EE C245 – ME C218
Introduction to MEMS Design
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Lecture 21: Gyros & Capacitive Transducers
Lecture Outline

• Reading: Senturia, Chpt. 5, Chpt. 6, Chpt. 21

• Lecture Topics:
  ◆ Project: Gyroscopes
  ◆ Deep Reactive-Ion Etching (revisited)
  ◆ Energy Conserving Transducers
    ◆ Charge Control
    ◆ Voltage Control
  ◆ Parallel-Plate Capacitive Transducers
    ◆ Linearizing Capacitive Actuators
    ◆ Electrical Stiffness
  ◆ Electrostatic Comb-Drive
    ◆ 1st Order Analysis
    ◆ 2nd Order Analysis
Gyroscopes
A gyroscope measures rotation rate, which then gives orientation → very important, of course, for navigation.

- Principle of operation based on conservation of momentum.
- **Example**: classic spinning gyroscope.

Rotor will preserve its angular momentum (i.e., will maintain its axis of spin) despite rotation of its gimbled chassis.
Vibratory Gyroscopes

• Generate momentum by vibrating structures
• Again, conservation of momentum leads to mechanisms for measuring rotation rate and orientation
• Example: vibrating mass in a rotating frame

Driven into vibration along the y-axis

Mass at rest

Get an x' component of motion

Rotate 30°

Capacitance between mass and frame = constant

y-displaced mass

C(t)

C(t1)

C(t2)

C(t2) > C(t1)
Basic Vibratory Gyroscope Operation

**Principle of Operation**

- Tuning Fork Gyroscope:

  - **Input Rotation**: \( \vec{\Omega} \)
  - **Driven Vibration**: \( @ f_o \)
  - **Coriolis Torque**: \( \vec{a}_c \)
  - **Coriolis (Sense) Response**: \( \vec{v} \)

  **Detect motion out-of-the plane of the tuning fork as rotation!**

  **Side View**:
  
  ![Side View Diagram]
  
  **Top View**:
  
  ![Top View Diagram]

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**Basic Vibratory Gyroscope Operation**

**Principle of Operation**

- **Tuning Fork Gyroscope:**

  - **Input Rotation:** $\vec{\Omega}$
  - **Driven Vibration:** $\vec{v}$
  - **Coriolis (Sense) Response:** $\vec{a}_c$
  - **Coriolis Torque:** $\vec{F}_c$
  - **Coriolis Force:** $\vec{x} = \frac{\vec{F}_c}{k} = \frac{m\vec{a}_c}{k} = \frac{\vec{a}_c}{\omega^2_r}$
  - **Beam Stiffness:** $k$
  - **Beam Mass:** $m$
  - **Sense Frequency:** $\omega_r$
  - **Driven Velocity:** $\vec{v}$
  - **Driven Acceleration:** $\vec{a}_c$
  - **Coriolis Acceleration:** $\vec{a}_c = 2\vec{v} \times \vec{\Omega}$
  - **Coriolis Displacement:** $x$
  - **Rotation Rate:** $\dot{\Omega}$

**Drive/Sense Response Spectra:**

- **Amplitude:**
  - **Drive Response:** $f_o (@ T_1)$
  - **Sense Response:**

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Vibratory Gyroscope Performance

**Principle of Operation**

- **Tuning Fork Gyroscope:**

\[
\dot{x} = \frac{F_c}{k} = \frac{m \ddot{a}_c}{k} = \frac{\ddot{a}_c}{\omega_r^2}
\]

\[
\ddot{a}_c = 2\dot{v} \times \Omega
\]

- To maximize the output signal \( x \), need:
  - Large sense-axis mass
  - Small sense-axis stiffness
  - (Above together mean low resonance frequency)
  - Large drive amplitude for large driven velocity (so use comb-drive)
  - If can match drive freq. to sense freq., then can amplify output by \( Q \) times

\[\nabla^2 x = \Box \nabla x \Rightarrow \nabla^2 \Box \text{ and } x = \text{large}\]
MEMS-Based Gyroscopes

Vibrating Ring Gyroscope

[Tuning Fork Gyroscope
[Ayazi, GA Tech.]]

[Najafi, Michigan]

[Laser
Polarizer
Rb/Xe Cell
Photodiode
3.2 mm
1 mm]

[Nuclear Magnetic Resonance Gyro [NIST]]

[Draper Labs.]
MEMS-Based Tuning Fork Gyroscope

- In-plane drive and sense modes pick up z-axis rotations
- Mode-matching for maximum output sensitivity
- From [Zaman, Ayazi, et al, MEMS’06]
MEMS-Based Tuning Fork Gyroscope

[Diagram of MEMS-based tuning fork gyroscope with labels for sense electrodes, drive voltage, oscillation sustaining amplifier, and differential transR sense amplifier.]

