Lecture 21: Gyros & Capacitive Transducers
• Reading: Senturia, Chpt. 5, Chpt. 6, Chpt. 21

• Lecture Topics:
  ➤ Project: Gyroscopes
  ➤ Deep Reactive-Ion Etching (revisited)
  ➤ Energy Conserving Transducers
    ➤ Charge Control
    ➤ Voltage Control
  ➤ Parallel-Plate Capacitive Transducers
    ➤ Linearizing Capacitive Actuators
    ➤ Electrical Stiffness
  ➤ Electrostatic Comb-Drive
    ➤ 1st Order Analysis
    ➤ 2nd Order Analysis
Gyroscopes
Classic Spinning Gyroscope

• A gyroscope measures rotation rate, which then gives orientation → very important, of course, for navigation
• Principle of operation based on conservation of momentum
• Example: classic spinning gyroscope

Rotor will preserve its angular momentum (i.e., will maintain its axis of spin) despite rotation of its gimbled chassis
Vibratory Gyroscopes

• Generate momentum by vibrating structures
• Again, conservation of momentum leads to mechanisms for measuring rotation rate and orientation
• Example: vibrating mass in a rotating frame

Driven into vibration along the y-axis

Mass at rest

Get an x' component of motion

Rotate 30°

Capacitance between mass and frame = constant

y-displaced mass
Basic Vibratory Gyroscope Operation

Principle of Operation

• Tuning Fork Gyroscope:

Input Rotation

Driven Vibration \( \Omega @ f_0 \)

Coriolis (Sense) Response

Coriolis Torque

Detect motion out-of-the plane of the tuning fork as rotation!
Basic Vibratory Gyroscope Operation

**Principle of Operation**

- **Tuning Fork Gyroscope:**

  - **Input Rotation:** ![Input Rotation](image)
  - **Driven Vibration @ f₀:** ![Driven Vibration](image)
  - **Coriolis Force:** ![Coriolis Force](image)
  - **Beam Mass:** ![Beam Mass](image)
  - **Rotation Rate:** ![Rotation Rate](image)
  - **Driven Velocity:** ![Driven Velocity](image)
  - **Coriolis Acceleration:** ![Coriolis Acceleration](image)

**Drive/Sense Response Spectra:**

- **Amplitude:** ![Amplitude](image)
- **ω:** ![ω](image)
- **f₀ (@ T₁):** ![f₀ (@ T₁)](image)
- **Driven Rotation Rate:** ![Driven Rotation Rate](image)
- **Coriolis Force:** ![Coriolis Force](image)
- **Beam Stiffness:** ![Beam Stiffness](image)
- **Coriolis Displacement:** ![Coriolis Displacement](image)
- **Sense Frequency:** ![Sense Frequency](image)

\[ \vec{a}_c = 2\vec{\nu} \times \vec{\Omega} \]

\[ \vec{x} = \frac{\vec{F}_c}{k} = \frac{m\vec{a}_c}{k} = \frac{\vec{a}_c}{\omega_r^2} \]

\[ \text{Input Rotation} \rightarrow \text{Driven Vibration @ f₀} \rightarrow \text{Coriolis Force} \rightarrow \text{Driven Rotation Rate} \rightarrow \text{Coriolis Acceleration} \rightarrow \text{Beam Mass} \rightarrow \text{Rotation Rate} \rightarrow \text{Sense Frequency} \]
**Vibratory Gyroscope Performance**

**Principle of Operation**

- Tuning Fork Gyroscope:

\[
\ddot{x} = \frac{F_c}{k} = \frac{m\ddot{a}_c}{k} = \frac{\ddot{a}_c}{\omega_r^2}
\]

\[
\ddot{a}_c = 2\dot{v} \times \Omega
\]

- To maximize the output signal \( x \), need:
  - Large sense-axis mass
  - Small sense-axis stiffness
  - (Above together mean low resonance frequency)
  - Large drive amplitude for large driven velocity (so use comb-drive)
    \( \Rightarrow \) \( N \approx C \omega \) and \( x \approx \text{large} \)
  - If can match drive freq. to sense freq., then can amplify output by \( Q \) times

\[ x = \frac{Q F_c}{k} \]
MEMS-Based Gyroscopes

Tuning Fork Gyroscope
[Draper Labs.]

Vibrating Ring Gyroscope
[Najafi, Michigan]

Tuning Fork Gyroscope
[Ayazi, GA Tech.]

3.2 mm

1 mm

Laser

Polarizer

Rb/Xe Cell

Photodiode

Nuclear Magnetic Resonance Gyro [NIST]
MEMS-Based Tuning Fork Gyroscope

- In-plane drive and sense modes pick up z-axis rotations
- Mode-matching for maximum output sensitivity
- From [Zaman, Ayazi, et al, MEMS'06]
MEMS-Based Tuning Fork Gyroscope

[Zaman, Ayazi, et al., MEMS'06]
MEMS-Based Tuning Fork Gyroscope

• Drive and sense axes must be stable or at least track one another to avoid output drift

Problem: if drive frequency changes relative to sense frequency, output changes ⇒ bias drift

Need: small or matched drive and sense axis temperature coefficients to suppress drift
Mode Matching for Higher Resolution

• For higher resolution, can try to match drive and sense axis resonance frequencies and benefit from Q amplification

Need: small or matched drive and sense axis temperature coefficients to make this work
**Issue: Zero Rate Bias Error**

- Imbalances in the system can lead to zero rate bias error

*Mass imbalance \(\Rightarrow \) off-axis motion of the proof mass

*Drive imbalance \(\Rightarrow \) off-axis motion of the proof mass

*Quadrature output signal that can be confused with the Coriolis acceleration

*Output signal in phase with the Coriolis acceleration
The ultimate in miniaturized spinning gyroscopes? from CSAC, we may now have the technology to do this.

