EE C245 – ME C218 Introduction to MEMS Design Fall 2007

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Lecture 13: Microstructural Elements

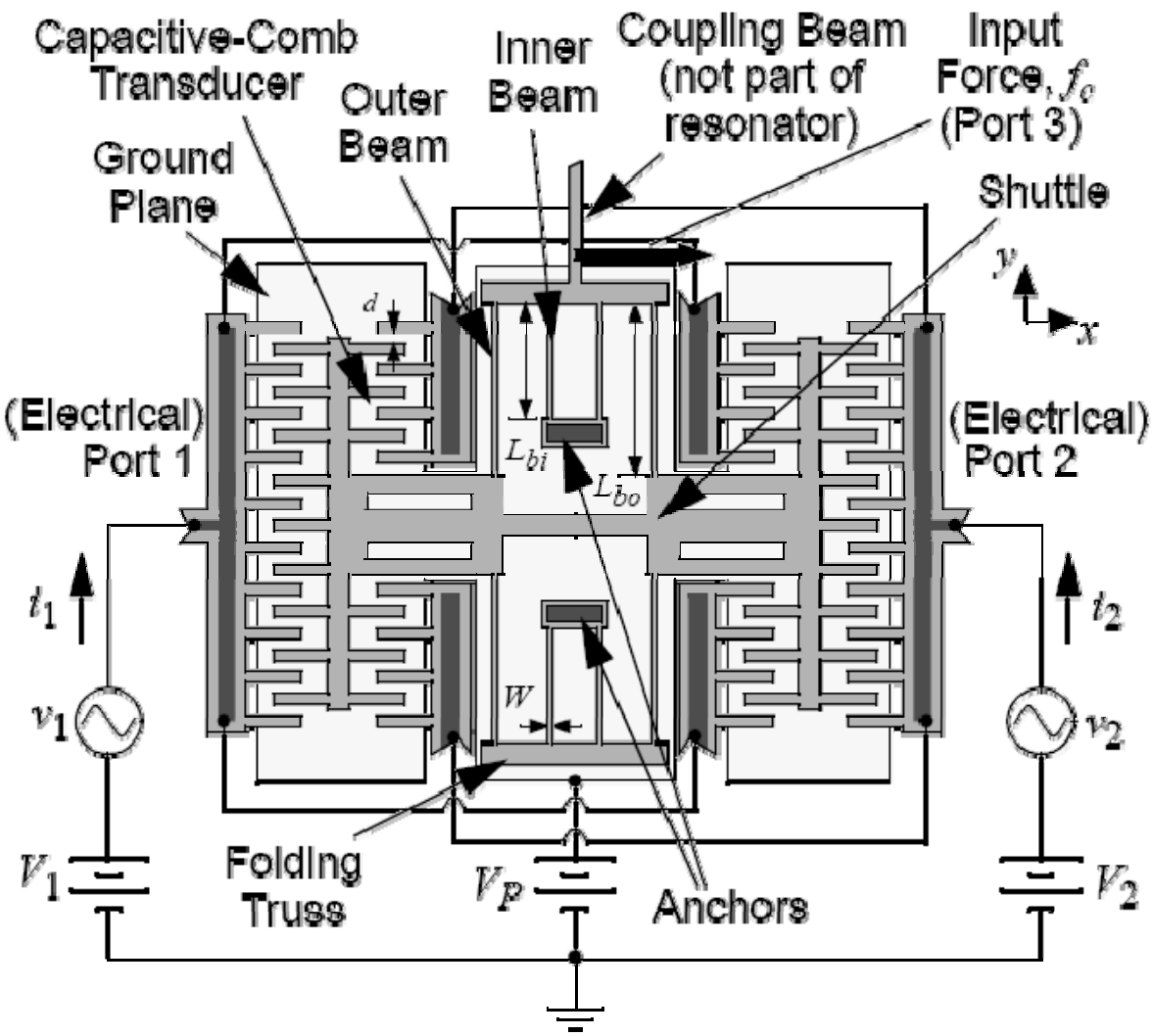
Lecture Outline

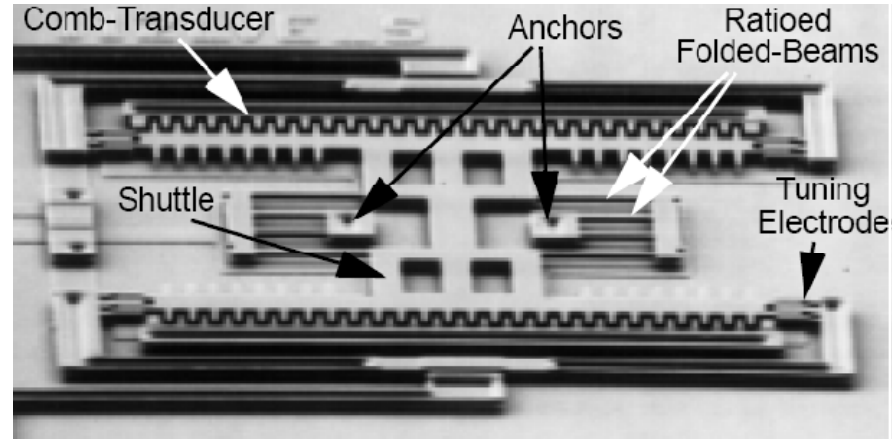
- Reading: Senturia, Chpt. 9
- Lecture Topics:
  - Bending of beams
  - Cantilever beam under small deflections
  - Combining cantilevers in series and parallel
  - Folded suspensions
  - Design implications of residual stress and stress gradients

