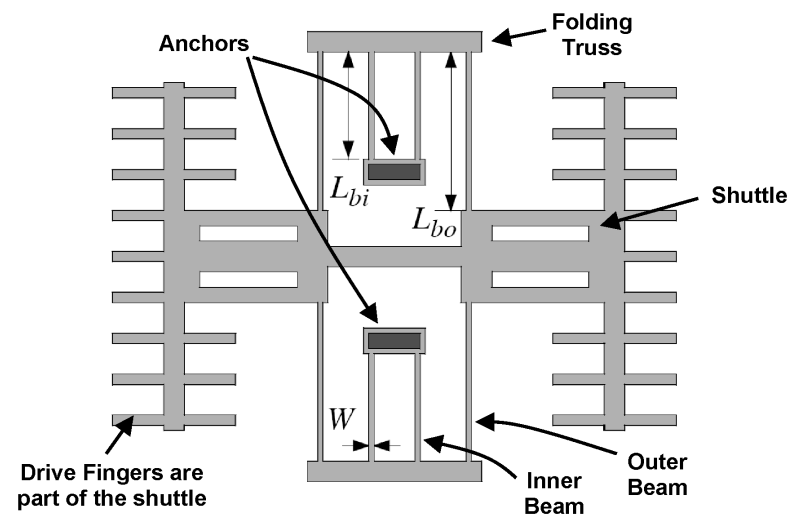
$$\therefore z = \frac{3\mu}{2} \pm \sqrt{\frac{(3\mu)^2}{4} - \frac{8(1050\mu^2)(72.75\times10^{-3})\cos(85°)}{4(6.4)}}$$

$z = 1.09\mu m, 1.9\mu m \Rightarrow$ both $< 3\mu m$ (i.e., < the gap)

$\therefore$ Folding Truss 3 will not stick.

Problem 3. **Total 37 points**

The figure below presents a shuttle mass suspended 2μm above the substrate by a ratioed folded-beam suspension, for which the inner and outer beams in the folding suspension have different lengths. The shuttle consists of an H-like structure with many fingers, all of which comprise one rigid body. In addition, the folding trusses are extremely rigid compared with the inner and outer beams. The whole structure is surface-micromachined in a 2μm-thick structural material.



Structural Material Properties:
Young's Modulus, E = 150 GPa; Density, ρ = 2,300 kg/m^3; Poisson ratio, ν = 0.226

Geometric Dimensions:
L_{bi} = 40μm; L_{bo} = 50μm; W = 2μm; Thickness, h = 2μm
Folding Truss Area = 5 × 50 μm^2; Shuttle Area = 4,000 μm^2

Use the data in the box above to answer the following questions.

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(a) If the outer beams of the structure were buckled after deposition at 700°C and cool-down to 25°C, is the thermal expansion coefficient of the substrate larger or smaller than that of the structural material? Why?

(b) Now, assume that there is no buckling and the structure is stress-free. Write an expression for the equivalent mass of this structure at a location on the shuttle and calculate its value.

(a) For buckling to occur, the stress on the outer beams must be compressive. This in turn requires that the substrate pull the device anchors together faster than contraction of the structural material when the temperature goes from 700°C to 25°C. This requires that

$$\alpha_{T\,substrate} > \alpha_{T\,film}$$

(b) The equivalent dynamic mass M_{eq} can be extracted from the maximum kinetic energy:

$$KE_{max} = \frac{1}{2} M_{eq} v_s^2 = \frac{1}{2} M_{eq} \omega_o^2 X_s^2 \longrightarrow \boxed{M_{eq} = \frac{2KE_{max}}{\omega_o^2 X_s^2}}$$

v_s: maximum shuttle velocity; X_s: maximum shuttle displacement

Find KE_{max}:

$$KE_{max} = \underbrace{KE_s}_{\text{shuttle}} + \underbrace{KE_t}_{\text{truss}} + \underbrace{KE_{bi}}_{\text{inner beam}} + \underbrace{KE_{bo}}_{\text{outer beam}}$$

must integrate to get these

$$KE_s = \frac{1}{2} v_s^2 M_s = \frac{1}{2}\omega_o^2 X_s^2 M_s$$

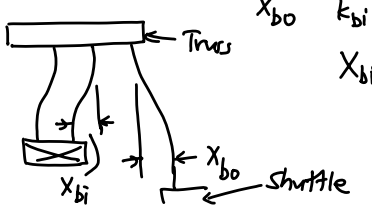
M_s: mass contribution from the shuttle

$$KE_t = \frac{1}{2} v_t^2 M_t$$

v_t: max velocity of the truss $= \omega_o X_{bi}$; X_{bi}: max displacement of the truss

Since force is divided evenly between the inner and outer beams of an inner-outer beam pair, we can write:

$$k_{bi} X_{bi} = k_{bo} X_{bo} \rightarrow \frac{X_{bi}}{X_{bo}} = \frac{k_{bo}}{k_{bi}} = \left(\frac{L_{bi}}{L_{bo}}\right)^3 = \frac{1}{\beta^3}, \text{ where } \beta = \frac{L_{bo}}{L_{bi}}$$



$$X_{bi} + X_{bo} = X_s = X_{bi} + \beta^3 X_{bi}$$

$$\Rightarrow X_{bi} = \frac{X_s}{1+\beta^3} \Rightarrow v_t = \frac{\omega_o^2 X_s^2}{(1+\beta^3)^2}$$

