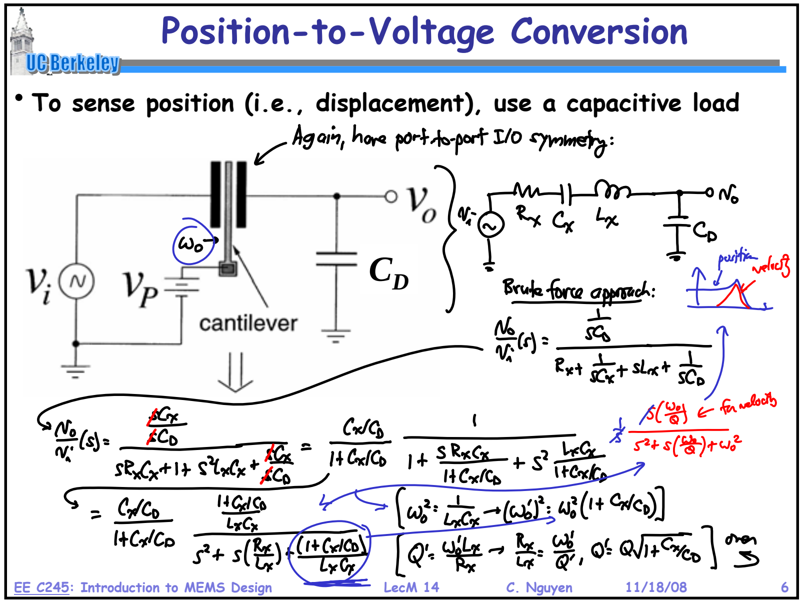

Position-to-Voltage Conversion

- To sense position (i.e., displacement), use a capacitive load

Again, have port-to-port I/O symmetry:

Brute force approach:

$$\frac{v_o}{v_i}(s)=\frac{\frac{1}{sC_D}}{R_x+\frac{1}{sC_x}+sL_x+\frac{1}{sC_D}}$$

$$\frac{v_o}{v_i}(s)=\frac{\frac{\cancel{s}C_x}{\cancel{s}C_D}}{sR_xC_x+1+s^2L_xC_x+\frac{\cancel{s}C_x}{\cancel{s}C_D}}=\frac{C_x/C_D}{1+C_x/C_D}\,\frac{1}{1+\frac{sR_xC_x}{1+C_x/C_D}+s^2\frac{L_xC_x}{1+C_x/C_D}}$$

$$=\frac{C_x/C_D}{1+C_x/C_D}\,\frac{\frac{1+C_x/C_D}{L_xC_x}}{s^2+s\left(\frac{R_x}{L_x}\right)+\frac{(1+C_x/C_D)}{L_xC_x}}$$

$$\cancel{\frac{1}{s}}\,\frac{\cancel{s}\left(\frac{\omega_o}{Q}\right)}{s^2+s\left(\frac{\omega_o}{Q}\right)+\omega_o^2}$$ ← for velocity

$$\left[\omega_o^2=\frac{1}{L_xC_x}\rightarrow(\omega_o')^2=\omega_o^2\left(1+C_x/C_D\right)\right]$$

$$\left[Q'=\frac{\omega_o'L_x}{R_x}\rightarrow\frac{R_x}{L_x}=\frac{\omega_o'}{Q'},\ Q'=Q\sqrt{1+C_x/C_D}\right]$$ over

