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EE C245 - ME C218 Introduction to MEMS Design Fall 2011

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Lecture Module 7: Mechanics of Materials

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Outline

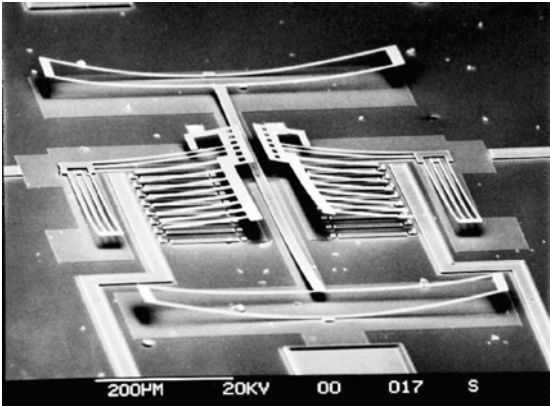
- Reading: Senturia, Chpt. 8
- Lecture Topics:
 - ↗ Stress, strain, etc., for isotropic materials
 - ↗ Thin films: thermal stress, residual stress, and stress gradients
 - ↗ Internal dissipation
 - ↗ MEMS material properties and performance metrics

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Vertical Stress Gradients

- Variation of residual stress in the direction of film growth
- Can warp released structures in z-direction



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Elasticity

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Normal Stress (1D)

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If the force acts normal to a surface, then the stress is called a **normal stress**

Force assumed uniform over the whole area A

Stress = $\left\{ \begin{array}{l} \text{Force per} \\ \text{unit area} \end{array} \right\} = \sigma = \frac{F}{A}$ [N/m² = Pa]
 ↙ standard mks unit

⇒ Microscopic Definition: force per unit area acting on the surface of a differential volume element of a solid body

⇒ Note: assume stress acts uniformly across the entire surface of the element, not at just a point

Differential volume element

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Strain (1D)

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Sometimes a unit called the "microstrain" is used, where 10^{-6}
 $1 \mu\epsilon = \frac{\Delta L}{L}$ of 1 part in 10^6

Strain = $\left\{ \begin{array}{l} \text{Fractional Change} \\ \text{in length} \end{array} \right\} = \epsilon = \frac{L' - L}{L} = \frac{\Delta L}{L}$ [unitless]

In the elastic regime (i.e., for "small" stresses at "low" temperatures), strain is found to be proportional to stress

σ ← stress For solids: MPa → GPa
 ↙ slope = E = Young's modulus of elasticity
 ε ← strain

σ = Eε → $\epsilon = \frac{\sigma}{E}$ [unitless]

Thus, the units of E are the same as σ → Pa

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The Poisson Ratio

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Apply normal stress to a free-standing object

- uniaxial strain
- but also get contraction in directions transverse to the uniaxial strain

⇒ contraction creates a (-) strain:

$$\epsilon_y = \frac{W' - W}{W} = \frac{\Delta W}{W} = -\nu \epsilon_x$$

ν = Poisson ratio [unitless]

- ↳ typical values: 0 → 0.5
- ⇒ inorganic solids: 0.2 → 0.3
- ⇒ elastomers (e.g., rubber): ~ 0.5

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Shear Stress & Strain (1D)

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Note: Assume compensating forces are applied to the vertical faces to avoid a net torque. (This by convention.)

Shear Stress = $\left\{ \begin{array}{l} \text{Force per Unit Area} \\ \text{Parallel to the Surfaces} \end{array} \right\} = \tau = \frac{F}{A}$ [Pa]

Generates a shear strain:

Shear Strain = $\theta = \frac{\tau}{G}$ ← G ≙ shear modulus

$$G = \frac{E}{2(1 + \nu)}$$

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2D and 3D Considerations

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- **Important assumption:** the differential volume element is in static equilibrium \rightarrow no net forces or torques (i.e., rotational movements)
 - \hookrightarrow Every σ must have an equal σ in the opposite direction on the other side of the element
 - \hookrightarrow For no net torque, the shear forces on different faces must also be matched as follows:

Stresses acting on a differential volume element

$\tau_{xy} = \tau_{yx}$
 $\tau_{xz} = \tau_{zx}$
 $\tau_{yz} = \tau_{zy}$

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2D Strain

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- In general, motion consists of
 - \hookrightarrow rigid-body displacement (motion of the center of mass)
 - \hookrightarrow rigid-body rotation (rotation about the center of mass)
 - \hookrightarrow Deformation relative to displacement and rotation

Area element experiences both displacement and deformation

- Must work with displacement vectors
- Differential definition of axial strain: $\epsilon_x = \frac{u_x(x + \Delta x) - u_x(x)}{\Delta x} = \frac{\partial u_x}{\partial x}$

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2D Shear Strain

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\Rightarrow For shear strains, must remove any rigid body rotation that accompanies the deformation

\hookrightarrow use a symmetric definition of shear strain:

$$\tau_{xy} = \theta_2 + \theta_1 \approx \left(\frac{\Delta u_x}{\Delta y} + \frac{\Delta u_y}{\Delta x} \right) = \left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x} \right)$$

For small amplitude deformations.

