EE C245 – ME C218
Introduction to MEMS Design
Fall 2007

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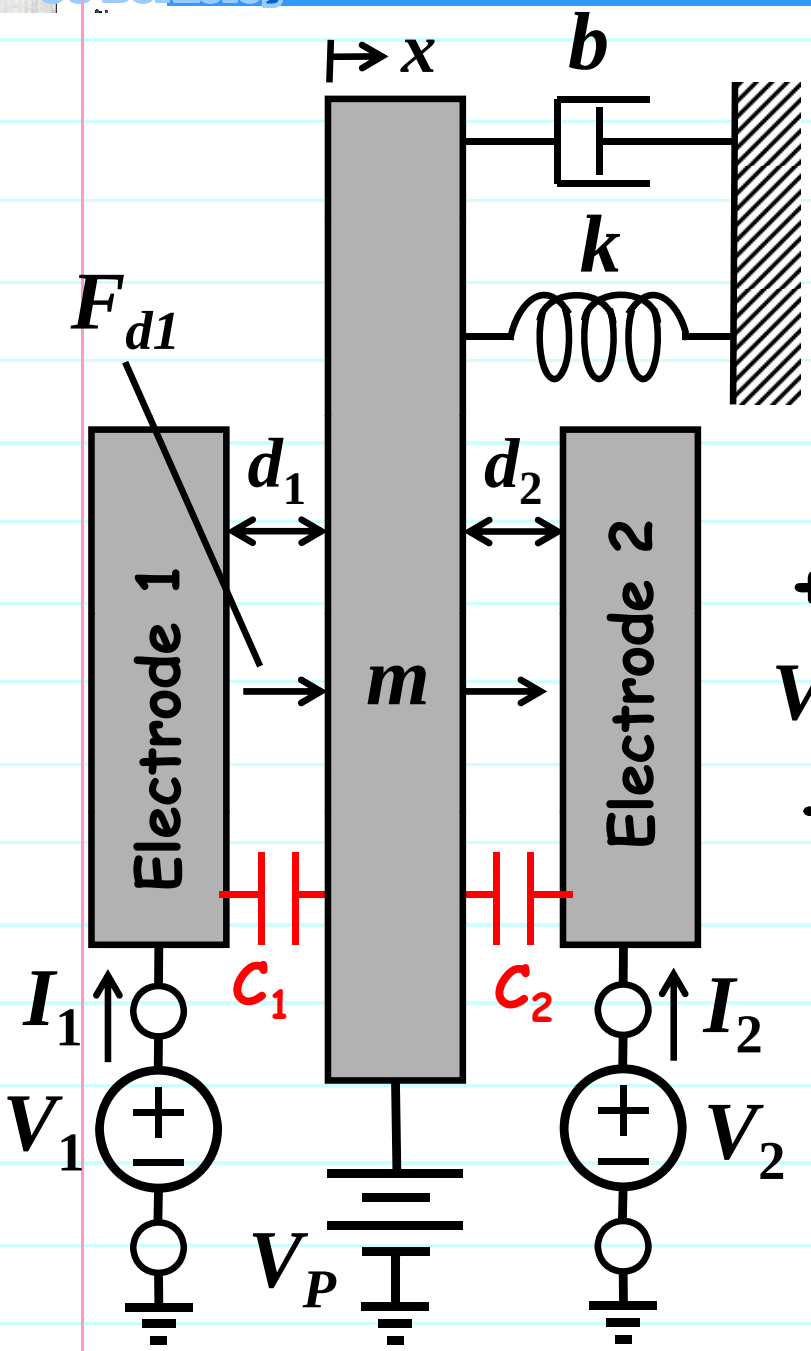
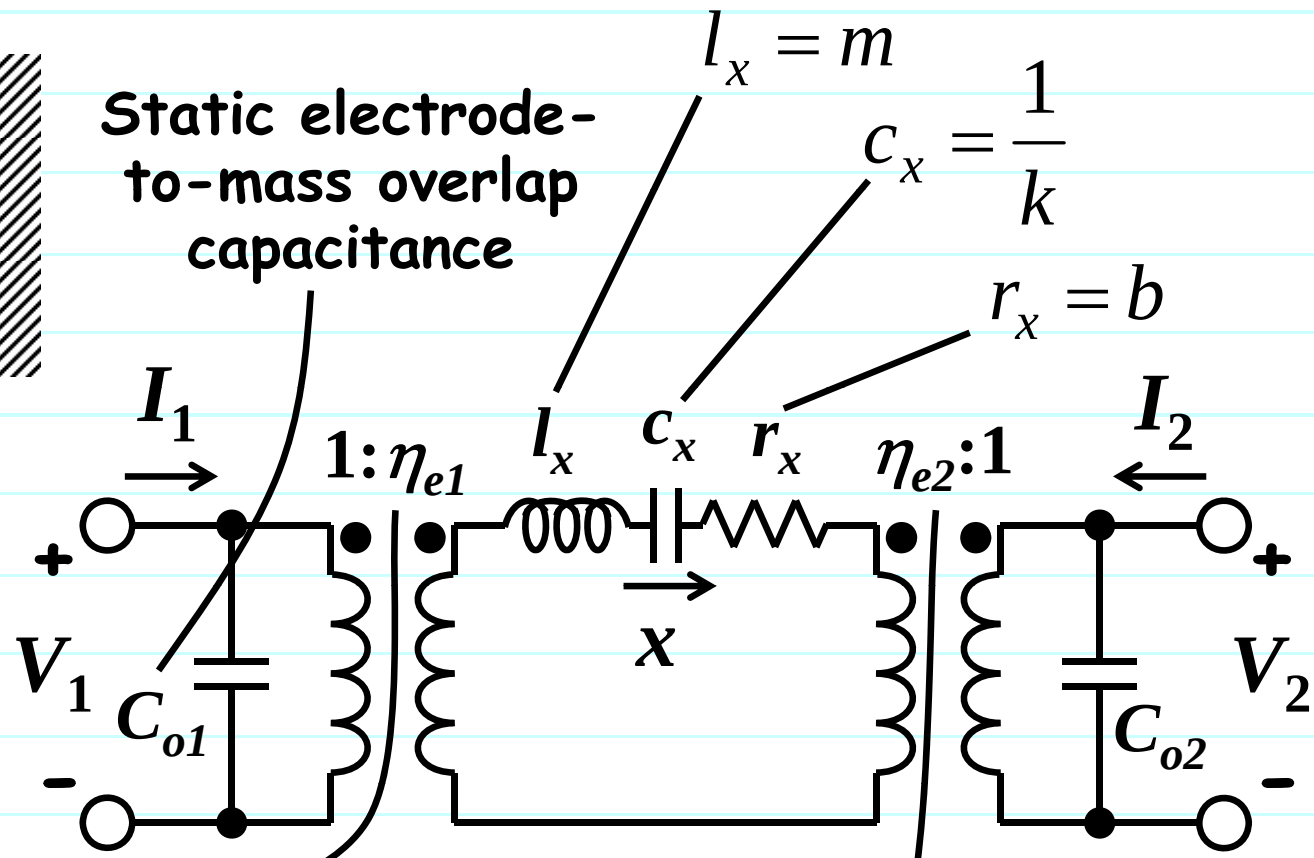
Lecture 25: Output Sensing