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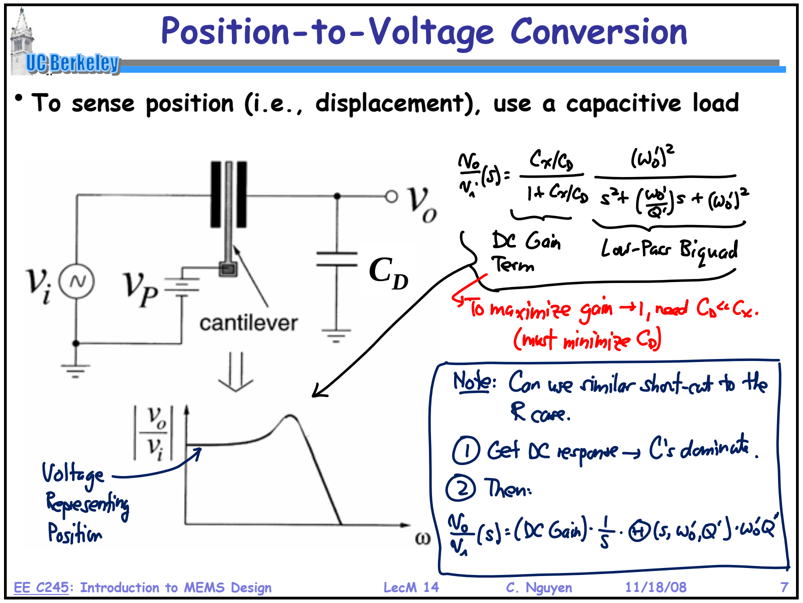

Position-to-Voltage Conversion

- To sense position (i.e., displacement), use a capacitive load

$$\frac{v_o}{v_i}(s)=\underbrace{\frac{C_x/C_D}{1+C_x/C_D}}_{\text{DC Gain Term}}\ \underbrace{\frac{(\omega_o')^2}{s^2+\left(\frac{\omega_o'}{Q'}\right)s+(\omega_o')^2}}_{\text{Low-Pass Biquad}}$$

To maximize gain → 1, need $C_D \ll C_x$. (must minimize C_D)

Voltage Representing Position

Note: Can use similar short-cut to the R case.

① Get DC response → C's dominate.

② Then:

$$\frac{v_o}{v_i}(s)=(\text{DC Gain})\cdot\frac{1}{s}\cdot\textcircled{H}(s,\omega_o',Q')\cdot\omega_o'Q'$$

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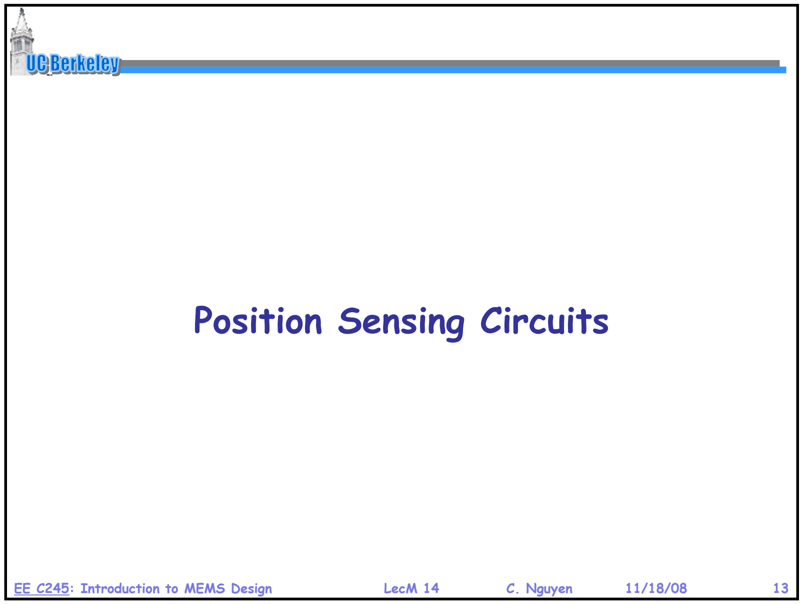

