Problem 1. Total 40 points

Please provide short written answers to the questions that follow.

(a) What is “footing” in deep reactive ion etching? Why does it occur?

“Footing” is an occurrence in deep reactive ion etching (DRIE) where etching occurs laterally into the silicon area to etch out an oxide (or other dielectric) film underlying the silicon. It occurs because the underlying dielectric does not provide enough charge to repel the incoming ions, thereby allowing buildup of charge on the dielectric surface. The charge repels incoming ions, sending them towards the sidewalls, and hence, promoting etching in the horizontal direction.

(b) Suppose you needed to integrate MEMS together with CMOS transistors in a single planar process. You have two choices on which to proceed: (i) one where steps from the MEMS process flow are intermixed with steps from the transistor process flow; and (ii) one where all of the transistors are fabricated first, in one module, and all the MEMS fabricated afterwards, in another module, where no intermixing of steps from the modules occurs. Which would you choose? Give two reasons why you made this choice.

Choose (ii), the modular one, because:

1. A modular process requires fewer passivation/protector steps than an intermixed one, which needs such steps every time the transistor flow is interrupted after the MEMS flow, and vice versa.
2. A modular process can adapt more easily to changes in either module, whereas a mixed process must be redesigned from scratch to accommodate changes in the transistor and/or MEMS module.
3. A modular process can more readily incorporate foundry transistors (e.g., CMOS), whereas intermixed processes must generally be realized in custom fabs.

(c) Suppose you wanted to integrate nickel micromechanical structures together with CMOS transistors in a single modular planar fabrication process. Would you do the nickel MEMS before the transistors or after them? Give two reasons why.

After the transistors, because:

1. Nickel can contaminate transistor if present during their fabrication, rendering them inoperable.
2. Nickel cannot withstand the high temperatures (>1000°C) often used in transistor fabrication.

(d) Transistor can withstand the relatively low temperatures (<500°C) required for nickel MEMS fabrication.

3. Doing nickel after transistor allows the use of less expensive foundry transistors.

(e) Give three reasons why electrostatic comb drive transducers are popular in MEMS designs.

1. They allow a large range of motion.
2. The force they generate is independent of displacement.
3. They do not require electrical stiffness, so they allow resonance frequencies independent of DC-bias voltage.
4. They provide a linear capacitance variation with displacement.

Any two of these received points.

Problem 2. Total 50 points

The figure below presents the top view of a shuttle mass free to move in the x-direction, as governed by the indicated mechanical spring with stiffness k_m. The spring is otherwise infinitely stiff in the y- and z-directions. The shuttle can be driven into motion by the electrode shown, which is rigidly anchored to a substrate at the shaded region. Mechanical and electrical data for this problem is given in the box below the figure.

Mechanical Data:
Geometrical Dimensions: L_o = 20 μm, g_o = 2 μm, θ = 30°, Thickness, h = 2 μm
Spring Constant, k_m = 1 N/m; Shuttle Mass, m = 4 × 10^{-11} kg

Electrical Data:
DC-Bias Voltage, V_p = 50V
Permittivity of Vacuum: 8.854 × 10^{-12} F/m

Electrode

Mechanical

Electrical
Answer the following questions regarding the above spring-mass system.

(a) Write an expression for the shuttle-to-electrode finger overlap capacitance \( C(x) \) as a function of displacement \( x \). Calculate its value at \( x=0 \).

(b) Write an expression for the change in shuttle-to-electrode finger overlap capacitance per unit displacement, \( \frac{dC}{dx} \). Calculate its value at \( x=0 \).

(c) Write an approximate expression for the electrical spring constant \( k_e \) generated between the shuttle and electrode. Calculate its value at \( V_p=50V \).

(d) Determine the pull-in voltage \( V_{py} \) of the shuttle (i.e., the voltage that causes the shuttle to collapse into the electrode).

\[ C(x) = \frac{2 \varepsilon_0 h (L_0 - 2x \cos \theta)\sin \theta}{\eta + x \sin \theta} \]

For \( x=0 \),

\[ C(0) = \frac{2 \varepsilon_0 h (L_0)\sin \theta}{\eta + 0 \sin \theta} = \frac{2 \varepsilon_0 h L_0 \sin \theta}{\eta} \]

\[ \frac{dC}{dx} = \frac{2 \varepsilon_0 h (-2x \cos \theta \sin \theta) + 2 \varepsilon_0 h (L_0 - 2x \cos \theta) \cos \theta}{\eta + x \sin \theta} \]

At \( x=0 \),

\[ \frac{dC}{dx} = \frac{2 \varepsilon_0 h (-2 \cos \theta \sin \theta) + 2 \varepsilon_0 h (L_0) \cos \theta}{\eta + 0 \sin \theta} = \frac{2 \varepsilon_0 h (L_0 - 2 \cos \theta \sin \theta)}{\eta} = \frac{2 \varepsilon_0 h (L_0 - 2 \cos \theta \sin \theta)}{\eta} \]

\[ k_e = \frac{\varepsilon_0 h}{\eta} \frac{2 \varepsilon_0 h (L_0 - 2 \cos \theta \sin \theta)}{\eta} \]

\[ V_{py} = \frac{k_m \sin \theta}{2 \varepsilon_0 h (L_0 - 2 \cos \theta \sin \theta) \sin \theta} \]

Find the pull-in gap, \( \eta_p \):

\[ F_e = \frac{1}{2} V_p^2 \frac{2 \varepsilon_0 h (L_0 - 2 \cos \theta \sin \theta) \sin \theta}{\eta_p} \]

\[ F_{net} = 0 = F_e - k_m \eta - g \eta_p \]

\[ \eta = \frac{1}{2} \frac{2 \varepsilon_0 h (L_0 - 2 \cos \theta \sin \theta)}{V_p} \sin \theta \]

(\text{continued on next page…})
Final Exam Solutions

Use the data in the box above to answer the following questions.