Lecture Outline

- Reading: Senturia, Chpt. 6, Chpt. 14
- Lecture Topics:
  - Input/Output Modeling
  - I/O Equivalent Circuit Models
    - Electromechanical Coupling
    - Mechanical Coupling
  - Detection Circuits
    - Position Sensing
    - Velocity Sensing

Complete Electrical-Port Equiv. Circuit

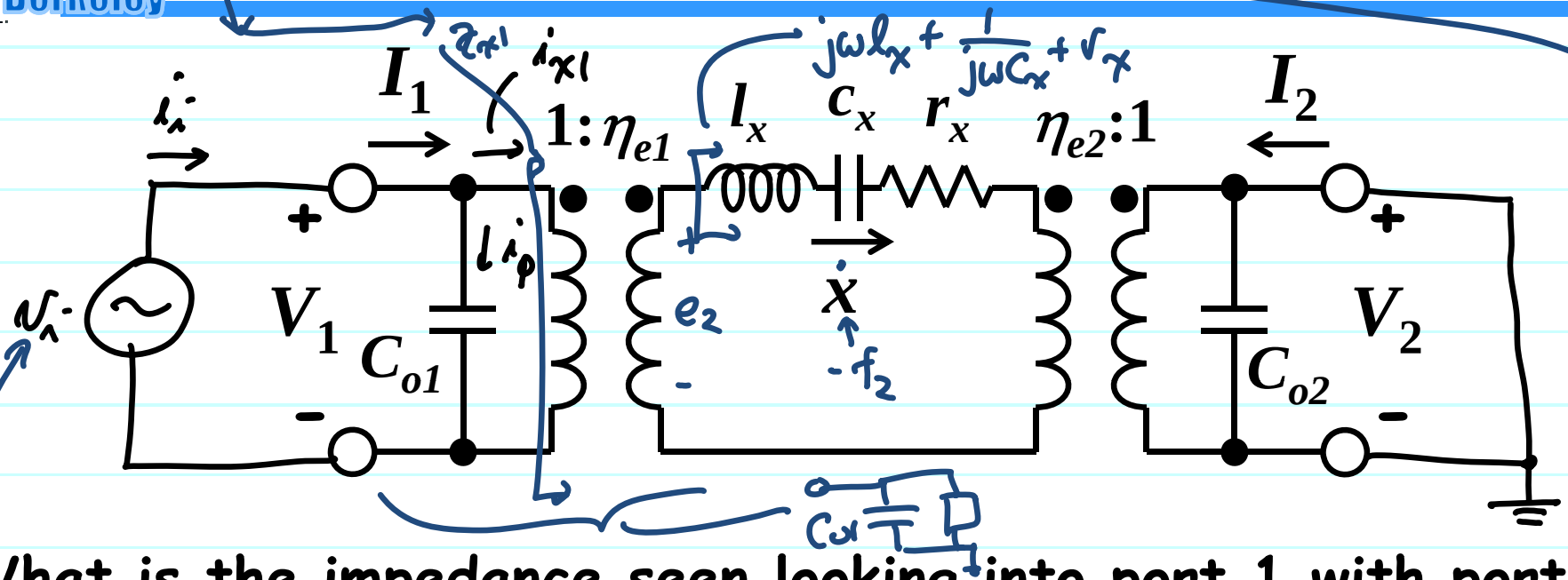




$$\eta_{e1} = V_P \frac{\partial C_1}{\partial x} = V_P \frac{C_{o1}}{d_1}$$

$$\eta_{e2} = V_P \frac{\partial C_2}{\partial x} = V_P \frac{C_{o2}}{d_2}$$

Input Impedance Into Port 1



- What is the impedance seen looking into port 1 with port 2 shorted to ground?

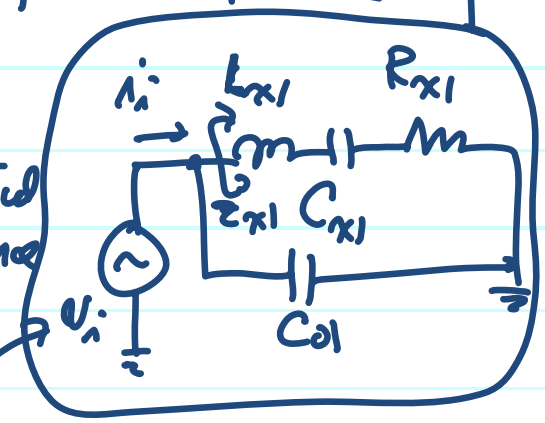
For our transformer model:

$$\begin{bmatrix} e_2 \\ f_2 \end{bmatrix} = \begin{bmatrix} \eta & 0 \\ 0 & -\frac{1}{\eta} \end{bmatrix} \begin{bmatrix} e_1 \\ f_1 \end{bmatrix} \Rightarrow \begin{array}{l} e_2 = \eta e_1 \rightarrow e_1 = \frac{e_2}{\eta} \\ f_2 = -\frac{1}{\eta} f_1 \rightarrow f_1 = -\eta f_2 \end{array}$$

