

Lecture 13: Mechanics of Materials II

- Announcements:
- HW#3 due today at 7 p.m. in the EE245 box
- HW#4 online
- Sound for last Friday's video recording is at <http://itunes.apple.com/itunes-u/electrical-engineering-c245/id460480361>
- They are putting videos on Youtube and sound on iTunes
-
- Reading: Senturia, Chpt. 8
- Lecture Topics:
 - ↳ Stress, strain, etc., for isotropic materials
 - ↳ Thin films: thermal stress, residual stress, and stress gradients
 - ↳ Internal dissipation
 - ↳ MEMS material properties and performance metrics
-
- Last Time:

Linear Thermal Expansion

temperature \uparrow \rightarrow solids expand in volume

Definition. linear thermal expansion coefficient

$$\left. \begin{array}{l} \text{Linear Thermal} \\ \text{Exp. Coefficient} \end{array} \right\} \triangleq \alpha_T = \frac{d\epsilon_x}{dT} \quad [\text{Kelvin}^{-1}]$$

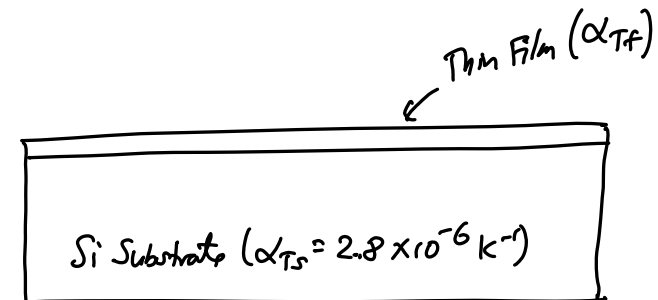
Remarks.

- ① α_T values tend to be in the 10^{-6} to 10^{-7} range
- ② $10^{-6} \text{ K}^{-1} = 1 \text{ } \mu\text{strain/K}$ \rightarrow
- ③ In 3D, get a volume thermal exp. coefficient

$$\frac{\Delta V}{V} = 3\alpha_T \Delta T$$

- ④ For moderate ΔT 's $\rightarrow \alpha_T \approx \text{constant}$
 \searrow for large ΔT , then $\alpha_T = f(T)$

Ex. Thin-Film Thermal Stress



Assume.

- ① Substrate is much thicker than the film.
- ② Film is deposited stress free @ $T_d \leftarrow$ deposition temperature
- ③ Then the whole thing is cooled to room temperature, T_r .

Thermal Strain of the Substrate: (in one plane dimension)

$$\epsilon_s = -\alpha_{TS} \Delta T, \text{ where } \Delta T = T_d - T_r$$

If the film were not attached to the substrate:

$$\epsilon_{f, \text{free}} = -\alpha_{Tf} \Delta T$$

But the film is attached to the substrate

↳ thickness sub \gg thickness film

∴ substrate c/w/r!

↳ therefore, the actual strain experienced by film is that of the substrate:

$$\epsilon_{f, \text{attached}} = -\alpha_{TS} \Delta T$$

Thus:

$$\text{Thermal Mismatch Strain} = \epsilon_{f, \text{mismatch}} = (\alpha_{Tf} - \alpha_{TS}) \Delta T$$

↳ Note this is biaxial strain (assuming the film is deposited isotropically onto the substrate)

$$\sigma_{f, \text{mismatch}} = \underbrace{\left(\frac{E}{1-\nu} \right)}_{E'} \epsilon_{f, \text{mismatch}}$$

Ex. Thin film is polyimide $\rightarrow \alpha_{Tf} = 70 \times 10^{-6} \text{ K}^{-1}$

↓

$$E' = 46 \text{ GPa}$$

deposited @ 250°C , then cooled to $RT = 25^\circ\text{C}$

$$\Delta T = 225 \text{ K}$$

$$\epsilon_{f, \text{mismatch}} = (70 - 2.8) \mu (225) = 1.5 \times 10^{-2}$$

$$\left[\mu = 10^{-6}, m = 10^{-3}, k = 10^3, G = 10^9 \right]$$

$$\sigma_{f, \text{mismatch}} = (4 \text{ G}) (1.5 \times 10^{-2}) = 60.5 \text{ MPa}$$

↑
 10^9

stress is (+) \rightarrow tensile

[-] would be compressive

↑
 SiO_2