Position Sensing Circuits

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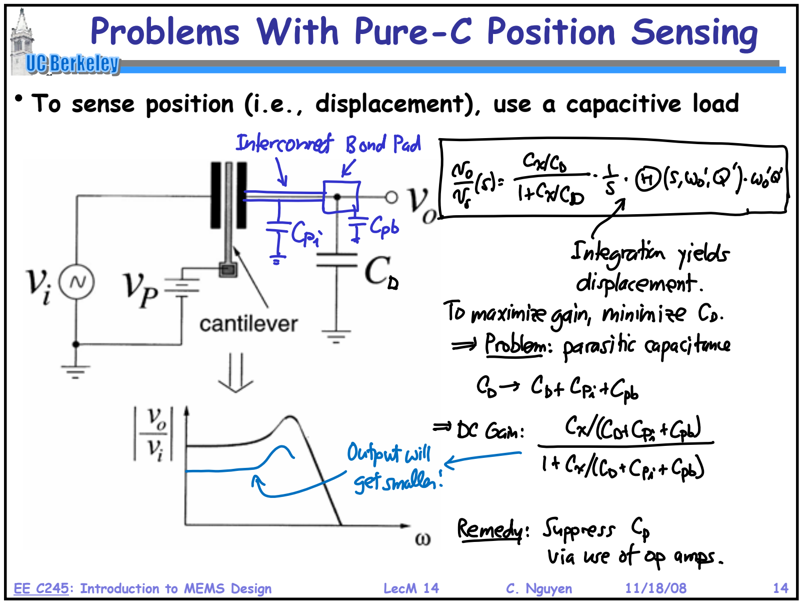

Problems With Pure-C Position Sensing

- To sense position (i.e., displacement), use a capacitive load

$$\boxed{\frac{v_o}{v_i}(s)=\frac{C_x/C_D}{1+C_x/C_D}\cdot\frac{1}{s}\cdot\textcircled{H}(s,\omega_o',Q')\cdot\omega_o'Q'}$$

Integration yields displacement.

To maximize gain, minimize C_D.

⇒ Problem: parasitic capacitance

$$C_D\rightarrow C_D+C_{pi}+C_{pb}$$

$$\Rightarrow \text{DC Gain: } \frac{C_x/(C_D+C_{pi}+C_{pb})}{1+C_x/(C_D+C_{pi}+C_{pb})}$$

Output will get smaller!

Remedy: Suppress C_p via use of op amps.

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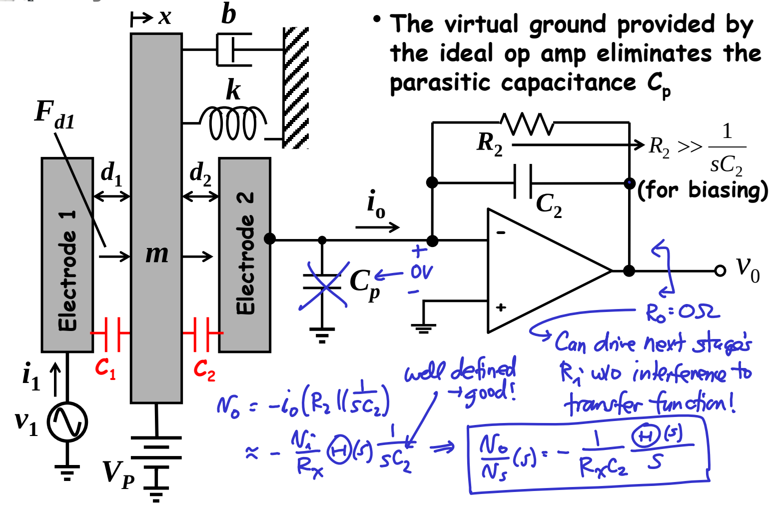
The Op Amp Integrator Advantage

- The virtual ground provided by the ideal op amp eliminates the parasitic capacitance C_p



Can drive next stage's R_i w/o interference to transfer function!