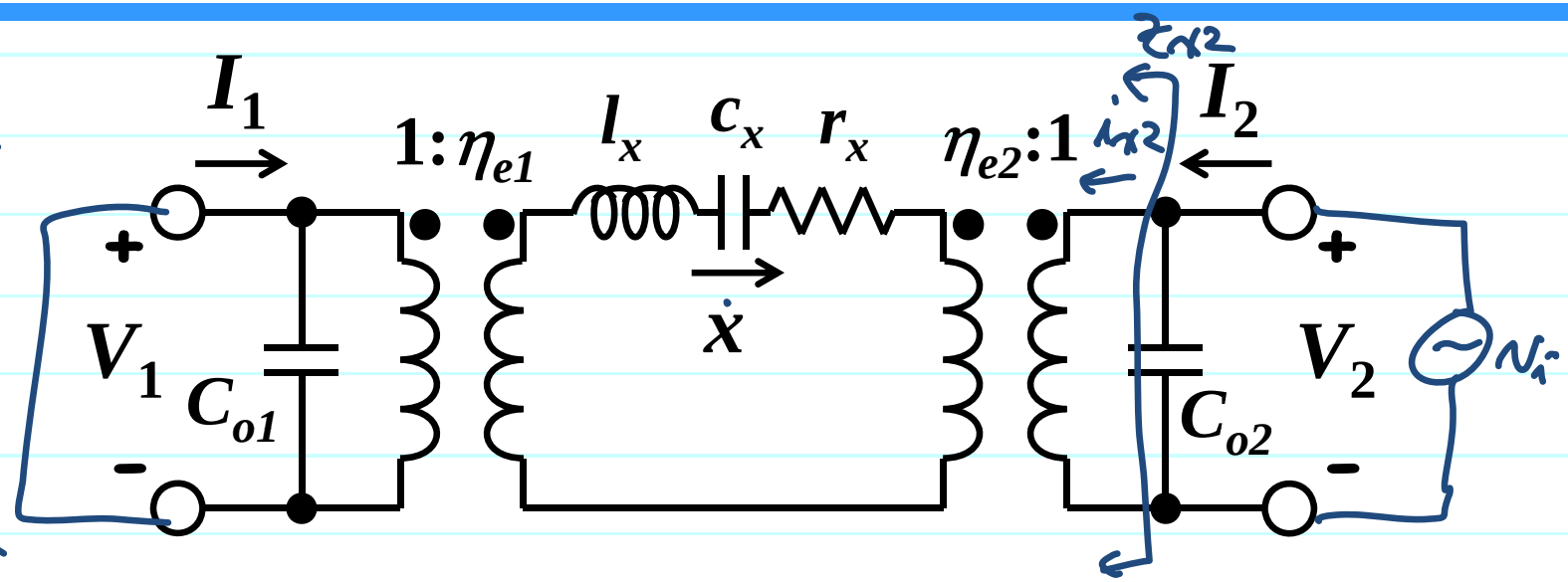
$$\frac{e_1}{f_1} = \frac{e_2}{\eta}\left(\frac{1}{-\eta f_2}\right) = -\frac{1}{\eta^2}\frac{e_2}{f_2} \rightarrow \frac{v_i}{i_{x1}} = Z_{x1} = -\frac{1}{\eta_{e1}^2}\frac{F_2}{(-\dot{x})} = +\frac{1}{\eta_{e1}^2} Z_x$$

m_{11} — electrical impedance — mechanical impedance

$$Z_{x1} = \frac{1}{\eta_{e1}^2}\left(j\omega l_x + \frac{1}{j\omega C_x} + r_x\right) = j\omega\underbrace{\left(\frac{l_x}{\eta_{e1}^2}\right)}_{L_{x1}} + \frac{1}{j\omega\underbrace{(\eta_{e1}^2 C_x)}_{C_{x1}}} + \underbrace{\frac{r_x}{\eta_{e1}^2}}_{R_{x1}}$$

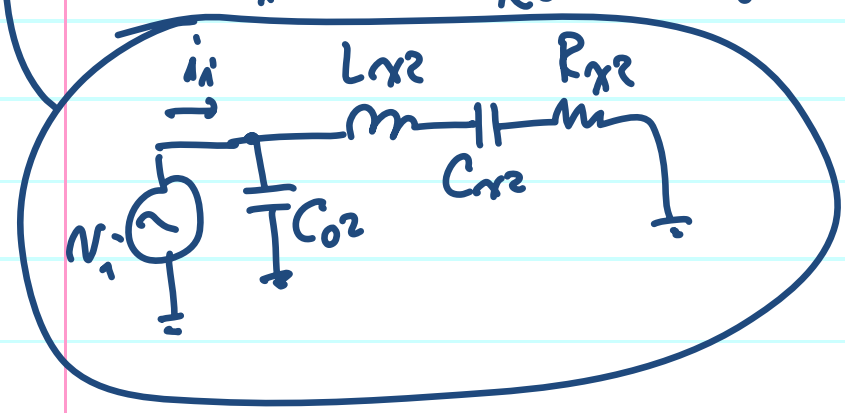


Input Impedance Into Port 2



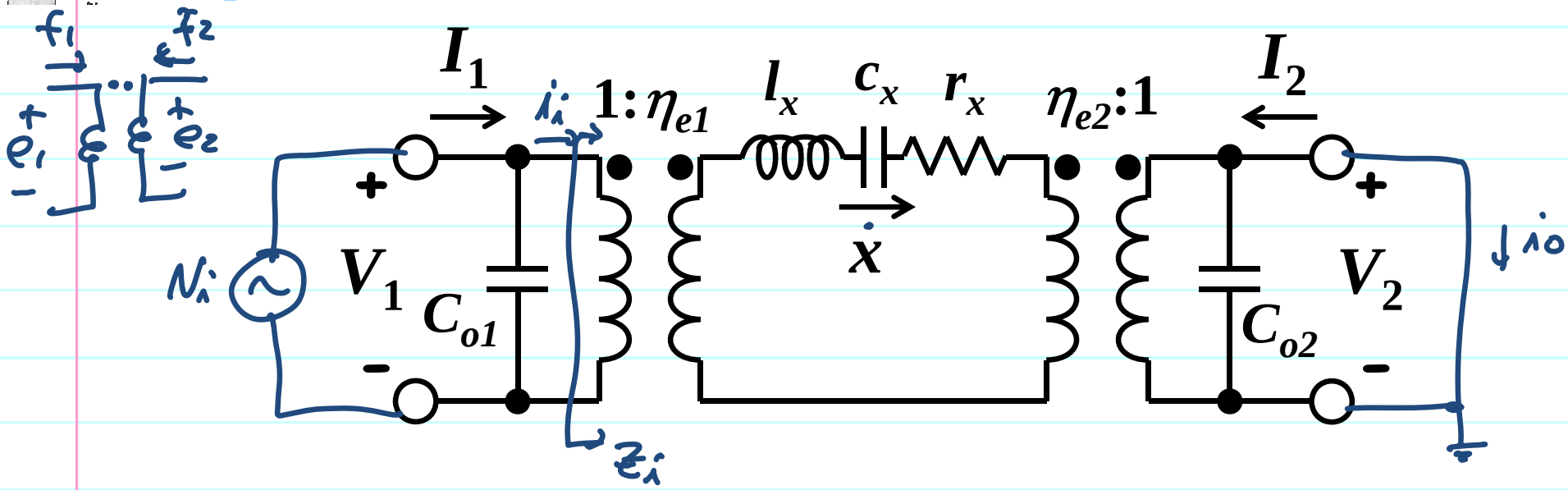
- What is the impedance seen looking into port 2 with port 1 shorted to ground?

$$\frac{v_i}{i_{x2}} = Z_{x2} = \frac{1}{\eta_{e2}^2}\left(j\omega l_x + \frac{1}{j\omega c_x} + r_x\right) = j\omega\underbrace{\left(\frac{l_x}{\eta_{e2}^2}\right)}_{L_{x2}} + \frac{1}{j\omega\underbrace{(\eta_{e2}^2 c_x)}_{C_{x2}}} + \underbrace{\frac{r_x}{\eta_{e2}^2}}_{R_{x2}}$$



Note: These are not the same as L_{x1}, C_{x1}, R_{x1}!

