Discussion: Review of Op Amps
Lecture Outline

• Reading: Senturia Chpt. 14

• Lecture Topics:
  ➤ Ideal Op Amps
  ➤ Non-Ideal Op Amps
     - Finite gain & bandwidth
     - Input offset voltage
Ideal Operational Amplifiers
Ideal Op Amp

• Equivalent Circuit of an Ideal Op Amp:

\[ v_0 = A(v_+ - v_-) \]

Differential input

\[ v_0 = A(v_2 - v_1) \]

Voltage-Controlled Voltage Source (VCVS)

Single-ended output

\[ i_1 = 0 \]

\[ i_2 = 0 \]

\[ R_{in} \]

\[ v_1 \]

\[ v_2 \]

\[ A(v_+ - v_-) \]

\[ R_0 \]

\[ v_0 \]

\[ v_+ \]

\[ v_- \]

Properties of Ideal Op Amps:

1. \( R_{in} = \infty \)

2. \( R_0 = 0 \)

3. \( A = \infty \)

4. \( i_+ = i_- = 0 \)

5. \( v_+ = v_- \), assuming \( v_0 \) finite

Why?
Ideal Op Amp (cont)

• Properties of Ideal Op Amps:

1. \( R_{in} = \infty \)
2. \( R_0 = 0 \)
3. \( A = \infty \)
4. \( i_+ = i_- = 0 \)
5. \( v_+ = v_- \) , assuming \( v_0 = \text{finite} \)

Why? Because for

\[ \infty (v_+ - v_-) = v_0 = \text{finite} \]
\[ \therefore v_+ - v_- = 0 \rightarrow v_+ = v_- \]

\[ \frac{v_0}{\infty} \Rightarrow \text{virtual short circuit (virtual ground)} \]

• Big assumption! \( (v_0 = \text{finite}) \)

• How can we assume this? We can assume this only when there is an appropriate negative feedback path!
Negative Feedback

Where $S$ could be a current, voltage, displacement, etc., ...

Negative feedback acts to oppose or subtract from input.

Overall transfer function.

(When there is negative FB around the amplifier.)

$S_0 = a S_\varepsilon$

$S_\varepsilon = S_i - \beta S_0$

$S_0 = a(S_i - \beta S_0)$

$S_0 (1 + a \beta) = a S_i \rightarrow \frac{S_0}{S_i} = \frac{1}{1 + a \beta}$

$[a \rightarrow \infty] \Rightarrow \frac{S_0}{S_i} \approx \frac{a}{a \beta} = \frac{1}{\beta} = \text{finite!} \quad \therefore S_0 = \frac{1}{\beta} S_i = \text{finite!}$
• **Comments:**

1. Negative FB can insure $S_0 = \text{finite even with } a = \infty$.

2. Overall gain dependent (or overall T.F.) dependent only on external components. (e.g., $\beta$).

3. Overall (Closed-loop) gain ($S_o/S_i$) is independent of amplifier gain $a$.

   → Very important, since amplifiers using transistors can be designed to have large gain, but it’s hard to get an exact gain. i.e., if you’re shooting for $a = 50,000$, you might get 47,000 or 60,000 instead.

   → Comment 3 makes this less consequential.
Positive Feedback

- Contrast with Positive Feedback:

\[ \Delta S_i \uparrow + \Delta S_S \uparrow \uparrow \Delta S_0 \uparrow \uparrow \]

Output blows up! (for \( a\beta > 1 \))

Will be the case for \( a = \infty \).

But for a bounded, controllable function, need negative FB around the op amp.

If \( \beta \) is a bandpass biquad transfer function → get oscillation at the resonance frequency
Inverting Amplifier

1. Verify that there is negative FB.
2. \( v_0 = \text{finite} \rightarrow v_+ = v_- \rightarrow \text{node attached to (-) terminal is virtual ground.} \)
3. \( i_- = 0 \therefore i_1 = i_2 \)

\[
\begin{align*}
i_1 &= \frac{v_i - 0}{R_1} = \frac{v_i}{R_1} = i_2 \\
v_0 &= 0 - i_2 R_2 = -i_2 R_2
\end{align*}
\]

\[
\Rightarrow v_0 = -\left( \frac{v_i}{R_1} \right) R_2 = -\frac{R_2}{R_1} v_i \therefore \frac{v_0}{v_i} = -\frac{R_2}{R_1}
\]

**Benefit:** Any shunt \( C \) at this node will be grounded out.

**NOTE:** Gain dependent only on \( R_1 \) & \( R_2 \) (external components), not on the op amp gain.
Flip the Op Amp Around

Put the feedback around the (+) terminal and see what happens

\[ R_1 \quad \Delta v \uparrow \quad \Delta v \uparrow \uparrow \]

\[ \text{This is not (-) FB : } v_0 \neq \text{finite}, \; v_+ \neq v_- \]

\[ \text{Cannot analyze using the ideal op amp method} \]

\[ \text{Circuit will “rail out”} \]

\[ v_0 = L^+ \text{ or } L^- \text{ depending on initial conditions} \]

\[ \begin{cases} 
  v_+ = (+) \rightarrow L^+ \\
  v_+ = (-) \rightarrow L^-
\end{cases} \]
An inverting amplifier is just a transresistance amplifier with an $R_1$ to convert voltage to current!