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Volume Change for a Uniaxial Stress

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Stresses acting on a differential volume element

Given an x -directed uniaxial stress, σ_x :

$\Delta x \rightarrow \Delta x(1 + \epsilon_x)$

$\Delta y \rightarrow \Delta y(1 - \nu\epsilon_x)$

$\Delta z \rightarrow \Delta z(1 - \nu\epsilon_x)$

\downarrow The resulting change in volume ΔV

$$\Delta V = \Delta x \Delta y \Delta z (1 + \epsilon_x)(1 - \nu\epsilon_x)^2 - \Delta x \Delta y \Delta z$$

$$= \Delta x \Delta y \Delta z [(1 + \epsilon_x)(1 - \nu\epsilon_x)^2 - 1]$$

$\{ \text{Assume small strains} \} \Rightarrow \Delta V = \Delta x \Delta y \Delta z [(1 + \epsilon_x)(1 - 2\nu\epsilon_x) - 1]$

$[(1 + m)x]^n \approx 1 + nm x \Rightarrow \approx \Delta x \Delta y \Delta z [1 + \epsilon_x - 2\nu\epsilon_x - 2\nu\epsilon_x^2 - 1]$

$\Delta V \approx \Delta x \Delta y \Delta z (1 - 2\nu) \epsilon_x$

For $\nu = 0.5$ (rubber) \rightarrow no ΔV !

$\nu < 0.5 \rightarrow$ finite ΔV

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Isotropic Elasticity in 3D

- Isotropic = same in all directions
- The complete stress-strain relations for an isotropic elastic solid in 3D: (i.e., a generalized Hooke's Law)

$$\varepsilon_x = \frac{1}{E} [\sigma_x - \nu(\sigma_y + \sigma_z)] \quad \gamma_{xy} = \frac{1}{G} \tau_{xy}$$

$$\varepsilon_y = \frac{1}{E} [\sigma_y - \nu(\sigma_z + \sigma_x)] \quad \gamma_{yz} = \frac{1}{G} \tau_{yz}$$

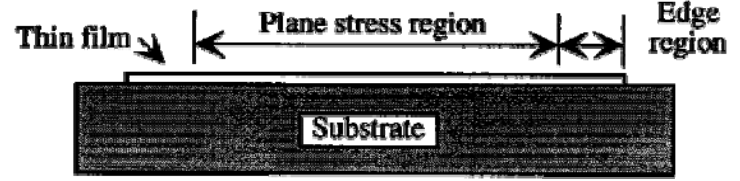
$$\varepsilon_z = \frac{1}{E} [\sigma_z - \nu(\sigma_x + \sigma_y)] \quad \gamma_{zx} = \frac{1}{G} \tau_{zx}$$

Basically, add in off-axis strains from normal stresses in other directions

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Important Case: Plane Stress

- Common case: very thin film coating a thin, relatively rigid substrate (e.g., a silicon wafer)



- At regions more than 3 thicknesses from edges, the top surface is stress-free $\rightarrow \sigma_z = 0$
- Get two components of in-plane stress:

$$\varepsilon_x = (1/E)[\sigma_x - \nu(\sigma_y + 0)]$$

$$\varepsilon_y = (1/E)[\sigma_y - \nu(\sigma_x + 0)]$$

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Important Case: Plane Stress (cont.)

- Symmetry in the xy-plane $\rightarrow \sigma_x = \sigma_y = \sigma$
- Thus, the in-plane strain components are: $\varepsilon_x = \varepsilon_y = \varepsilon$ where

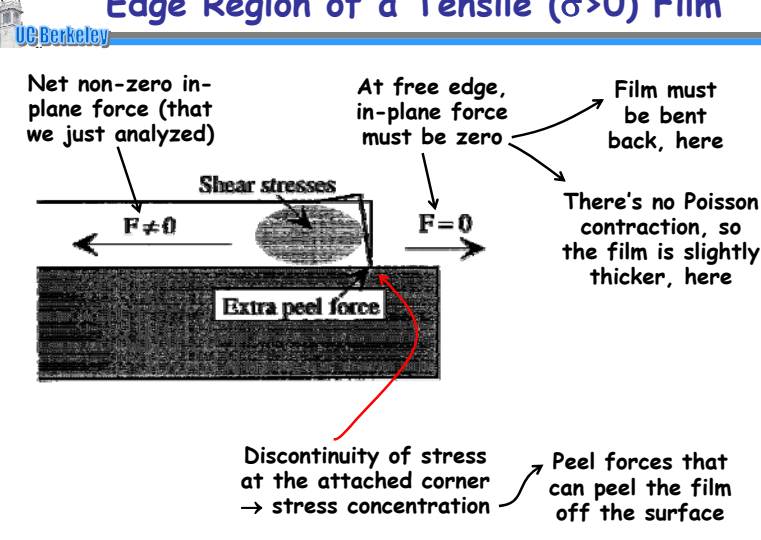
$$\varepsilon_x = (1/E)[\sigma - \nu\sigma] = \frac{\sigma}{[E/(1-\nu)]} = \frac{\sigma}{E'}$$

and where

$$\text{Biaxial Modulus } \triangleq E' = \frac{E}{1-\nu}$$

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Edge Region of a Tensile ($\sigma > 0$) Film



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