EE C245 - ME C218
Introduction to MEMS Design
Fall 2007

Prof. Clark T.-C. Nguyen

Dept. of Electrical Engineering & Computer Sciences
University of California at Berkeley
Berkeley, CA 94720

Lecture 13: Microstructural Elements
Lecture Outline

• Reading: Senturia, Chpt. 9

• Lecture Topics:
  ⇨ Bending of beams
  ⇨ Cantilever beam under small deflections
  ⇨ Combining cantilevers in series and parallel
  ⇨ Folded suspensions
  ⇨ Design implications of residual stress and stress gradients
MEMS Material Properties
**Folded-Beam Comb-Drive Resonator**

- **Issue w/ Wine-Glass Resonator:** non-standard fab process
- **Solution:** use a folded-beam comb-drive resonator

\[ f_0 = 342.5 \text{kHz} \]

\[ Q = \frac{342,500}{8.3} \]

\[ Q = 41,000 \]
Measurement of Young’s Modulus

- Use micromechanical resonators
  - Resonance frequency depends on $E$
  - For a folded-beam resonator:

\[
\text{Resonance Frequency} = f_0 = \frac{4Eh(W/L)^3}{M_{eq}} \left(\frac{1}{2} \right)
\]

$E$ = Young’s modulus
$M_{eq}$ = Equivalent mass
$h$ = thickness
$W$, $L$ = Dimensions

- Extract $E$ from measured frequency $f_0$
- Measure $f_0$ for several resonators with varying dimensions
- Use multiple data points to remove uncertainty in some parameters
Bending of Beams
• Springs and suspensions very common in MEMS

º Coils are popular in the macro-world; but not easy to make in the micro-world
º Beams: simpler to fabricate and analyze; become “stronger” on the micro-scale → use beams for MEMS

Comb-Driven Folded Beam Actuator
**Objective:** Find relation between tip deflection \( y(x=L_c) \) and applied load \( F \)

**Assumptions:**
1. Tip deflection is small compared with beam length
2. Plane sections (normal to beam’s axis) remain plane and normal during bending, i.e., “pure bending”
3. Shear stresses are negligible
Reaction Forces and Moments

Point Load

- Reaction Moment: $M_R = M_1$
- Reaction Force: $F_R = F$

Moment due to $F$, here:
$M_1 = FL$

Moment due to $F$ at $x$:
$M_2 = F(L-x)$

For equilibrium (i.e., to prevent the beam from falling apart):
$V_{x,r} = F$
$M_{x, r} = M_3 = F(L-x)$

Reaction

(Senturia gives expressions)
Beam Segment in Pure Bending

Small section of a beam bent in response to a transverse load.

Find the axial stress $\sigma_x$ as a function of $z$:

Consider a segment bounded by the dashed lines defined by $d\theta$:

At $z=0$: (i.e., at the neutral axis): segment length $= dx = Rd\theta \quad (1)$

At any $z$: segment length $= dl = (R-z)d\theta \quad (2)$

Combining (1) and (2): $dl = dx - zd\theta = dx - \frac{z}{R}dx$
Thus, the axial strain @ z:
\[ d \varepsilon_x = \frac{dL}{dx} = -\frac{z}{R} \]
\[ \varepsilon_x = -\frac{z}{R} \]

Thus, the strain varies linearly along beam thickness, and has a maximum value:
\[ \varepsilon_{x,\text{max}} = \frac{h/2}{R} \]

Of course, there is a corresponding axial stress:
\[ \sigma_x = \varepsilon_x E = \left( -\frac{zE}{R} \right) = \sigma_x \]

This gradient in stress then generates a bending moment in response to the applied moment!
Internal Bending Moment

Small section of a beam bent in response to a transverse load

To get the internal bending moment:
⇒ integrate the stress through the thickness of the beam:

\[ M = \int_{-h/2}^{h/2} (Wdx) \cdot z = -\int_{-h/2}^{h/2} \frac{EWx^2}{R} \, dx \]

\[ \sigma_x = -\frac{zE}{R} \]

\[ \frac{1}{R} = -\frac{M}{EI} \]

\[ \frac{1}{12} Wh^3 = I = \text{Moment of Inertia} \]

Note: (R) radius of curvature
                     (-1) internal bending moment!
Differential Beam Bending Equation

Write out geometric relationships:

\[
\cos \theta = \frac{dx}{ds} \quad \Rightarrow \quad ds = \frac{dx}{\cos \theta} \quad \Rightarrow \quad ds = dx
\]

\[
\tan \theta = \frac{dw}{dx} = \text{slope of the beam} \quad \text{at any point}
\]

\[
ds = Rd\theta \quad \Rightarrow \quad \frac{1}{R} = \frac{d\theta}{ds}
\]

Inserting (1) in (2):

\[
\frac{1}{R} = \frac{d^2w}{dx^2} = -\frac{M}{EI}
\]
Example: Cantilever Beam w/ a Concentrated Load
Cantilever Beam w/ a Concentrated Load

Clamped end condition:
At \( x=0 \):
\( w=0 \)
\( \frac{dw}{dx} = 0 \)

Internal Moment @ position \( x \):
\[ M = -M_0 = -F(L-x) \]
Thus:
\[ \frac{d^2w}{dx^2} = \frac{F}{EI} (L-x) \]
\( \{ \text{Clamped End B.C.'s: } W(x=0) = 0, \frac{dw}{dx}(x=0) = 0 \} \)
\( \{ \text{Free End B.C.'s: none} \} \)

Solve to get expression for \( w \):
- Use Laplace; or use trial solution: \( w = A + Bx + Cx^2 + Dx^3 \), then apply B.C.'s:
\[ w = \frac{FL}{2EI} x^2 \left( 1 - \frac{x}{3L} \right) \]

[Deflection @ \( x \) due to a point load]
F applied at \( x=L \)