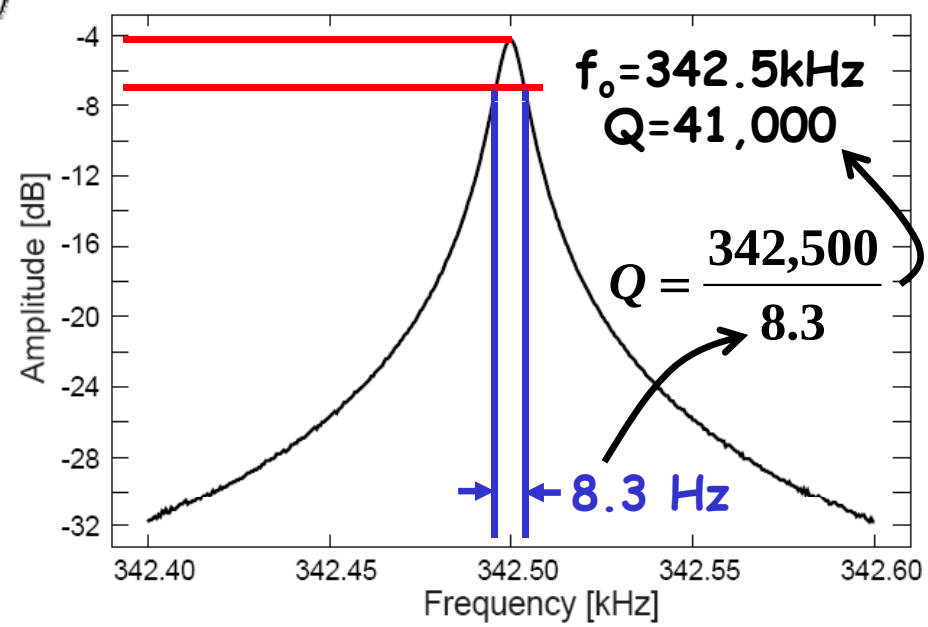
MEMS Material Properties

Folded-Beam Comb-Drive Resonator

- **Issue w/ Wine-Glass Resonator**: non-standard fab process
- **Solution**: use a folded-beam comb-drive resonator







Measurement of Young's Modulus

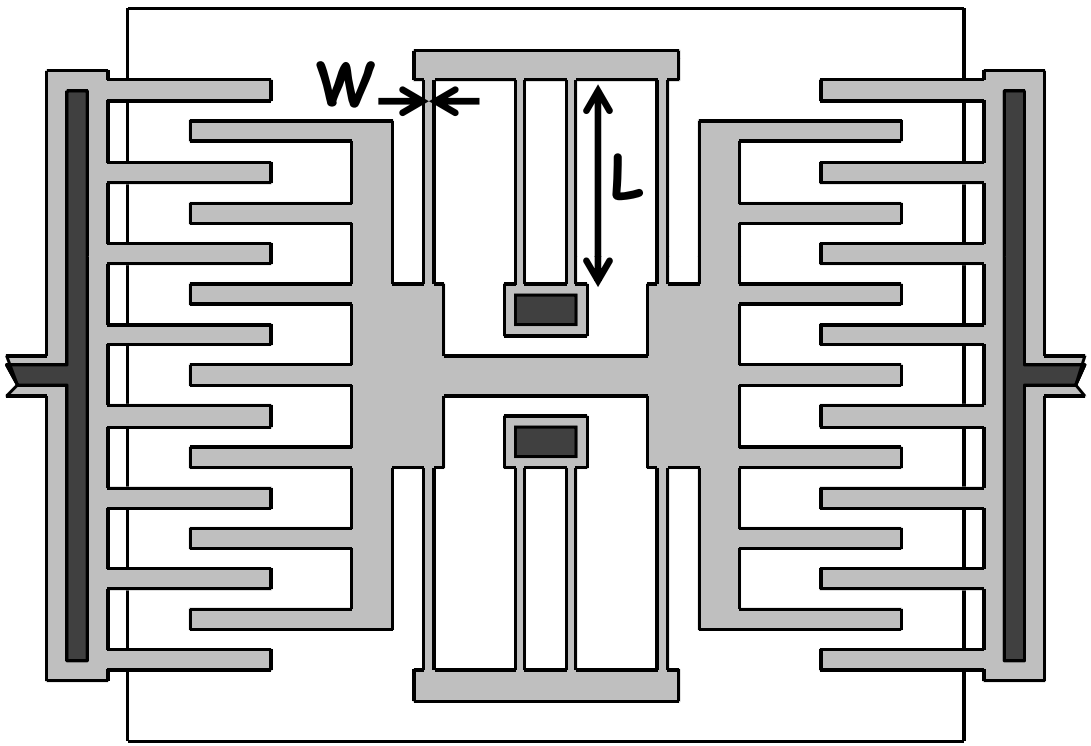
- Use micromechanical resonators
  - Resonance frequency depends on E
  - For a folded-beam resonator:

Young's modulus

$$\text{Resonance Frequency} = f_o = \left[\frac{4Eh(W/L)^3}{M_{eq}}\right]^{1/2}$$

Equivalent mass

h = thickness

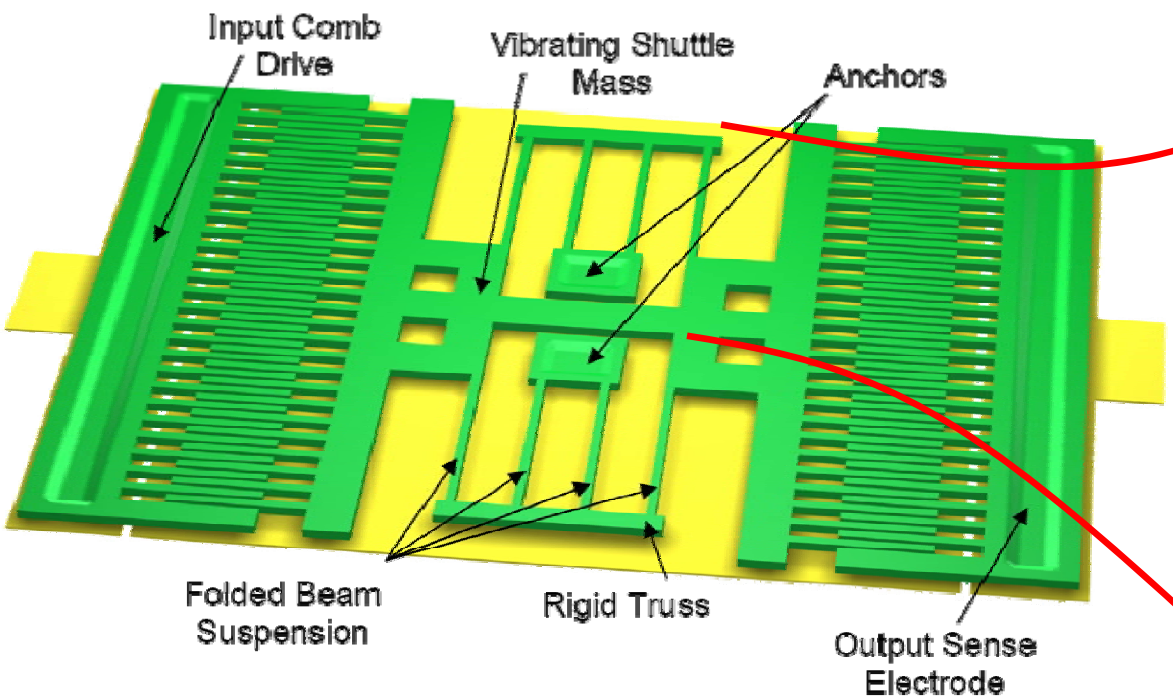


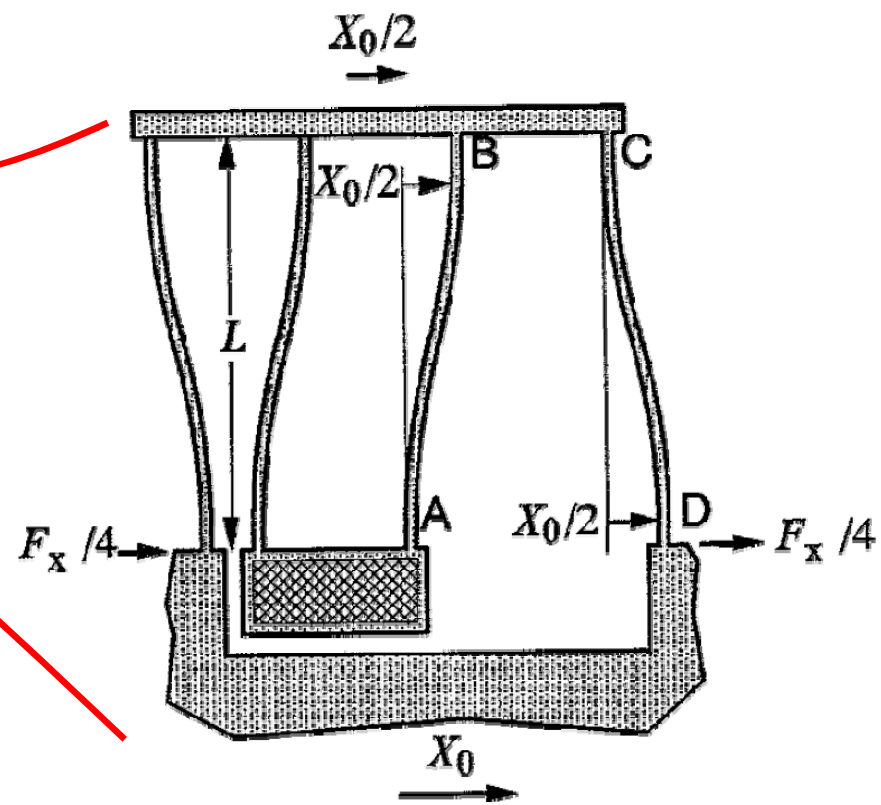
- Extract E from measured frequency f_o
- Measure f_o for several resonators with varying dimensions
- Use multiple data points to remove uncertainty in some parameters

