Lecture 12: Mechanics of Materials I

- Announcements:
- Module 7 on Mechanics of Materials online
- New version of HW#3 (not different, just makes things clearer in part (d))
- Problem with video recording last Friday, but sound is fine; used module, so sound should be enough
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- Reading: Senturia Chpt. 3, Jaeger Chpt. 11, Handouts: "Bulk Micromachining of Silicon"
- Lecture Topics:
  - Bulk Micromachining
  - Anisotropic Etching of Silicon
  - Boron-Doped Etch Stop
  - Electrochemical Etch Stop
  - Isotropic Etching of Silicon
  - Deep Reactive Ion Etching (DRIE)
  - Wafer Bonding
- ----------------------------------
- Finish up bulk micromachining Module 6
- Start through material of Module 7: Mechanics of Materials, but lectures themselves will be mostly handwritten
- Reading: Senturia, Chpt. 8
- Lecture Topics:
  - Stress, strain, etc., for isotropic materials
  - Thin films: thermal stress, residual stress, and stress gradients
  - Internal dissipation
  - MEMS material properties and performance metrics
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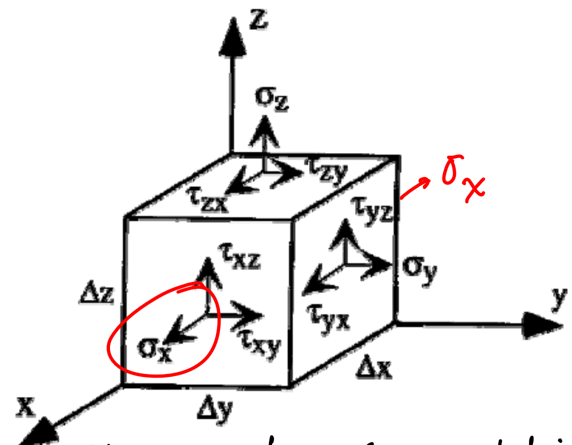
- Last Time:
- Going thru Bulk Micromachining Module 6
- Finish this now

Example. Exercise the "terms"

→ determine the volume change ΔV for a uniaxial stress (along the x-direction)



Upon application of σ_x, what is ΔV?

$$\Delta x \longrightarrow \Delta x(1+\epsilon_x)$$
$$\Delta y \longrightarrow \Delta y(1-\nu\epsilon_x)$$
$$\Delta z \longrightarrow \Delta z(1-\nu\epsilon_x)$$

assuming isotropic material & same ν along y & z

The resulting change in volume: ΔV

$$\Delta V = \underbrace{\Delta x \Delta y \Delta z (1+\epsilon_x)(1-\nu\epsilon_x)^2}_{\text{volume after application of } \sigma_x} - \Delta x \Delta y \Delta z$$

$= \Delta x \Delta y \Delta z \left[(1+\epsilon_x)(1-\nu\epsilon_x)^2 - 1\right]$

[Assume small strains] $\Rightarrow (1+mx)^n \approx 1 + nmx$

$\Delta V = \Delta x \Delta y \Delta z \left[(1+\epsilon_x)(1-2\nu\epsilon_x) - 1\right]$

$$\boxed{\Delta V = \Delta x \Delta y \Delta z (1-2\nu)\epsilon_x}$$

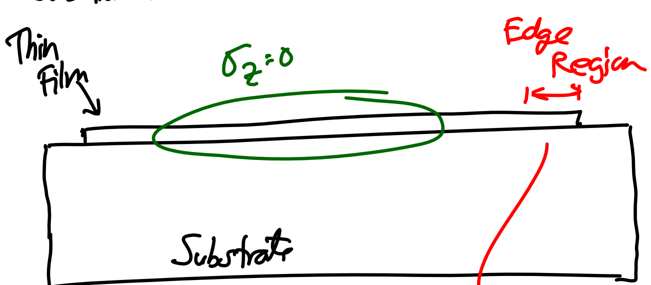
For $\nu = 0.5$ (rubber) $\longrightarrow$ no ΔV!

$\nu < 0.5 \rightarrow$ finite ΔV

For isotropic materials $\rightarrow$ Module 7, pg. 13

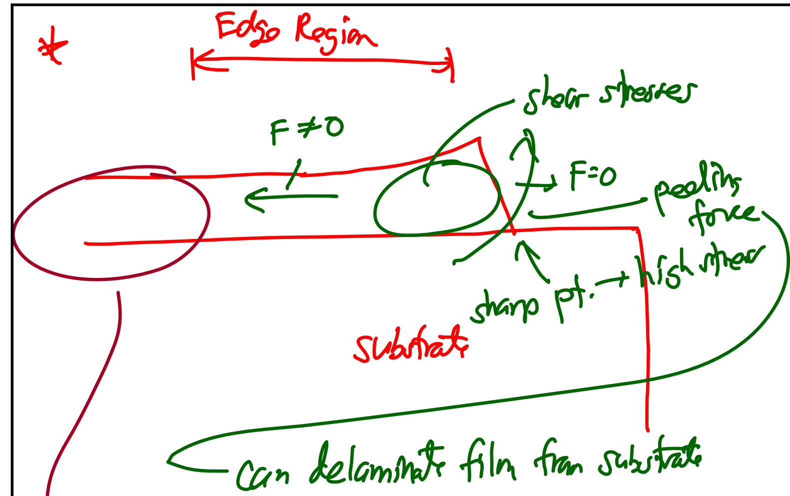
Important Case: Plane Stress

$\Rightarrow$ common case for a thin-film coating on a rigid substrate.



3 thicknesses from the edge

* zoom-in



Take a closer look @ this region: $\sigma_z = 0$

Get two components of stress:

$$\epsilon_x = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + 0)\right]$$

$$\epsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_x + 0)\right]$$

Assume: Plane Stress $\rightarrow$ isotropic $\rightarrow \sigma_x = \sigma_y = \sigma$

(symmetry in the xy-plane) $\rightarrow \epsilon_x = \epsilon_y = \epsilon$

$$\epsilon_x = \frac{1}{E}\left[\sigma - \nu\sigma\right] = \frac{\sigma}{\left(\frac{E}{1-\nu}\right)} \rightarrow \epsilon_x = \frac{\sigma}{E'}$$

where $\boxed{E' \triangleq \text{Biaxial Modulus} = \frac{E}{1-\nu}}$

Linear Thermal Expansion

temperature ↑ → solids expand in volume

Definition. linear thermal expansion coefficient

$$\left.\begin{matrix}\text{Linear Thermal} \\ \text{Exp. Coefficient}\end{matrix}\right\} \triangleq \alpha_T = \frac{d\epsilon_x}{dT} \quad [\text{kelvin}^{-1}]$$