$v_o = -i_o\left(R_2 \| \frac{1}{sC_2}\right)$

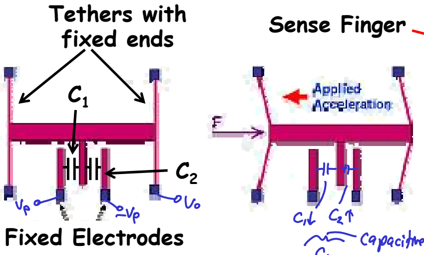
$\approx -\frac{v_i}{R_x}\,\textcircled{H}(s)\,\frac{1}{sC_2} \Rightarrow \boxed{\frac{v_o}{v_s}(s) = -\frac{1}{R_xC_2}\frac{\textcircled{H}(s)}{s}}$

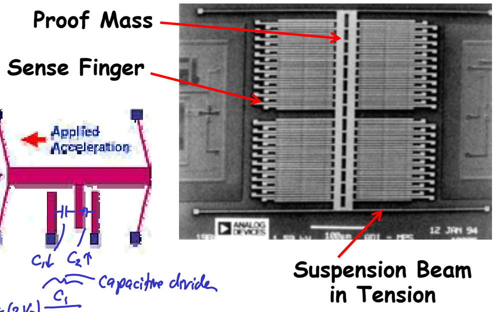
well defined → good!

Differential Position Sensing

Differential Position Sensing

- **Example**: ADXL-50

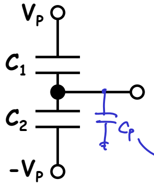




capacitive divide

$V_o = -V_p + (2V_p)\frac{C_1}{C_1+C_2}$

$= \frac{-V_pC_1 - V_pC_2 + 2V_pC_1}{C_1+C_2}$



$\boxed{V_o = V_p\left(\frac{C_1-C_2}{C_1+C_2}\right)}$ (ideal)

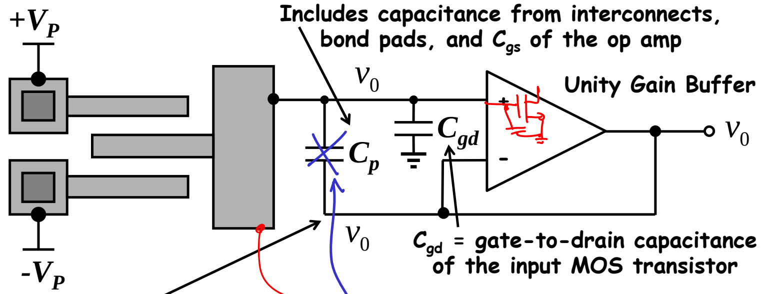
Issue: parasitic C_p!

$V_o = \left(\frac{C_1-C_2}{C_1+C_2+C_p}\right)V_p$

lowers the gain! problem!!!

Soln: use an op amp!

Buffer-Bootstrapped Position Sensing

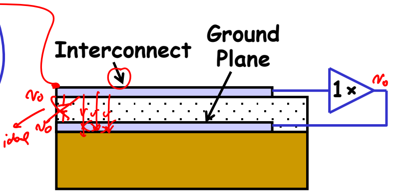


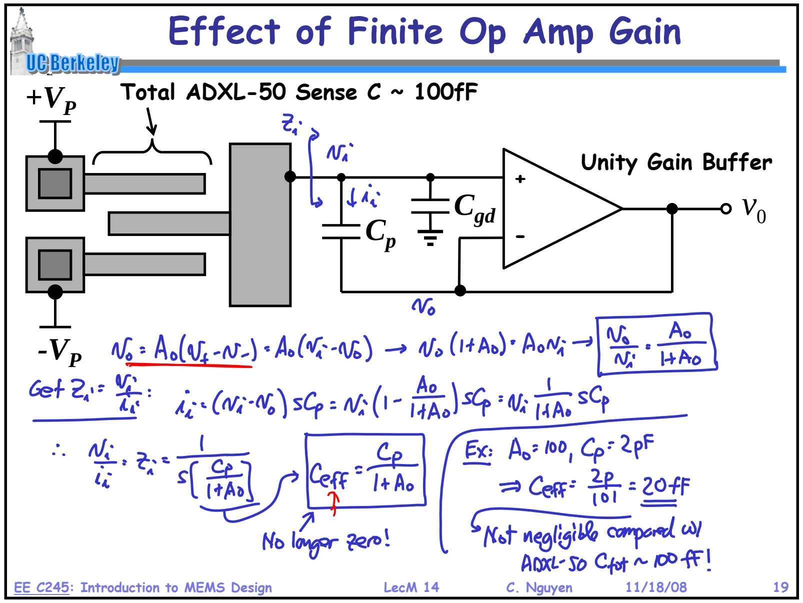
Includes capacitance from interconnects, bond pads, and C_{gs} of the op amp