Thus:

$$\frac{KE_t}{2} = \frac{1}{2}\frac{\omega_o^2 X_s^2}{(1+\beta^3)^2} M_t \longrightarrow KE_t = \frac{1}{2}\omega_o^2 X_s^2 \underbrace{\left[\frac{2M_t}{(1+\beta^3)^2}\right]}_{\text{mass contribution from the two trusses}}$$

M_t: static mass of a single truss

From lecture, the static displacement functions for the inner & outer beams can be written:

$$x_{bi}(y) = \frac{X_s}{(1+\beta^3)}\left[3\left(\frac{y}{L_{bi}}\right)^2 - 2\left(\frac{y}{L_{bi}}\right)^3\right]$$

$$x_{bo}(y) = X_s\left\{1 - \frac{\beta^3}{1+\beta^3}\left[3\left(\frac{y}{L_{bo}}\right)^2 - 2\left(\frac{y}{L_{bo}}\right)^3\right]\right\}$$

Using these:

$$KE'_{bi} = \frac{1}{2}\int_0^{L_{bi}} [\omega_o x_{bi}(y)]^2 \frac{M_{bi}}{L_{bi}} dy = \frac{1}{2}\frac{M_{bi}}{\cancel{L_{bi}}}\frac{\omega_o^2 X_s^2}{(1+\beta^3)^2}\left(\frac{13}{70}\cancel{L_{bi}}\right) \quad \text{(for one beam)}$$

KE'_{bi}: one beam; $\frac{13}{70}$: from lecture

$$KE_{bi} = \frac{1}{2}\omega_o^2 X_s^2\left[\frac{13}{35}\frac{M_{bi}}{(1+\beta^3)^2}\right](4) = \frac{1}{2}\omega_o^2 X_s^2 \underbrace{\left[\frac{52}{35}\frac{M_{bi}}{(1+\beta^3)^2}\right]}_{M_{eq}\text{ contribution from all inner beams}}$$

KE_{bi}: all 4 beams; M_{bi}: static mass of one inner beam

$$KE'_{bo} = \frac{1}{2}\int_0^{L_{bo}} [\omega_o x_{bo}(y)]^2 \frac{M_{bo}}{L_{bo}} dy = \frac{M_{bo}}{2}\frac{\omega_o^2 X_s^2}{L_{bo}}\int_0^{L_{bo}}\left\{1 - \frac{\beta^3}{(1+\beta^3)}\left[3\left(\frac{y}{L_{bo}}\right)^2 - 2\left(\frac{y}{L_{bo}}\right)^3\right]\right\}^2 dy$$

KE'_{bo}: one beam

$$\int_0^{L_{bo}}\left\{1 - \frac{2\beta^3}{(1+\beta^3)}\left[3\left(\frac{y}{L_{bo}}\right)^2 - 2\left(\frac{y}{L_{bo}}\right)^3\right] + \frac{\beta^6}{(1+\beta^3)^2}\left[3\left(\frac{y}{L_{bo}}\right)^2 - 2\left(\frac{y}{L_{bo}}\right)^3\right]^2\right\} dy$$

$$= y\Big|_0^{L_{bo}} - \frac{2\beta^3}{(1+\beta^3)}\left[\frac{y^3}{L_{bo}^2} - \frac{1}{2}\frac{y^4}{L_{bo}^3}\right]\Bigg|_0^{L_{bo}} + \frac{\beta^6}{(1+\beta^3)^2}\left(\frac{13}{35}L_{bo}\right)\Bigg|_0^{L_{bo}}$$

$$\therefore KE_{bo} = \frac{1}{2}\omega_o^2 X_s^2 \underbrace{M_{bo}\left(\frac{1}{1+\beta^3} + \frac{\beta^6}{(1+\beta^3)^2}\frac{13}{35}\right)(4)}_{M_{eq}\text{ contribution from all 4 outer beams}}$$

M_{bo}: static mass of one outer beam

Thus: (combining all mass contributions)

$$\boxed{M_{eq} = M_s + \frac{2M_t}{(1+\beta^3)^2} + \frac{13}{35}\frac{4M_{bi}}{(1+\beta^3)^2} + \left(\frac{1}{1+\beta^3} + \frac{\beta^6}{(1+\beta^3)^2}\frac{13}{35}\right)(4M_{bo})} \qquad \beta = \frac{L_{bo}}{L_{bi}}$$

Plugging in numbers: $\beta = \frac{50}{40} = 1.25$

$$M_{eq} = \left\{(4000\mu^2)(2\mu) + \frac{2(5\mu)(50\mu)}{(1+1.25^3)^2} + \frac{13}{35}\frac{4(40\mu)(20\mu)}{(1+1.25^3)^2} + \left(\frac{1}{1+1.25^3} + \frac{1.25^6}{(1+1.25^3)^2}\frac{13}{35}\right)(4)(50\mu)(2\mu)\right\}(2300) \Rightarrow M_{eq} = 1.97\times10^{-11}\,\text{kg}$$