Bending of Beams

Beams: The Springs of Most MEMS

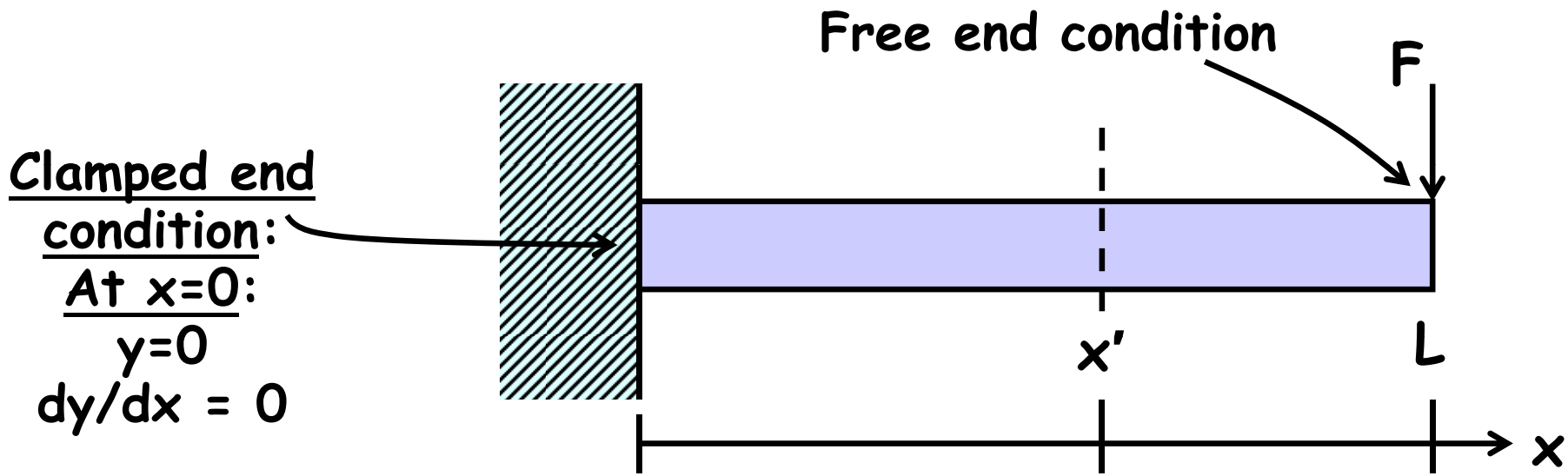
- Springs and suspensions very common in MEMS
  - Coils are popular in the macro-world; but not easy to make in the micro-world
  - Beams: simpler to fabricate and analyze; become "stronger" on the micro-scale → use beams for MEMS