Port 1 to 2 TransG Across the Circuit



- What is the transconductance from port 1 to port 2 with port 2 shorted to ground? $\frac{i_o}{v_i}(j\omega)$

$$\dot{x} = \frac{1}{\eta_{e1}} i_i \qquad i_o = \eta_{e2}\dot{x}$$

$$i_o = \frac{\eta_{e2}}{\eta_{e1}} i_i = \frac{\eta_{e2}}{\eta_{e1}}\left(\frac{v_i}{Z_i}\right) = \frac{\eta_{e2}}{\eta_{e1}} v_i \left[\eta_{e1}^2 \frac{1}{j\omega l_x + \frac{1}{j\omega C_x} + r_x}\right]$$

$$\therefore \frac{i_o}{v_i}(j\omega) = \frac{\eta_{e1}\eta_{e2}}{j\omega l_x + \frac{1}{j\omega C_x} + r_x} = \left[j\omega L_{x12} + \frac{1}{j\omega C_{x12}} + R_{x12}\right]^{-1}$$

$$\begin{cases} L_{x12} = \dfrac{l_x}{\eta_{e1}\eta_{e2}} \\ C_{x12} = \eta_{e1}\eta_{e2} C_x \\ R_{x12} = \dfrac{r_x}{\eta_{e1}\eta_{e2}} \end{cases}$$

Port 1 to 2 v_i-to-i_o Transfer Function

$$\therefore \frac{i_o}{v_i}(j\omega) = \frac{\eta_{e1}\eta_{e2}}{j\omega l_x + \frac{1}{j\omega C_x} + r_x} = \left[j\omega L_{x12} + \frac{1}{j\omega C_{x12}} + R_{x12}\right]^{-1} \quad \begin{cases} L_{x12} = \dfrac{l_x}{\eta_{e1}\eta_{e2}} \\ C_{x12} = \eta_{e1}\eta_{e2} C_x \\ R_{x12} = \dfrac{r_x}{\eta_{e1}\eta_{e2}} \end{cases}$$

Separate freq. response & magnitude:

$$\frac{i_o}{v_i}(s) = \frac{1}{sL_x + \frac{1}{sC_x} + R_x} = \frac{sC_x}{s^2 L_x C_x + 1 + sC_x R_x} = \frac{s\left(\frac{1}{L_x}\right)}{s^2 + \frac{1}{L_x C_x} + s\left(\frac{R_x}{L_x}\right)}$$

$$\left[\frac{1}{L_x C_x} = \omega_o^2,\ Q = \frac{\omega_o L_x}{R_x} \rightarrow \frac{R_x}{L_x} = \frac{\omega_o}{Q}\right] \Rightarrow \frac{i_o}{v_i}(s) = \underbrace{\frac{1}{R_x}}_{\text{Gain Term}} \underbrace{\frac{s(\omega_o/Q)}{s^2 + s(\omega_o/Q) + \omega_o^2}}_{\text{Freq. Shaping Term (Bandpass Biquad)}} = \frac{1}{R_x} H(s)$$

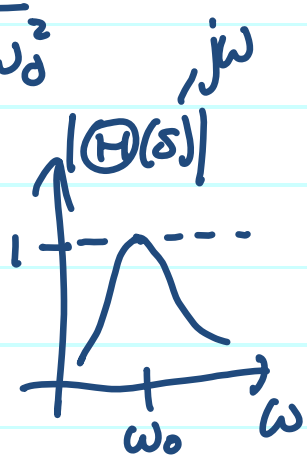
always the same

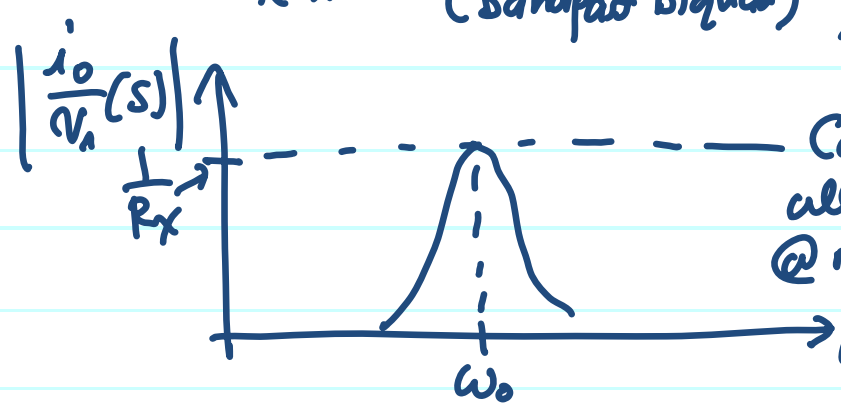
$$H(s) = \frac{s(\omega_o/Q)}{s^2 + s(\omega_o/Q) + \omega_o^2}$$