Unity Gain Buffer

C_{gd} = gate-to-drain capacitance of the input MOS transistor

- Bootstrap the ground lines around the interconnect and bond pads
  - No voltage across C_p
  - It's effectively not there!





Effect of Finite Op Amp Gain

$v_o = A_o(v_+ - v_-) = A_o(v_i - v_o) \rightarrow v_o(1+A_o) = A_o v_i \rightarrow \boxed{\frac{v_o}{v_i} = \frac{A_o}{1+A_o}}$

Get $Z_i = \frac{v_i}{i_i}$: $i_i = (v_i - v_o)sC_p = v_i\left(1 - \frac{A_o}{1+A_o}\right)sC_p = v_i\frac{1}{1+A_o}sC_p$

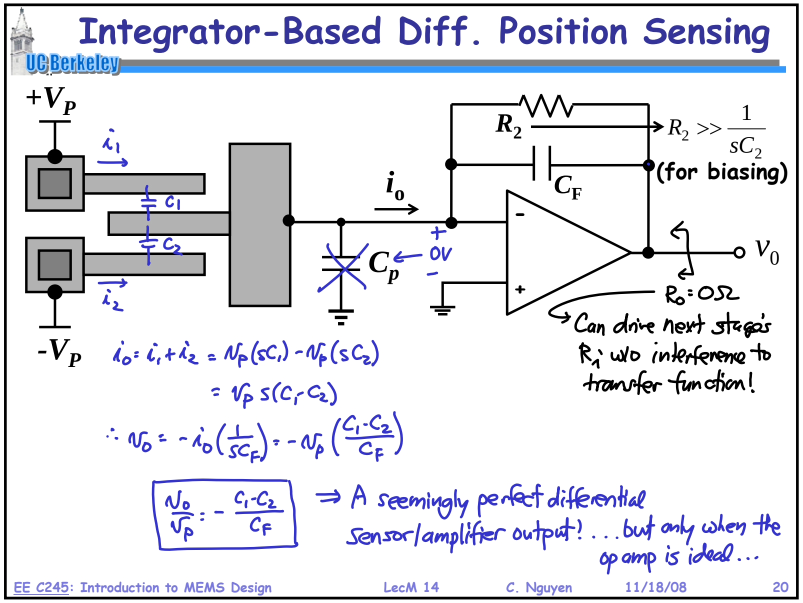
$\therefore \frac{v_i}{i_i} = Z_i = \frac{1}{s\left[\frac{C_p}{1+A_o}\right]}$ $\boxed{C_{eff} = \frac{C_p}{1+A_o}}$

No longer zero!

Ex: $A_o = 100$, $C_p = 2pF$

$\Rightarrow C_{eff} = \frac{2p}{101} = \underline{\underline{20fF}}$

Not negligible compared w/ ADXL-50 C_{tot} ~ 100fF!



Integrator-Based Diff. Position Sensing

Can drive next stage's R_i w/o interference to transfer function!

$i_o = i_1 + i_2 = v_P(sC_1) - v_P(sC_2)$

$= v_P s(C_1 - C_2)$

$\therefore v_o = -i_o\left(\frac{1}{sC_F}\right) = -v_P\left(\frac{C_1 - C_2}{C_F}\right)$

$\boxed{\frac{v_o}{v_P} = -\frac{C_1 - C_2}{C_F}}$ ⇒ A seemingly perfect differential sensor/amplifier output! . . . but only when the op amp is ideal . . .