[Reference: Zaman, Ayazi, et al., MEMS'06]
MEMS-Based Tuning Fork Gyroscope

• Drive and sense axes must be stable or at least track one another to avoid output drift

Problem: if drive frequency changes relative to sense frequency, output changes ⇒ bias drift

Need: small or matched drive and sense axis temperature coefficients to suppress drift
Mode Matching for Higher Resolution

- For higher resolution, can try to match drive and sense axis resonance frequencies and benefit from Q amplification.

Problem: mismatch between drive and sense frequencies ⇒ even larger drift!

Need: small or matched drive and sense axis temperature coefficients to make this work.
**Issue: Zero Rate Bias Error**

- Imbalances in the system can lead to zero rate bias error

- Drive imbalance $\Rightarrow$ off-axis motion of the proof mass

- Output signal in phase with the Coriolis acceleration

- Mass imbalance $\Rightarrow$ off-axis motion of the proof mass

- Quadrature output signal that can be confused with the Coriolis acceleration
Nuclear Magnetic Res. Gyroscope

• The ultimate in miniaturized spinning gyroscopes?
  \(\Rightarrow\) from CSAC, we may now have the technology to do this

Better if this is a noble gas nucleus (rather than e-), since nuclei are heavier \(\Rightarrow\) less susceptible to B field

**Soln:** Spin polarize \(\text{Xe}^{129}\) nuclei by first polarizing e- of \(\text{Rb}^{87}\) (a la CSAC), then allowing spin exchange

**Challenge:** suppressing the effects of B field

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**Diagram:**
- Atoms aligned with nuclear spins
- Laser
- Polarizer
- Rb/Xe Cell
- Photodiode
- \(\dot{\theta}_t\)
- 3.2 mm
- 1 mm
- Ch ll

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MEMS-Based Tuning Fork Gyroscope

[Diagram showing the components and connections of a MEMS-based tuning fork gyroscope]

(-) Sense Output Current

(+) Sense Output Current

[Zaman, Ayazi, et al., MEMS'06]
Deep Reactive-Ion Etching
Deep Reactive-Ion Etching (DRIE)

The Bosch process:

- Inductively-coupled plasma
- Etch Rate: 1.5-4 μm/min
- Two main cycles in the etch:
  - Etch cycle (5-15 s): SF₆ (SFₓ⁺) etches Si
  - Deposition cycle: (5-15 s): C₄F₈ deposits fluorocarbon protective polymer (CF₂⁻)ₙ
- Etch mask selectivity:
  - SiO₂ ~ 200:1
  - Photoresist ~ 100:1
- Issue: finite sidewall roughness
  - Scalloping < 50 nm
- Sidewall angle: 90° ± 2°
DRIE Issues: Etch Rate Variance

- Etch rate is diffusion-limited and drops for narrow trenches
  - Adjust mask layout to eliminate large disparities
  - Adjust process parameters (slow down the etch rate to that governed by the slowest feature)
DRIE Issues: “Footing”

• Etch depth precision
  ➔ Etch stop: buried layer of SiO₂
  ➔ Due to 200:1 selectivity, the (vertical) etch practically just stops when it reaches SiO₂

• Problem: Lateral undercut at Si/SiO₂ interface → “footing”
  ➔ Caused by charge accumulation at the insulator

Poor charge relaxation and lack of neutralization of e⁻'s at insulator

Ion flux into substrate builds up (+) potential

Charging-induced potential perturbs the E-field

Distorts the ion trajectory

Result: strong and localized damage to the structure at Si-SiO₂ interface → “footing”
Recipe-Based Suppression of “Footing”

• Use higher process pressure to reduce ion charging [Nozawa]
  ➡ High operating pressure → concentration of (-) ions increases and can neutralize (+) surface charge
  ➡ Issue: must introduce as a separate recipe when the etch reaches the Si-insulator interface, so must be able to very accurately predict the time needed for etching

• Adjust etch recipe to reduce overetching [Schmidt]
  ➡ Change $C_4F_8$ flow rate, pressure, etc., to enhance passivation and reduce overetching
  ➡ Issue: Difficult to simultaneously control footing in a narrow trench and prevent grass in wide trenches

• Use lower frequency plasma to avoid surface charging [Morioka]
  ➡ Low frequency → more ions with low directionality and kinetic energy → neutralizes (-) potential barrier at trench entrance
  ➡ Allows e-’s to reach the trench base and neutralize (+) charge → maintain charge balance inside the trench
Metal Interlayer to Prevent “Footing”

Pre-defined metal interlayer grounded to substrate supplies e’s to neutralize (+) charge and prevent charge accumulation at the Si-insulator interface.
Footing Prevention (cont.)