$\underline{s=0}$: $H(0) = 0$

$\underline{s=j\omega_o}$: $H(j\omega_o) = 1$

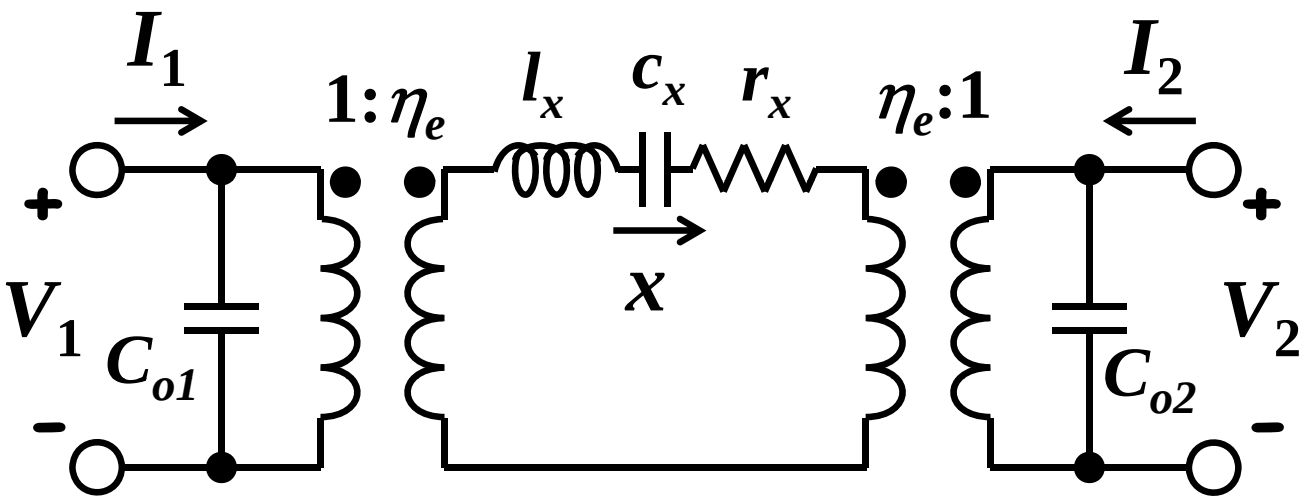
$s=\infty$: $H(\infty) = 0$





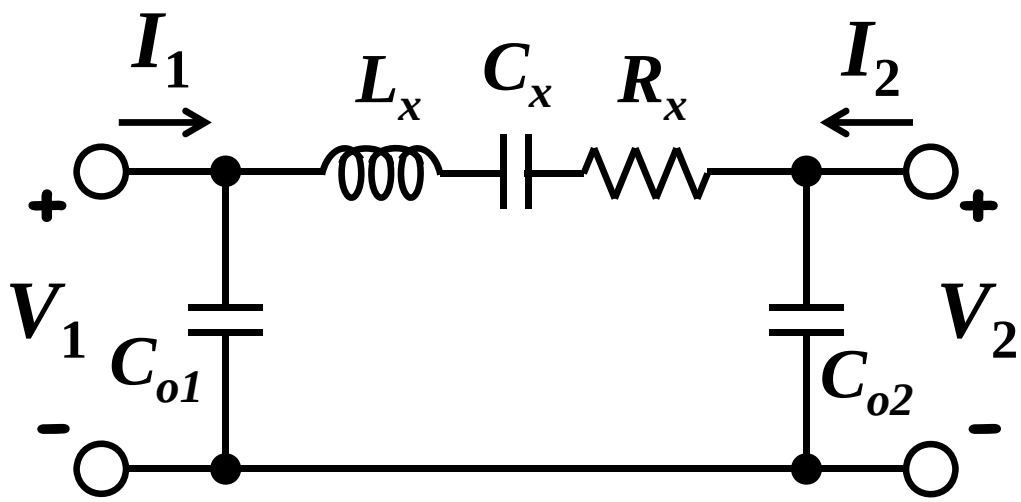
Cause: just do all calculations @ resonance & just multiply by $H(s)$.

Condensed Equiv. Circuit (Symmetrical)



