PROBLEM SET #1

Issued: Wednesday, Sept.10, 2008

Due (at 5 p.m.): Thursday, Sept. 18, 2008, during class

This homework assignment is intended to give you some early practice playing with dimensions and exploring how scaling can greatly improve certain performance characteristics of mechanical systems. Don't worry at this point if you do not understand fully some of the physical expressions used. Some of them will be revisited later in the semester.

1. Suppose you are asked to design a polycrystalline silicon clamped-clamped beam resonator, such as discussed in lecture. For polycrystalline silicon, assume the following material properties: Young's modulus E = 150 GPa, density ρ = 2300 kg/m^3, and Poisson ratio ν = 0.226.

(a) Consider a beam with width W_r = 8 μm and thickness h = 2 μm. Use Euler-Bernoulli theory (i.e., the formulation covered in class) to determine the length of the beam L_r that allows it to mechanically resonate in a direction perpendicular to the substrate at:

(i) 10 MHz, (ii) 100 MHz, (iii) 1 GHz.

(b) Use Euler-Bernoulli theory to determine the length of the beam L_r that allows it to mechanically resonate perpendicular to the substrate at 1 GHz if the beam width W_r and thickness h are as follows:

(i) W_r = 8μm, h = 2μm; (ii) W_r = 1μm, h = 1μm; and (iii) W_r = 300nm, h = 100nm.

(c) Euler-Bernoulli theory is actually not very accurate when the length of the beam begins to approach its thickness, mainly because it ignores shear displacements and rotary inertias. (These are things that you will learn more about later in the course.) For cases where thickness approaches length, the more complicated Timoshenko design procedure should be used to model a beam's resonance characteristics. For a clamped-clamped beam, Timoshenko's design procedure uses the following equation:

$$\tan\frac{\beta}{2}+\frac{\beta}{\alpha}\left(\frac{\alpha^2+g^2\left(\frac{\kappa G}{E}\right)}{\beta^2-g^2\left(\frac{\kappa G}{E}\right)}\right)\tanh\frac{\alpha}{2}=0 \tag{1}$$

where

$$g^2=\omega_0^2L_r^2\left(\frac{\rho}{E}\right) \tag{2}$$

$$\left.\begin{matrix}\alpha^2\\ \beta^2\end{matrix}\right\}=\frac{g^2}{2}\left[\mp\left(1+\frac{E}{\kappa G}\right)+\sqrt{\left(1-\frac{E}{\kappa G}\right)^2+\frac{4L_r^2hW_r}{g^2I_r}}\right] \tag{3}$$

(The first term is negative for α, positive for β.)

$$I_r=\frac{W_rh^3}{12} \tag{4}$$

$$G = \frac{E}{2(1+\nu)} \tag{5}$$

and where for a rectangular beam, $\kappa = 2/3$.

Use Timoshenko's formulas above to determine the actual frequencies of the beams you designed in parts (a) and (b) above. Can you suggest a rule for scaling of beams to attain higher frequencies that insures Euler-Bernoulli theory works reasonably well?

2. This problem concerns the micro-atomic cell summarized in Figs. 1-3, with all relevant dimensions and materials indicated, and with material properties summarized in the table below.

	C_p [J / kg K]	k [W / m K]	ρ [kg / m^3]
Glass	500	1.05	2500
Kapton	755	0.12	1420
Aluminum	903	136	2700

(a) Determine the power required to maintain a cell temperature of 80^{o}C if the cell itself operates under a vacuum environment.

(b) Determine the warm-up time (i.e., the time constant) of the system. In other words, what is the time constant with which the temperature rises when a step function in input power is applied to the system?

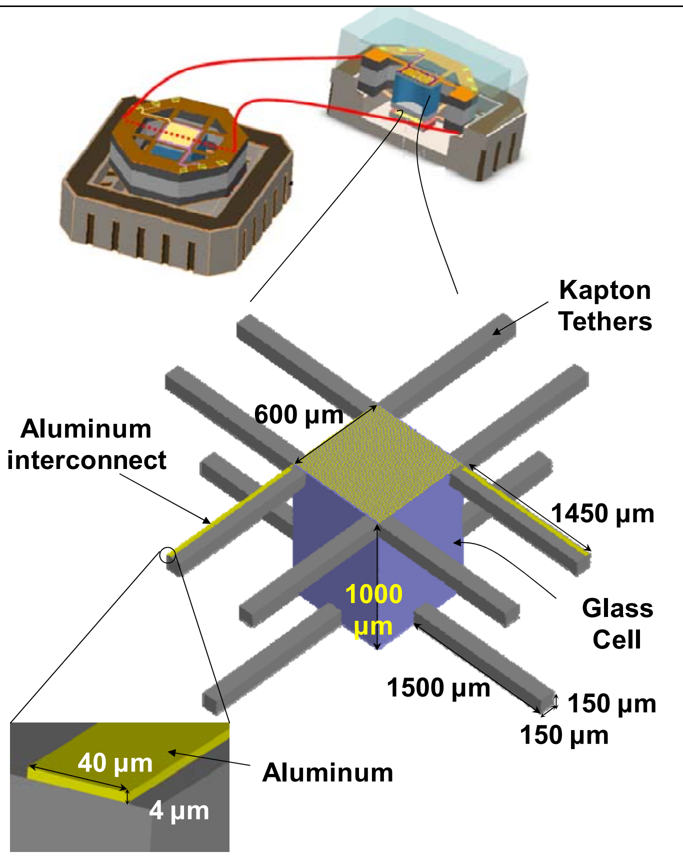

Fig. 1: Draper / Symmetricom Cell Schematic (The yellow area on top is a long, folded resistor.)

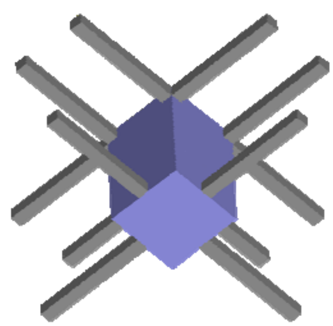

Fig. 2: Bottom view.

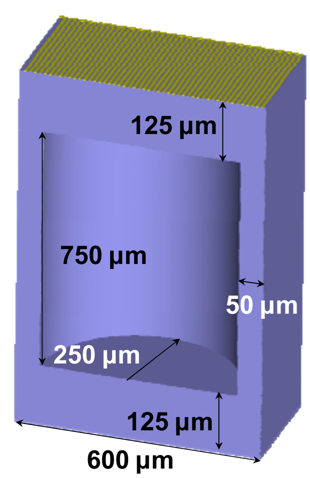

Fig. 3: Cell cross-section.