• **Below:** DRIE footing over an oxide stop layer

• **Right:** efficacy of the metal interlayer footing prevention approach

[Kim, Stanford]

[Kim, Seoul Nat. Univ.]
DRIE Examples

High aspect-ratio gear

Tunable Capacitor
[Yao, Rockwell]

Microgripper
[Keller, MEMS Precision Instruments]
Energy Conserving Transducers
• **Goal:** Determine gap spacing $g$ as a function of input variables

• First, need to determine the energy of the system

• Two ways to change the energy:
  - Change the charge $q$
  - Change the separation $g$

\[
\Delta W(q, g) = V \Delta q + F_e \Delta g
\]

\[
dW = V dq + F_e dg
\]

• **Note:** We assume that the plates are supported elastically, so they don't collapse
Here, the stored energy is the work done in increasing the gap after charging capacitor at zero gap.

\[ W = 0 + \int_{0}^{g} F_e \, dg' \]

\[ W(q, g) \]

Zero gap \( \rightarrow \) zero stored energy
• Having found stored energy, we can now find the force acting on the plates and the voltage across them:
Voltage-Control Case

• Practical situation: We control V
  ➤ Charge control on the typical sub-pF MEMS actuation capacitor is difficult
  ➤ Need to find $F_e$ as a partial derivative of the stored energy $\mathcal{W} = \mathcal{W}(V,g)$ with respect to $g$ with $V$ held constant? But can’t do this with present $\mathcal{W}(q,g)$ formula
  ➤ Solution: Apply Legendre transformation and define the co-energy $\mathcal{W}^*(V,g)$
Co-Energy Formulation

• For our present problem (i.e., movable capacitive plates), the co-energy formulation becomes

\[ \mathcal{W}'(V, g) = qV - \mathcal{W} \]
• Find co-energy in terms of voltage

\[ W' = \int_0^V q(g, V')dV' = \int_0^V \left( \frac{\varepsilon A}{g} \right) V'dV' = \frac{1}{2} \left( \frac{\varepsilon A}{g} \right) V^2 = \frac{1}{2} CV^2 \]  

(as expected)

• Variation of co-energy with respect to gap yields electrostatic force:

\[ F_e = -\frac{\partial W'(V, g)}{\partial g} \bigg|_{V} = -\frac{1}{2} \left( -\frac{\varepsilon A}{g^2} \right) V^2 = \frac{1}{2} \frac{C}{g} V^2 \]

• Variation of co-energy with respect to voltage yields charge:

\[ q = \frac{\partial W'(V, g)}{\partial V} \bigg|_{g} = \left( \frac{\varepsilon A}{g} \right) V = CV \]  

as expected
Spring-Suspended Capacitive Plate
Charge Control of a Spring-Suspended Cylinder

Diagram showing a spring suspended cylinder with charges +q and -q, forces $F_e$, and current I.
Voltage Control of a Spring-Suspended C
• Net attractive force on the plate:

• An increment in gap dg leads to an increment in net attractive force \( dF_{net} \)
Pull-In Voltage $V_{PI}$

- $V_{PI}$ = voltage at which the plates collapse
- The plate goes unstable when

$$k = \frac{\varepsilon A V_{PI}^2}{g_{PI}^3} \quad (1) \quad \text{and} \quad F_{net} = 0 = \frac{\varepsilon A V_{PI}^2}{2 g_{PI}^2} - k (g_o - g_{PI}) \quad (2)$$

- Substituting (1) into (2):

$$0 = \frac{\varepsilon A V_{PI}^2}{2 g_{PI}^2} - \frac{\varepsilon A V_{PI}^2}{g_{PI}^2} (g_o - g_{PI})$$

$$g_o - g_{PI} = \frac{1}{2} \Rightarrow g_o = \frac{3}{2} g_{PI}$$

$$\therefore \frac{g_o - g_{PI}}{g_{PI}} = \frac{1}{2} \Rightarrow g_{PI} = \frac{2}{3} g_o$$

When a gap is driven by a voltage to (2/3) its original spacing, collapse will occur!
Below: Plot of normalized electrostatic and spring forces vs.
normalized displacement $1-(g/g_0)$
Advantages of Electrostatic Actuators

• Easy to manufacture in micromachining processes, since conductors and air gaps are all that’s needed → low cost!
• Energy conserving → only parasitic energy loss through $I^2R$ losses in conductors and interconnects
• Variety of geometries available that allow tailoring of the relationships between voltage, force, and displacement
• Electrostatic forces can become very large when dimensions shrink → electrostatics scales well!
• Same capacitive structures can be used for both drive and sense of velocity or displacement
• Simplicity of transducer greatly reduces mechanical energy losses, allowing the highest Q's for resonant structures
Problems With Electrostatic Actuators

• Nonlinear voltage-to-force transfer function
• Relatively weak compared with other transducers (e.g., piezoelectric), but things get better as dimensions scale