(a) Calculate the resonance frequency of this structure (i) when $V_p = 0V$; and (ii) when $V_p = 50V$.

(b) Assume that terminal 7 is tied to a dc-bias $V_p = 50V$ and all other terminals except 1 and 2 are grounded. Draw the electrical equivalent circuit for the ensuing micromechanical device (ignoring grounded ports) and provide numerical element values for the equivalent circuit.

(c) If terminal 7 is tied to a dc-bias $V_p = 50V$, terminal 1 is driven by an ac voltage at the resonance frequency with magnitude 30 mV, and all other terminals are grounded, what is the magnitude of motional current coming out of terminal 4 (into ground)?

(d) If terminals 1 and 2 of the micromechanical device are tied to terminals 1 and 2 of the sensing circuit, terminal 7 is tied to a dc-bias $V_p = 50V$, and all other terminals are grounded, what rms output noise voltage density (in V/√Hz) ensues at a frequency equal to half the resonance frequency? For this part, assume that the op amp is ideal in all respects, except that it exhibits noise modeled by the following uncorrelated input-reflected noise sources.

\[
\sqrt{\frac{v^2}{\Delta f}} = 12 \text{ nV/√Hz} \quad \text{and} \quad \sqrt{\frac{I^2}{\Delta f}} = 0.01 \text{ pA/√Hz}
\]

(a) This device is just a folded-beam cantilever resonator with added mass on its folding trusses. Thus, from lecture:

\[
\omega_0 = \left(\frac{k}{M_e}\right)^{1/2} \quad k_p = 2EIb(\frac{L}{2})^3 = 2(150 GPa)(20 \mu m)(\frac{20 \mu m}{2})^3 = 75 N/m
\]

\[
M_e = \frac{1}{4} M_0 + \frac{35}{32} M_b
\]

\[
= \left(3200 \mu m\right) \left\{ 4000 + \frac{35}{32} (2) (500) + \frac{15}{32} (4)(\frac{20 \mu m}{2}) (40 \mu m) \right\} = 2.06 \times 10^{-11} kg
\]

\[
\omega_0 = \left(\frac{75}{2.06 \times 10^{-11}}\right)^{1/2} = 1.91 \times 10^6 \text{ rad/s} \quad \Rightarrow \quad f_0 = \frac{1.91 \times 10^6}{2\pi} = 302.98 kHz \quad \text{(for } \omega_0 \text{)}
\]

Since cantilevered devices do not have electrical stiffness (in the lateral direction).
(b) With only port 1 and 2 active, the equivalent circuit reduces to that of a two-port mechanical device:

\[
C_x = \frac{1}{k_x} = \frac{1}{0.012} = 83.33 \frac{F}{N}
\]

\[
I_x = \frac{M_x}{k_x} = \frac{2.05 \times 10^{-11}}{0.012} = 1.71 \times 10^{-9} \frac{A}{N}
\]

\[
\tau = \frac{J_x}{k_x} = \frac{2.93 \times 10^{-12}}{0.012} = 2.45 \times 10^{-11} \frac{N\cdotm}{N}
\]

\[
\eta_{out} = \frac{V_{out}}{\frac{d\phi}{dt}} = \frac{V_{out}}{2.45 \times 10^{-11} \frac{N\cdotm}{N}} = 4.09 \times 10^{-8} \frac{V}{N\cdotm} 
\]

(c) The displacement \(X_x\) at the primary shuttle induced by the excitation \(N_x\) is:

\[
X_x = \frac{\eta_{out} N_x}{k_x} = \frac{(4.09 \times 10^{-8}) N_m}{0.012} \approx 3.49 \times 10^{-6} \text{m} = 0.349 \text{nm}
\]

Folding the shuttle displacement:

\[
X_x = \frac{\partial X_x}{\partial N_x} = \frac{\partial X_x}{\partial N_m} = \frac{(0.349 \times 10^{-6})}{(0.012)} = 2.91 \times 10^{-5} \text{m/N}
\]

The noise circuit is as follows:

By superposition:

\[
\frac{N_x}{\tau} = -R_x \cdot \frac{I_1}{2}
\]

\[
\frac{I_1}{2} = \frac{N_x}{R_x} = \frac{-1}{2} \cdot \frac{N_x}{R_x}
\]

The op amp circuit differentially amplifies

\[
\frac{N_x}{\tau} = \frac{2.93 \times 10^{-10}}{(1.25 \times 10^{-9})} = 2.32 \text{ kHz}
\]

\[
\Phi\left(\frac{1}{2}\right) = \frac{(\omega/\delta)(\omega/\delta)}{2} = \frac{2 \omega}{\delta} \frac{\omega}{\delta} + \frac{\omega}{\delta} = \frac{2 \omega}{\delta} + \frac{\omega}{\delta} = \frac{3 \omega}{\delta} + \frac{\omega}{\delta}
\]

\[
\left|\Phi\left(\frac{1}{2}\right)\right|^2 = \left(\frac{3 \omega}{\delta} + \frac{\omega}{\delta}\right)^2 = 6.67 \times 10^{-6}
\]