Transresistance Amplifier

- Take $R_1$ away

Virtual ground

1. Verify that there is neg. FB $\rightarrow$ yes, since same FB as inverting amplifier

2. Thus, $v_o = \text{finite} \rightarrow v_+ = v_- \rightarrow (-)$ terminal is virtual ground

3. $i_- = 0 \rightarrow i_1 = i_2$

$$v_0 = -i_2 R_2 = -i_i R_2 \quad \Rightarrow \quad \frac{v_0}{i_i} = -R_2$$
Integrator

- Replace $R_2$ with $C_2$

Virtual ground

Again, shunt elements at this node will be grounded out.

1. Verify that there is neg. FB $\rightarrow$ need a large resistor $R_2$ to DC connect the output and (-) terminal

2. With $R_2$, $v_o = \text{finite} \rightarrow v_+ = v_- \rightarrow (-) \text{terminal} = \text{virtual gnd}$

3. $i_- = 0 \rightarrow i_1 = i_2$

$$v_0 = -\frac{i_2}{sC_2} = -\frac{i_i}{sC_2} \quad \Rightarrow \quad \frac{v_0}{i_i} = -\frac{1}{sC_2}$$
Non-Inverting Amplifier

1. Verify that there is negative FB $\rightarrow$ same feedback circuit as for inverting amplifier, so neg. FB

2. $v_0 = \text{finite} \rightarrow v_+ = v_- \quad i_1 = \frac{v_i}{R_1} = i_2$

3. Analysis:

$$i_- = 0 \quad \therefore \quad v_0 = i_1 R_1 + i_2 R_2 = \frac{v_i}{R_1} (R_1 + R_2)$$

$$\therefore \quad \frac{v_0}{v_i} = 1 + \frac{R_2}{R_1}$$

Again, gain depends only on $R_1$ & $R_2$ (external or ratioed components), not on the op amp gain.
Unity Gain Buffer

Non-Inverting Amplifier

\[ \frac{v_0}{v_i} = 1 + \frac{R_2}{R_1} \]

Unity Gain Buffer

[Equations and circuit diagrams]
Non-Ideal Operational Amplifiers
Actual Op Amps Are Not Ideal

• Actual op amps, of course, are not ideal; rather, they ...
  ✔ Generate noise
  ✔ Have finite gain, $A_o$
  ✔ Have finite bandwidth, $\omega_b$
  ✔ Have finite input resistance, $R_i$
  ✔ Have finite input capacitance, $C_i$
  ✔ Have finite output resistance, $R_o$
  ✔ Have an offset voltage $V_{OS}$ between their (+) and (-) terminals
  ✔ Have input bias currents
  ✔ Have an offset $I_{OS}$ between the bias currents into the (+) and (-) terminals
  ✔ Have finite slew rate
  ✔ Have finite output swing (governed by the supply voltage used, -L to +L)

• And what’s worse: All of the above can be temperature (or otherwise environmentally) dependent!
Finite Op Amp Gain and Bandwidth

- For an ideal op amp: \( A = \infty \)
- In reality, the gain is given by: \( A(s) = \frac{A_0}{1 + \frac{s}{\omega_b}} \)
- For \( \omega \gg \omega_b \):
  \[
  A(s) \approx \frac{A_0}{(s/\omega_b)} = \frac{A_0 \omega_b}{s} = \frac{\omega_T}{s}
  \]

20 dB/dec

Unity gain frequency: \( \omega_T = A_0 \omega_b \)

This pole actually designed in for some op amps.

Open-loop response of the amplifier.

Integrator w/ time const. \( 1/\omega_T \)

Finite Gain

Finite Bandwidth
Example:

Inverting Amplifier

Signals from \( v_I \) and \( v_0 \) are summed at this node!

How much gets to this node is determined by the values of \( \alpha \) and \( \beta \).
Frequency response of Closed Loop Amplifiers

Example:

Non-Inverting Amplifier -

\[ v_i = v_\varepsilon + v_\beta \]

\[ \rightarrow v_\varepsilon = v_i - v_\beta \]

![Non-Inverting Amplifier diagram](image)

\[ v_\beta = \frac{R_1}{R_1 + R_2} v_0 \]

\[ \therefore \beta = \frac{v_\beta}{v_0} = \frac{R_1}{R_1 + R_2} \]

Negative FB Block Diagram -

![Negative FB Block Diagram](image)
Finite Gain BW of Op Amps

Brute Force $\frac{v_0}{v_I}(S)$ Derivation.