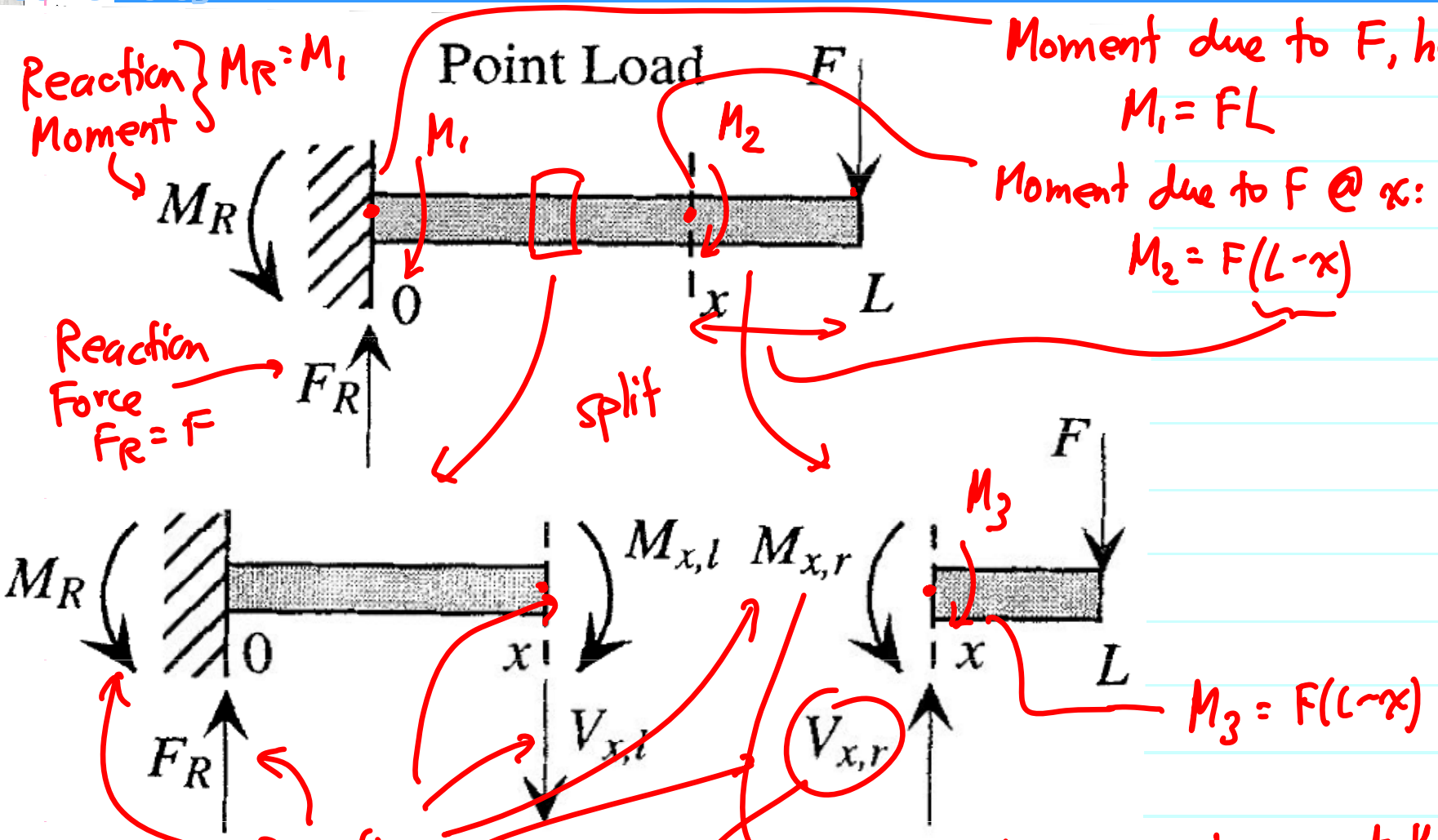
Comb-Driven Folded Beam Actuator

Bending a Cantilever Beam



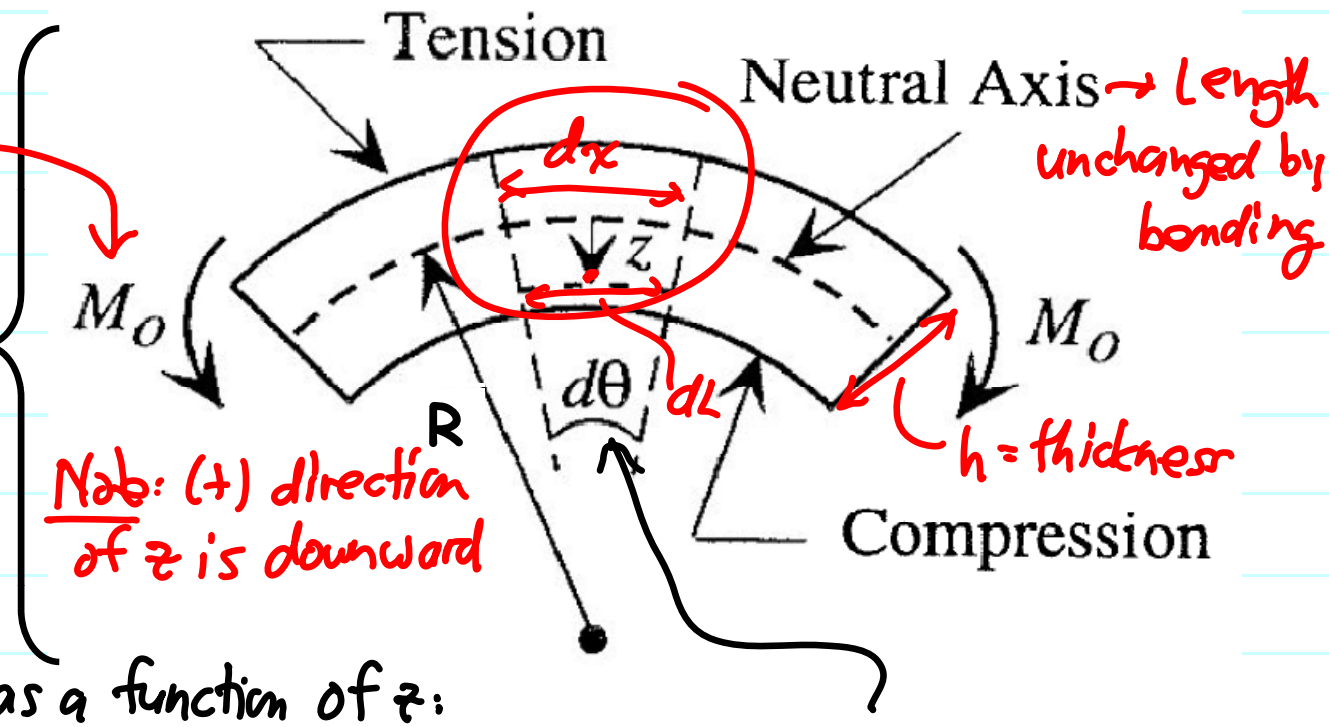
- Objective: Find relation between tip deflection $y(x=L_c)$ and applied load F
- Assumptions:
  1. Tip deflection is small compared with beam length
  2. Plane sections (normal to beam's axis) remain plane and normal during bending, i.e., "pure bending"
  3. Shear stresses are negligible