Better if this is a noble gas nucleus (rather than e-), since nuclei are heavier ⇒ less susceptible to B field.

**Soln:** Spin polarize Xe\(^{129}\) nuclei by first polarizing e- of Rb\(^{87}\) (a la CSAC), then allowing spin exchange.

**Challenge:** suppressing the effects of B field.
MEMS-Based Tuning Fork Gyroscope

Drive Voltage
Signal

(-) Sense
Output
Current

(+) Sense
Output
Current

[Zaman, Ayazi, et al, MEMS'06]
Deep Reactive-Ion Etching
Deep Reactive-Ion Etching (DRIE)

The Bosch process:

• Inductively-coupled plasma
• Etch Rate: 1.5-4 μm/min
• Two main cycles in the etch:
  ➤ **Etch cycle** (5-15 s): SF$_6$ (SF$_x^+$) etches Si
  ➤ **Deposition cycle**: (5-15 s): C$_4$F$_8$ deposits fluorocarbon protective polymer (CF$_2^-$)$_n$
• Etch mask selectivity:
  ➤ SiO$_2$ ~ 200:1
  ➤ Photoresist ~ 100:1
• **Issue**: finite sidewall roughness
  ➤ scalloping < 50 nm
• Sidewall angle: 90° ± 2°
Etch rate is diffusion-limited and drops for narrow trenches
- Adjust mask layout to eliminate large disparities
- Adjust process parameters (slow down the etch rate to that governed by the slowest feature)
**DRIE Issues: “Footing”**

- **Etch depth precision**
  - Etch stop: buried layer of SiO$_2$
  - Due to 200:1 selectivity, the (vertical) etch practically just stops when it reaches SiO$_2$

- **Problem**: Lateral undercut at Si/SiO$_2$ interface $\rightarrow$ “footing”
  - Caused by charge accumulation at the insulator

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**Diagram**: Poor charge relaxation and lack of neutralization of e-'s at insulator $\downarrow$ Ion flux into substrate builds up (+) potential $\downarrow$ Charging-induced potential perturbs the E-field $\downarrow$ Distorts the ion trajectory $\downarrow$ Result: strong and localized damage to the structure at Si-SiO$_2$ interface $\rightarrow$ “footing”
Recipe-Based Suppression of “Footing”

- Use higher process pressure to reduce ion charging [Nozawa]
  - High operating pressure → concentration of (-) ions increases and can neutralize (+) surface charge
  - Issue: must introduce as a separate recipe when the etch reaches the Si-insulator interface, so must be able to very accurately predict the time needed for etching

- Adjust etch recipe to reduce overetching [Schmidt]
  - Change $C_4F_8$ flow rate, pressure, etc., to enhance passivation and reduce overetching
  - Issue: Difficult to simultaneously control footing in a narrow trench and prevent grass in wide trenches

- Use lower frequency plasma to avoid surface charging [Morioka]
  - Low frequency → more ions with low directionality and kinetic energy → neutralizes (-) potential barrier at trench entrance
  - Allows e-’s to reach the trench base and neutralize (+) charge → maintain charge balance inside the trench
Metal Interlayer to Prevent “Footing”

Pre-defined metal interlayer grounded to substrate supplies e’s to neutralize (+) charge and prevent charge accumulation at the Si-insulator interface.
Footing Prevention (cont.)

• Below: DRIE footing over an oxide stop layer
• Right: efficacy of the metal interlayer footing prevention approach

[Kim, Stanford]

[Kim, Seoul Nat. Univ.]
DRIE Examples

High aspect-ratio gear

Tunable Capacitor
[Yao, Rockwell]

Microgripper
[Keller, MEMS Precision Instruments]
Energy Conserving Transducers
Basic Physics of Electrostatic Actuation

- **Goal:** Determine gap spacing $g$ as a function of input variables
- First, need to determine the energy of the system
- Two ways to change the energy:
  - Change the charge $q$
  - Change the separation $g$

\[ \Delta W(q, g) = V \Delta q + F_e \Delta g \]

\[ \delta W = V dq + F_e dg \]

- **Note:** We assume that the plates are supported elastically, so they don't collapse
Here, the stored energy is the work done in increasing the gap after charging capacitor at zero gap.

\[ W = 0 + \int_{0}^{g} F_e \, dg' \]

\[ F_e = \left( \frac{q}{2} \right) \varepsilon_0 \frac{q^2}{A} \]

\[ W(g) = \frac{1}{2} \frac{q^2}{\varepsilon_0 A} g \]

No change in charge: \( dq = 0 \)

\[ W(q,g) \]

Zero gap \( \rightarrow \) zero stored energy