\[ v_I = v_+ \]

\[ v_- = v_\pm - \frac{v_0}{A(S)} \]

\[ \frac{v_0 - v_-}{R_2} = \frac{v_-}{R_1} \rightarrow \frac{v_0}{R_2} = v_- \left( \frac{1}{R_1} + \frac{1}{R_2} \right) \]

\[ = \left( v_I - \frac{v_0}{A(S)} \right) \left( \frac{1}{R_1} + \frac{1}{R_2} \right) \]

\[ v_0 = A(S)(v_+ - v_-) \]

\[ v_0 = A(S)v_+ - A(S)v_- \]

\[ A(S)v_- = A(S)v_+ - v_0 \]

\[ v_- = v_I - \frac{v_0}{A(S)} \]
Brute Force $\frac{v_0}{v_I}(S)$ Derivation (cont.).

\[ v_0 \left( \frac{1}{R_2} + \frac{1}{A(S)} \left( \frac{1}{R_1} + \frac{1}{R_2} \right) \right) = v_I \left( \frac{1}{R_1} + \frac{1}{R_2} \right) \]

\[ \frac{v_0}{v_I} = 1 + \frac{R_2}{A(S)} \left( 1 + \frac{R_2}{R_1} \right) \]
Finite Gain BW of Op Amps (cont.)

Brute Force $\frac{v_0}{v_I}(S)$ Derivation (cont.).

\[
A(S) = \frac{A_0}{1 + \frac{S}{w_b}} \Rightarrow \frac{v_0}{v_I}(S) = \frac{1 + \frac{R_2}{R_1}}{1 + \frac{S}{w_b} \left(1 + \frac{R_2}{R_1}\right)}
\]

\[
= \frac{A_0 \left(1 + \frac{R_2}{R_1}\right)}{1 + \frac{S}{w_b} \left(1 + \frac{R_2}{R_1}\right)}
\]
Finite Gain BW of Op Amps (cont.)

Brute Force $\frac{V_0}{V_I}$ Derivation (cont.).

\[
\frac{V_0}{V_I} = \frac{1 + \frac{R_2}{R_1}}{1 + \left(1 + \frac{S}{w_b}\right) \frac{1}{A_0} \left(1 + \frac{R_2}{R_1}\right)} = 1 + \frac{R_2}{R_1} \frac{1}{\frac{1}{A_0}\left(1 + \frac{R_2}{R_1}\right) + \frac{S}{w_b} \frac{1}{A_0} \left(1 + \frac{R_2}{R_1}\right)}
\]

\[
= \left(1 + \frac{R_2}{R_1}\right) \frac{1}{1 + \frac{S}{w_b} \frac{1}{A_0} \left(1 + \frac{R_1}{R_1 + R_2}\right)}
\]
Finite Gain/BW of Op Amps

Recall from our previous feedback analysis:

\[ \frac{v_0}{v_i} (S) = \frac{A(S)}{1 + \beta A(S)} \]

\[ A(S) = \frac{A_0}{1 + \frac{S}{w_b}} \]

\[ \begin{aligned}
&\left\{ \begin{array}{l}
\frac{v_0}{v_I} (S) = \frac{A_0}{1 + S/w_b} \\
T(S) = \beta A(S) \Delta \text{"loop gain"}
\end{array} \right. \\
&= \frac{A_0}{1 + A_0 \beta} \left( 1 + \frac{1}{w_b (1 + A_0 \beta)} \right) = \frac{v_0}{v_I} (S)
\end{aligned} \]

Frequency-shaping term

\[ T_0 = A_0 \beta = \"loop gain\" \text{ at } w = 0 \]

(i.e., at dc)
Input Offset Voltage
Input Offset Voltage, $V_{0S}$:

$$v_0 = A(v_+ - v_-)$$

**Ideal case:** $v_0 = 0$

**Reality:** $v_0 \neq 0$ (usually, $v_0 = L^+$ or $L^-$: it rails out!)

Why? Internal mismatches within the op amp → cause a dc offset. Model this with an equivalent input offset voltage $V_{0S}$.

Typically, $V_{0S} = 1\text{mV} - 5\text{mV}$
**Effect of $V_{0S}$ on Op Amp Circuits**

**Example:** Non-Inverting Amplifier

$$V_0 = V_{0S} \left(1 + \frac{R_2}{R_1}\right)$$

e.g., $\frac{R_2}{R_1} = 9$, $V_{0S} = 5mV \rightarrow V_0 = 50mV$

(not so bad ...)

$$R_2$$

$$R_1$$
Effect of $V_{0S}$ on Op Amp Circuits (cont.)

**Example: Integrator**

To fix this, place a resistor in shunt with the $C \rightarrow$ then:

$$v_0 = V_{0S} \left(1 + \frac{R_f}{R}\right)$$

$$v_0 = V_{0S} + \frac{1}{C} \int_0^t i_1 dt$$

$$= V_{0S} + \frac{1}{C} \int_0^t \frac{V_{0S}}{R} dt$$

$$= V_{0S} \left(1 + \frac{t}{RC}\right) + v_C \bigg|_{t=0}$$

Will continue to increase until op amp saturates