Reaction Forces and Moments



Beam Segment in Pure Bending

Small section of a beam bent in response to a tranverse load



Find the axial stress σ_x as a function of z:

Consider a segment bounded by the dashed lines defined by $d\theta$:

At $z=0$: (i.e., at the neutral axis): segment length $= dx = R\,d\theta$ (1)

At any z: segment length $= dL = (R-z)\,d\theta$ (2)

Combining (1) & (2): $dL = dx - z\,d\theta = dx - \frac{z}{R}dx$

Beam Segment in Pure Bending (cont.)

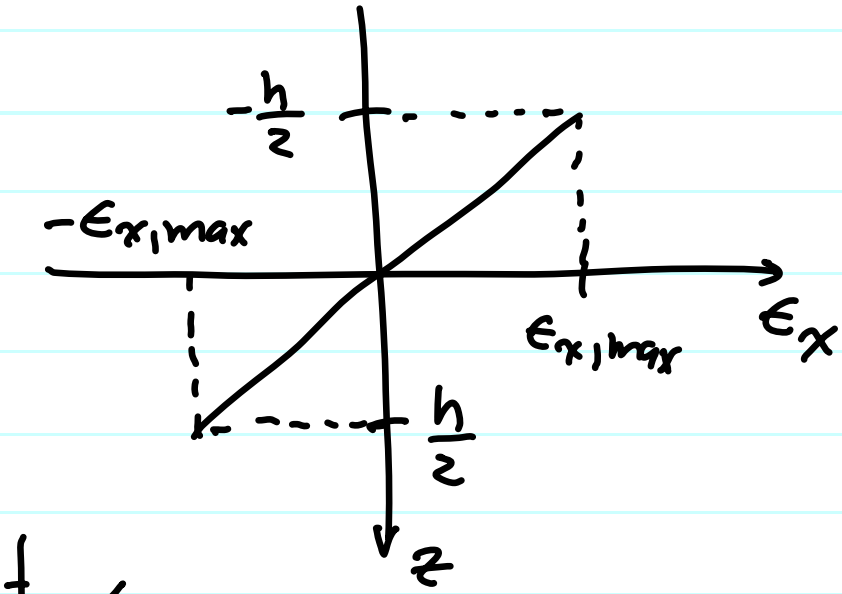
Thus, the axial strain @ z:

[dx: original (unstressed) segment length]

$$\epsilon_x = \frac{dL - dx}{dx} = -\frac{z}{R} \rightarrow \boxed{\epsilon_x = -\frac{z}{R}}$$

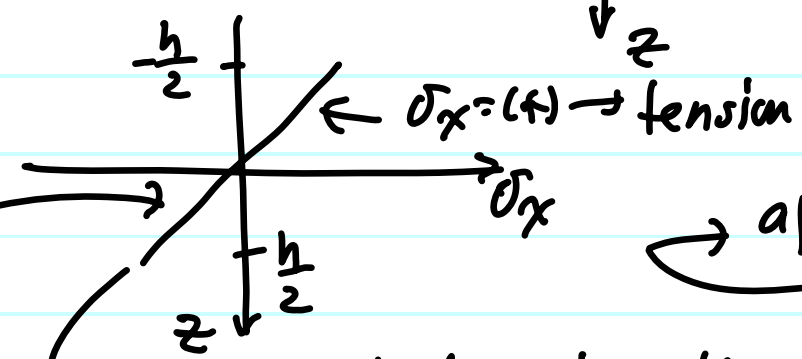
Thus, the strain varies linearly along beam thickness, and has a maximum value

$$\epsilon_{x,max} = \frac{h/2}{R}$$



Of course, there is a corresponding axial stress:

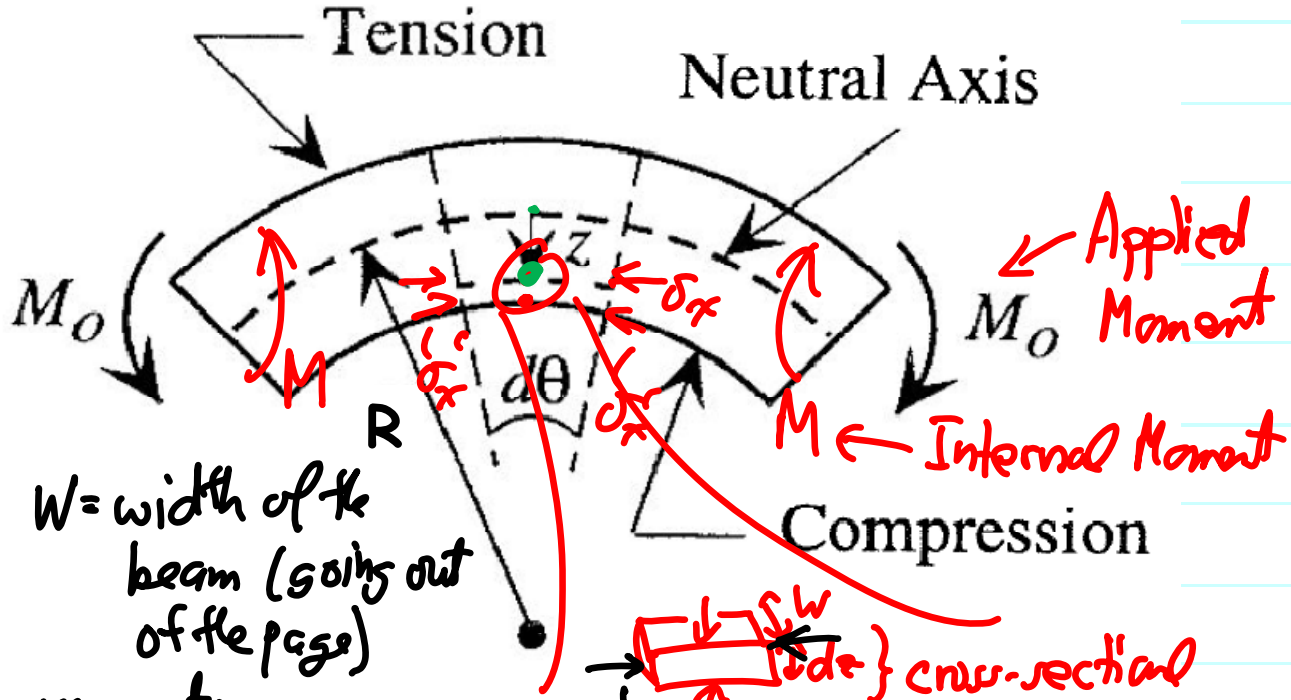
$$\sigma_x = \epsilon_x E = \boxed{-\frac{zE}{R} = \sigma_x}$$



This gradient in stress then generates a bending moment in response to the applied moment!

Internal Bending Moment

Small section of a beam bent in response to a transverse load



W = width of the beam (going out of the page)

} cross-sectional area = $W dz$

$(W dz)\cdot\sigma_x$ = force

<u>To get the internal bending moment:</u>

⇒ integrate the stress through the thickness of the beam:

$$M = \int_{-h/2}^{h/2} \underbrace{[(W dz)\sigma_x]}_{\text{force}} \cdot \underset{\text{moment arm}}{z} = -\int_{-h/2}^{h/2} \frac{EWz^2}{R}\, dz \Rightarrow \boxed{M = -\left(\frac{1}{12}Wh^3\right)\frac{E}{R}}$$

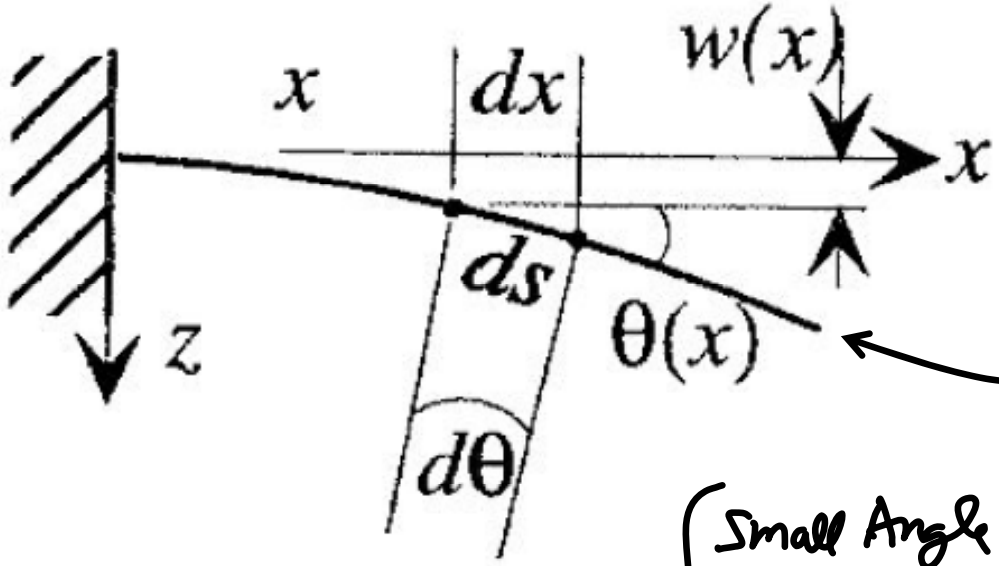
$$\left[\sigma_x = -\frac{zE}{R}\right]$$