Holds for the symmetrical case, where port 1 and port 2 are identical

If $\eta_{e1} = \eta_{e2}$, then …



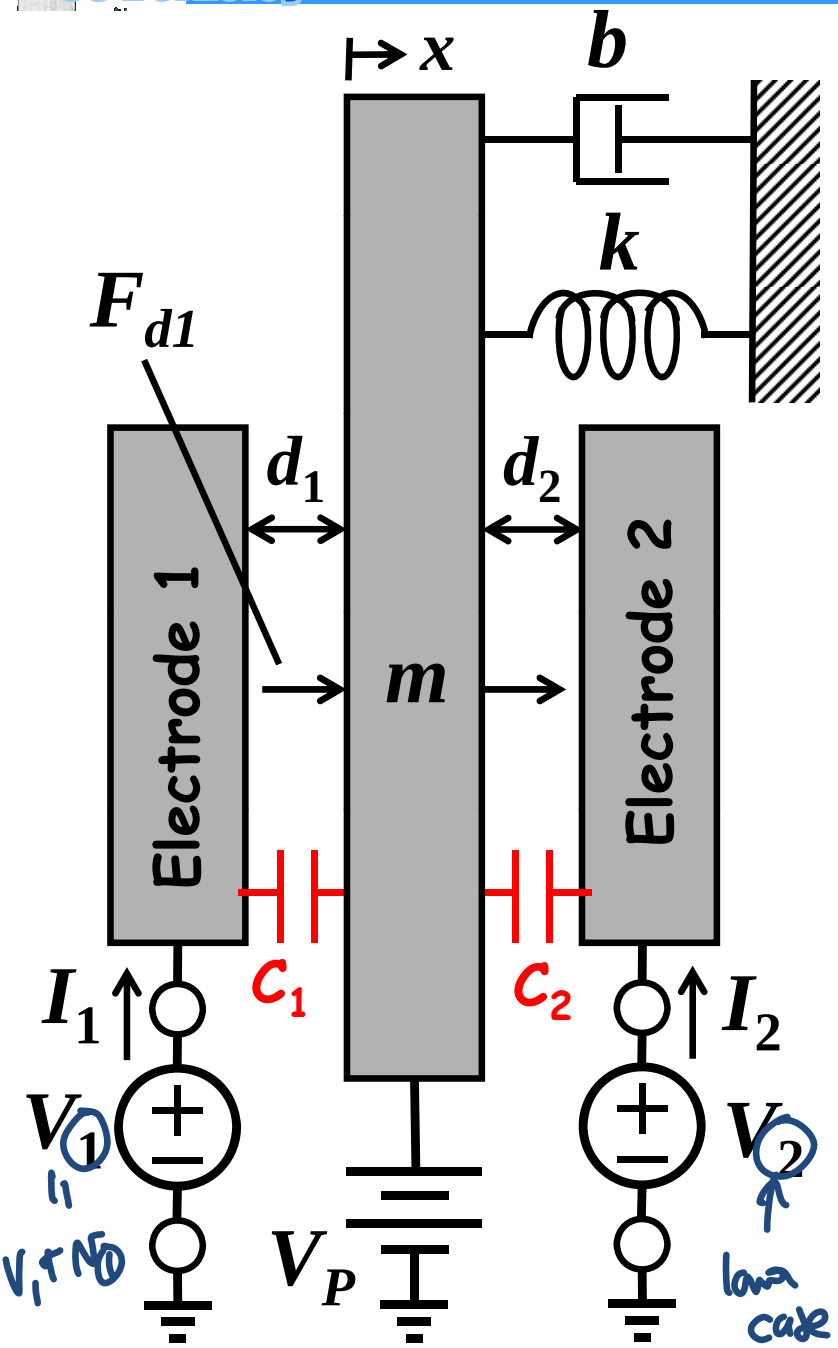
where

$$L_x = \frac{m}{\eta_e^2}$$

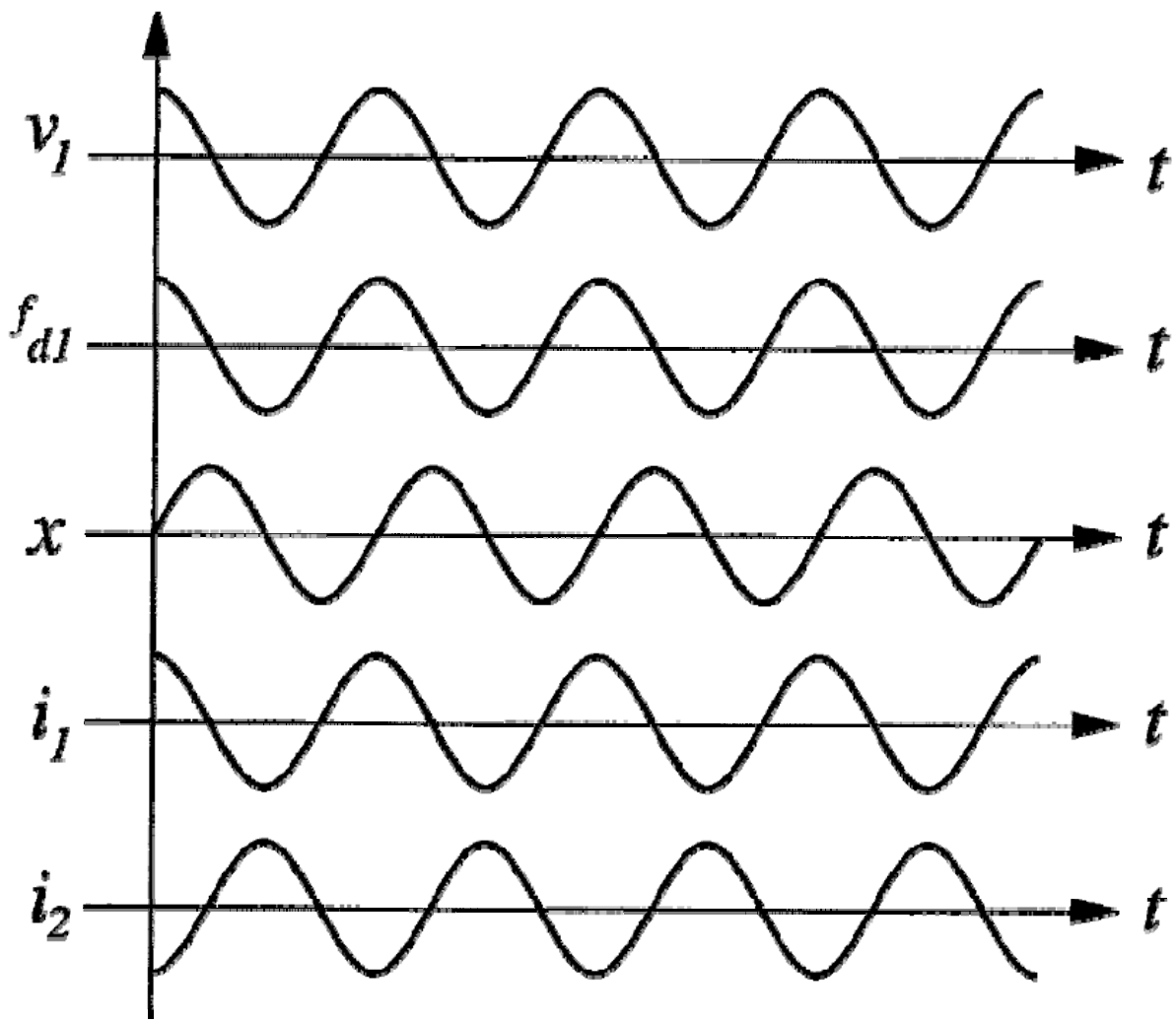
$$C_x = \frac{\eta_e^2}{k}$$

$$R_x = \frac{b}{\eta_e^2}$$

Phasings of Signals

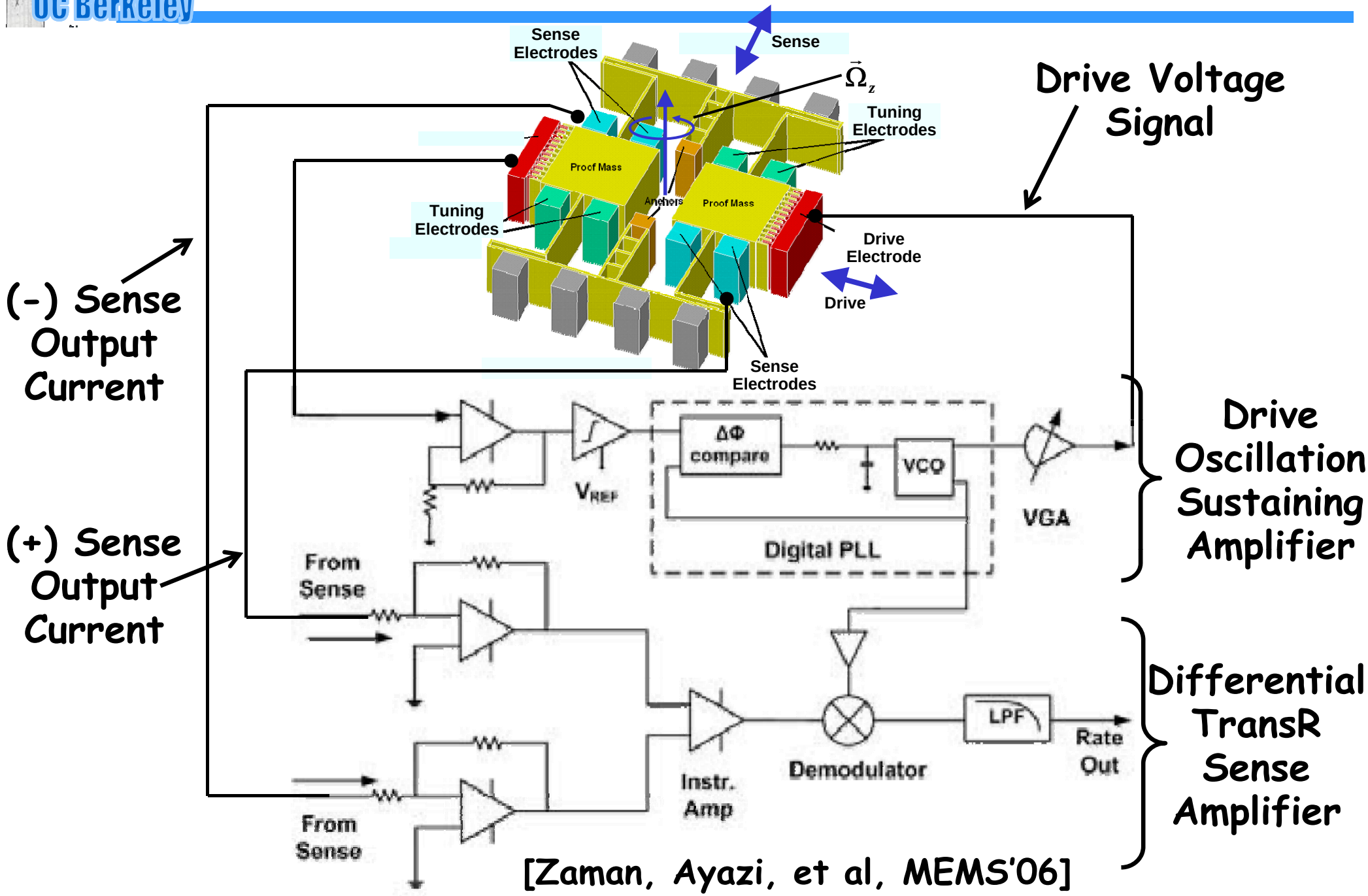


- <u>Below</u>: plots of resonance electrical and mechanical signals vs. time, showing the phasings between them



Sensing Circuits

MEMS-Based Tuning Fork Gyroscope



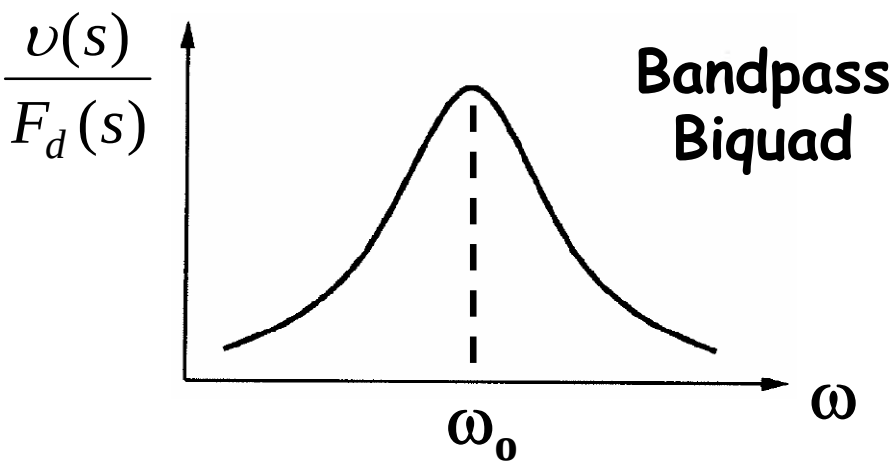
[Zaman, Ayazi, et al, MEMS'06]

Detecting Velocity Versus Position

Velocity

- Transfer Function:

$$\frac{\upsilon(s)}{F_d(s)} = \frac{1}{k}\frac{\omega_o^2 s}{s^2 + (\omega_o/Q)s + \omega_o^2}$$

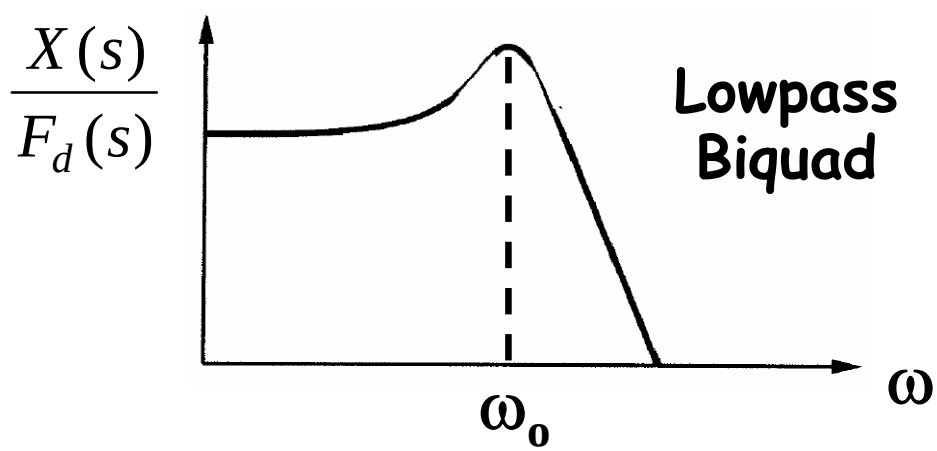


- Detect velocity when the output is at resonance or when a bandpass response is required

