

Lecture 2w: Benefits of Scaling I

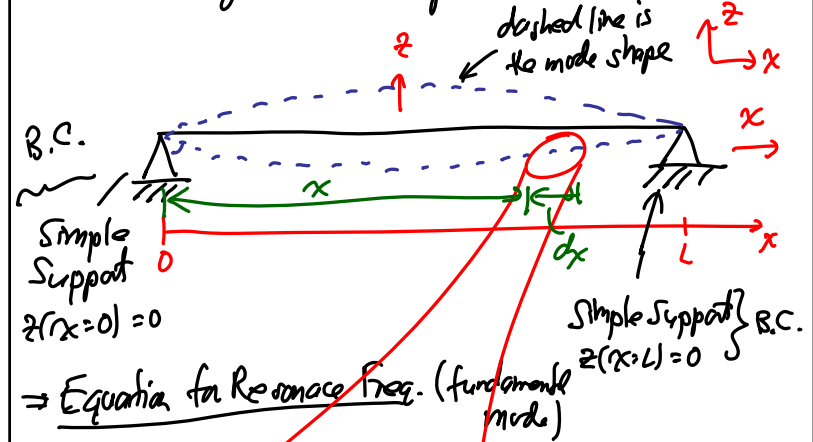
Lecture 2: Benefits of Scaling I

- Announcements:
- Henry Barrow Office Hour Change
 - ↳ Tu 10:30-12 noon → Tu 10-11:30 a.m.
 - ↳ Decided last time
 - ↳ Brian Pepin's office hours remain Th 10:30-12 noon
- The notes from last time are online
- Modules 1 & 2 are online
- HW#1 is online
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- Today:
- Reading: Senturia, Chapter 1
- Lecture Topics:
 - ↳ Benefits of Miniaturization
 - ↳ Examples
 - GHz micromechanical resonators
 - Chip-scale atomic clock
 - Micro gas chromatograph
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- Start going through module 2

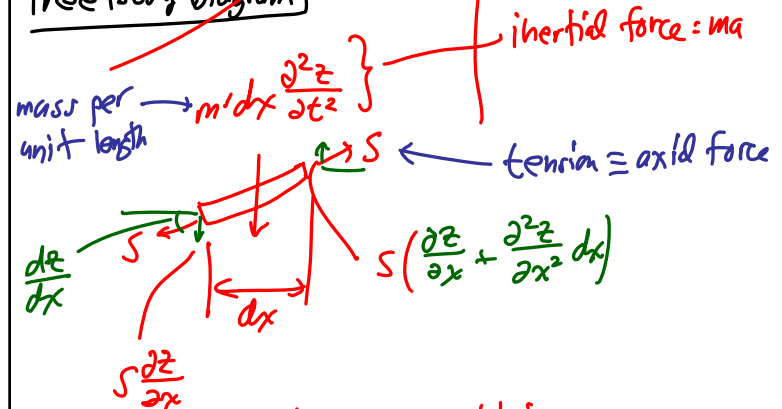
↪ over

Scaling of Guitar Strings

guitar strings \equiv transversely vibrating stretched wire



Free Body Diagram



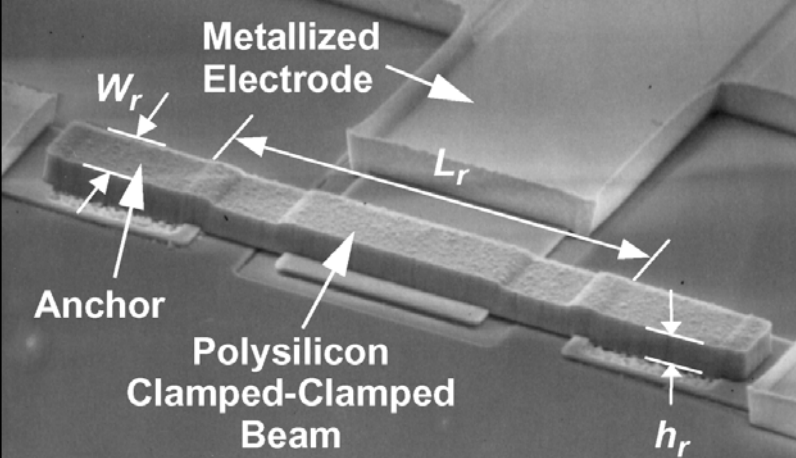
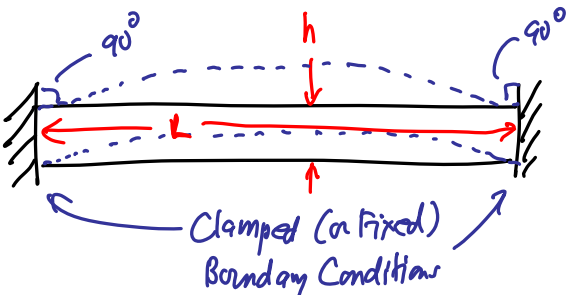
\Rightarrow condition for dynamic equilibrium:

$$S(\frac{\partial z}{\partial x} + \frac{\partial^2 z}{\partial x^2} dx) - S \frac{\partial z}{\partial x} - m'dx \frac{\partial^2 z}{\partial t^2} = 0$$

Solve \downarrow $f_i = \frac{i}{2L} \sqrt{\frac{S}{m'}}$ ← frequency $i = \text{mode} = 1, 2, 3, \dots$

if $L \downarrow \rightarrow f_i \uparrow$

Clamped-Clamped Beam

Anchor
Polysilicon Clamped-Clamped Beam
Clamped (or fixed) Boundary Conditions

⇒ Eq. for resonance:

$$f_0 = \frac{1}{2\pi} \sqrt{\frac{k}{m}} = 1.03 \sqrt{\frac{E}{\rho}} \frac{h}{L^2} \quad (1)$$

where $E \triangleq$ Young's modulus [GPa]
 $\rho \triangleq$ density [kg/m³]
 $h \triangleq$ thickness [m]
 $L \triangleq$ length [m]

Example. $L=40\mu\text{m}, h=2\mu\text{m}$
 polySi $\rightarrow E=150\text{ GPa}, \rho=2300\text{ kg/m}^3$
 $\therefore f_0 = (1.03) \sqrt{\frac{150\text{ G}}{2300}} \frac{2\mu}{(40\mu)^2} \Rightarrow \boxed{f_0 = 10.4\text{ MHz}}$
 acoustic velocity $2x, 5x$

Scaling:

① Scale all dimension equally by a factor S
 $f_0 \sim \frac{S}{S^2} = \frac{1}{S}$

② If scale L only: $f_0 \sim \frac{1}{S^2}$ → even farther nice in f_0 (but problems...)

Example:
 $L=4\mu\text{m} \rightarrow f_0 = (1.03)(2076) \frac{2\mu}{(4\mu)^2}$
 ignore width effects (for now)
 questionable things to do...
 $f_0 = 1.04\text{ GHz!}$
 $\sim 800\text{ MHz}$

Remarks.

① Eq. (1) not accurate when $L \approx h$.
 ② When $L \approx h$ (or when it isn't more than $10xh$)
 → get anchor loss problems