$\frac{1}{12}Wh^3 = I \triangleq$ Moment of Inertia

$$\boxed{\frac{1}{R} = -\frac{M}{EI}}$$

<u>Note:</u> (+) radius of curvature → (−) internal bending moment!

Differential Beam Bending Equation



Neutral axis of a bent cantilever beam

Write out geometric relationships: [Small Angle Approximation]

$\cos\theta = \frac{dx}{ds} \rightarrow ds = \frac{dx}{\cos\theta} \longrightarrow ds \approx dx$

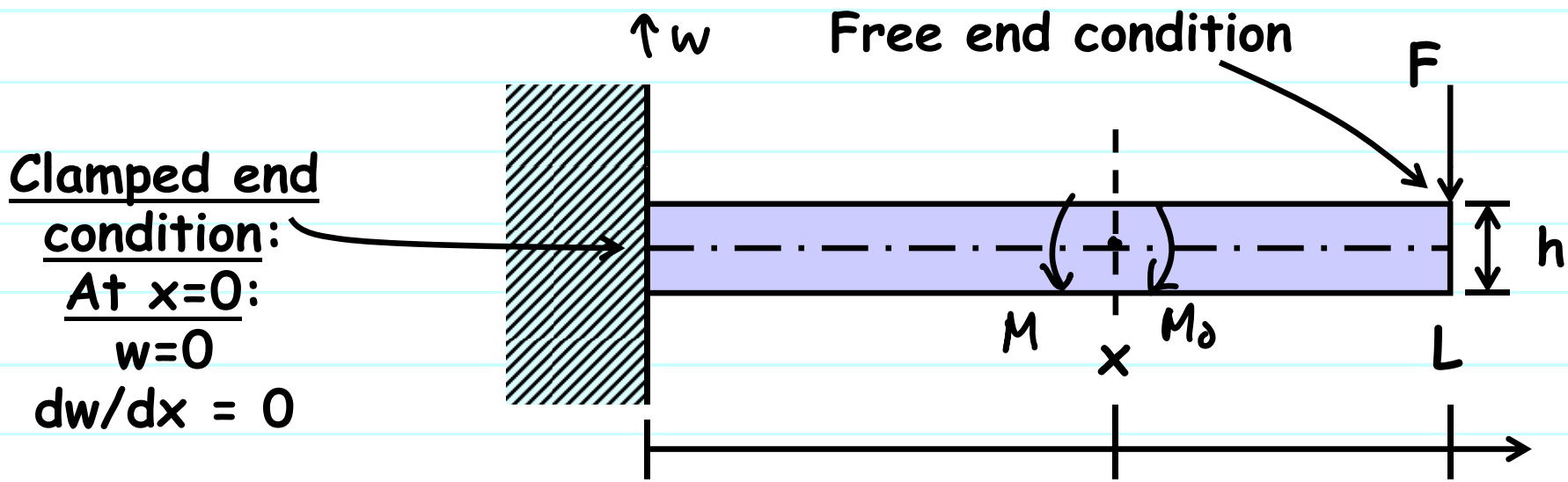
$\tan\theta = \frac{dw}{dx}$ = slope of the beam @ any point $\longrightarrow [\tan\theta \simeq \theta] \rightarrow \theta = \frac{dw}{dx}$ (1)

$ds = R\,d\theta \rightarrow \frac{1}{R} = \frac{d\theta}{ds} \longrightarrow \frac{1}{R} = \frac{d\theta}{dx}$ (2)

Inserting (1) in (2): $\frac{1}{R} = \boxed{\frac{d^2w}{dx^2} = -\frac{M}{EI}}$ [Differential Equation for Small Angle Bending of Beams]

Example: Cantilever Beam w/ a Concentrated Load

Cantilever Beam w/ a Concentrated Load



Internal Moment @ position x: $M = -M_0 = -F(L-x)$

Thus: $\frac{d^2w}{dx^2} = \frac{F}{EI}(L-x)$, w/ $\begin{cases} \text{Clamped End B.C.'s: } w(x=0)=0,\ \frac{dw}{dx}(x=0)=0 \\ \text{Free End B.C.'s: none} \end{cases}$

Solve to get expression for w:

⇒ use Laplace; or use trial solution: $w = A + Bx + Cx^2 + Dx^3$, then apply B.C.'s:

$$w = \frac{FL}{2EI}x^2\left(1 - \frac{x}{3L}\right)$$ [deflection @ x due to a point load F applied at x=L]