Position (i.e., displacement)

- Transfer Function:

$$\frac{X(s)}{F_d(s)} = \frac{1}{k}\frac{\omega_o^2}{s^2 + (\omega_o/Q)s + \omega_o^2}$$



- Detect position when the output is varying slowly, i.e., at low frequencies

Detecting Velocity Versus Position

Velocity

- **Transfer Function:**

$$\frac{\upsilon(s)}{F_d(s)} = \frac{1}{k}\frac{\omega_o^2 s}{s^2 + (\omega_o/Q)s + \omega_o^2}$$

$$= \frac{\omega_o Q}{k}\frac{(\omega_o/Q)s}{s^2 + (\omega_o/Q)s + \omega_o^2}$$

$$= \underbrace{\frac{\omega_o Q}{k}}_{\text{Gain Factor}}\ \underbrace{H(s)}_{\text{Bandpass Biquad Freq. Response}}$$

(Just integrate to get position)

$$\therefore\ @\omega_o:\ \frac{v}{F_d} = \frac{\omega_o Q}{k}$$

Position (i.e., displacement)

- **Transfer Function:**

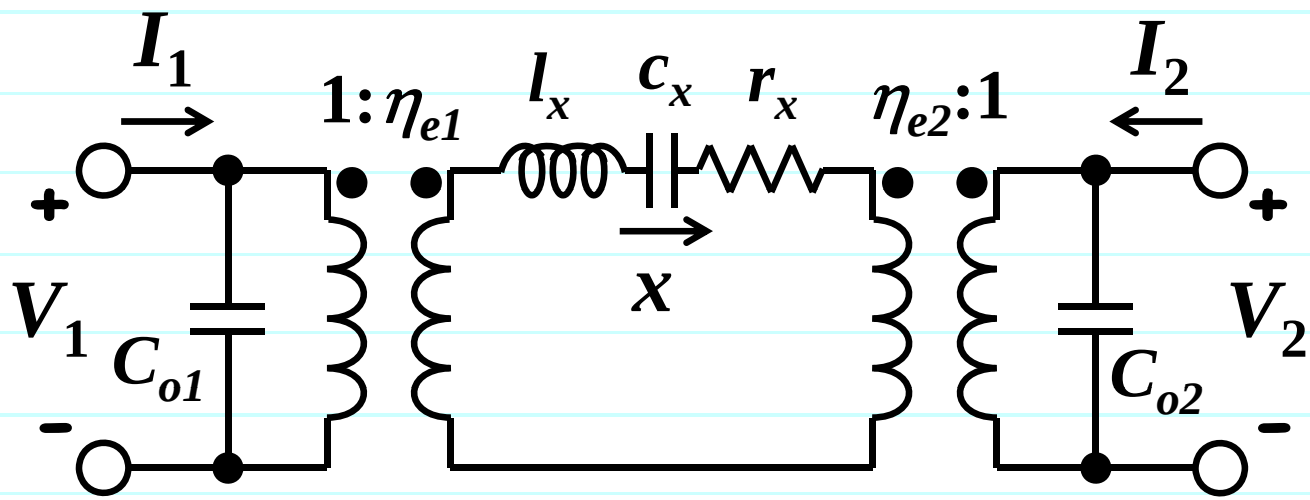
$$\frac{X(s)}{F_d(s)} = \frac{1}{k}\frac{\omega_o^2}{s^2 + (\omega_o/Q)s + \omega_o^2}$$

$$= \frac{1}{k}\frac{(\omega_o/Q)s}{s^2 + (\omega_o/Q)s + \omega_o^2}\frac{\omega_o Q}{s}$$

$$= \underbrace{\frac{\omega_o Q}{k}}_{\text{Gain Factor}}\ \underbrace{\frac{1}{s}}_{\text{Integrator}}\ \underbrace{H(s)}_{\text{Bandpass Biquad Freq. Response}}$$

$$@\omega_o:\ \frac{X}{F_d} = \frac{\cancel{\omega_o} Q}{k}\frac{1}{j\cancel{\omega_o}}(1) = \frac{Q}{jk}\ \checkmark$$

Output Current Measures Velocity



- Relationship between output current and velocity:

$$i_o = \eta_{e2}\dot{x}$$

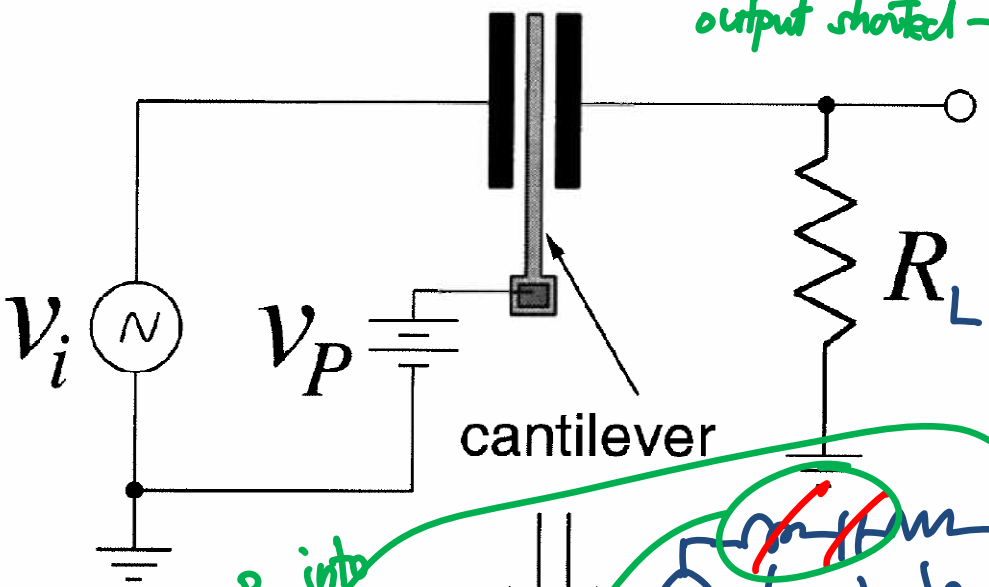
Output current is proportional to velocity, and thus, directly measures velocity

- To turn current into voltage (for a voltage output), send the current into a resistor R_L
- To get position, must integrate → send the current into a capacitor C_L

Velocity-to-Voltage Conversion

- To convert velocity to a voltage, use a resistive load

Consider the mechanical device by itself first, w/ output shorted



$$\frac{\dot{x}}{F_{d1}}(s) = \frac{\omega_o Q}{k}\,\Theta(s)$$

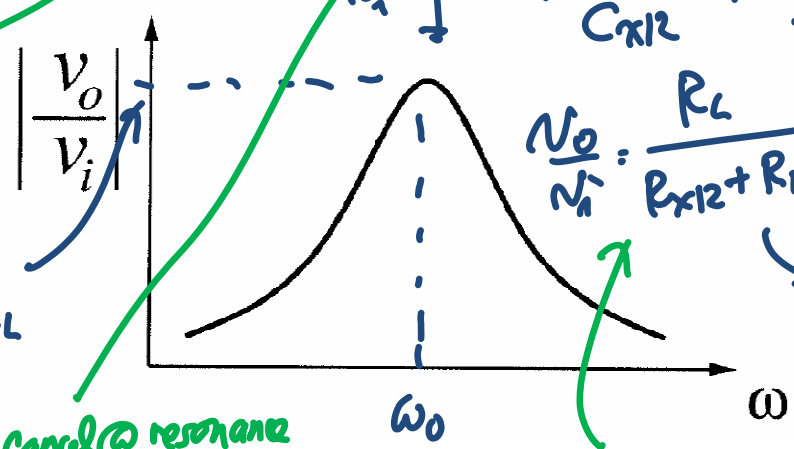
$$\left[F_{d1} = \eta_{e1} v_i\right] \Rightarrow \frac{\dot{x}}{v_i}(s) = \eta_{e1}\frac{\omega_o Q}{k}\,\Theta(s)$$

$$\left[i_o = \eta_{e2}\dot{x}\right] \Rightarrow \frac{i_o}{v_i}(s) = \eta_{e1}\eta_{e2}\frac{\omega_o Q}{k}\,\Theta(s)$$

$$= \frac{\eta_{e1}\eta_{e2} Q}{m\omega_o}\,\Theta(s)$$

$$\frac{1}{R_{x12}}$$

Now, taking R_L into account



$$\frac{R_L}{R_{x12}+R_L}$$

$$\frac{v_o}{v_i} = \frac{R_L}{R_{x12}+R_L}$$

$$\frac{i_o}{v_i}(s) = \frac{1}{R_{x12}}\,\Theta(s)$$

$$\boxed{\frac{v_o}{v_i}(s) = \frac{R_L}{R_{x12}+R_L}\,\Theta(s)}$$

$$Q' = Q\left(\frac{R_{x12}}{R_{x12}+R_L}\right)$$

w/ altered Q

L_x & C_x cancel @ resonance

Solve the problem @ resonance first, then multiply by $\Theta(s)$