Lecture 3: Benefits of Scaling II

- Announcements:
- None
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- Today:
- Reading: Senturia, Chapter 1
- Lecture Topics:
  - Benefits of Miniaturization
  - Examples
    - GHz micromechanical resonators
    - Chip-scale atomic clock
    - Thermal Circuits
    - Micro gas chromatograph
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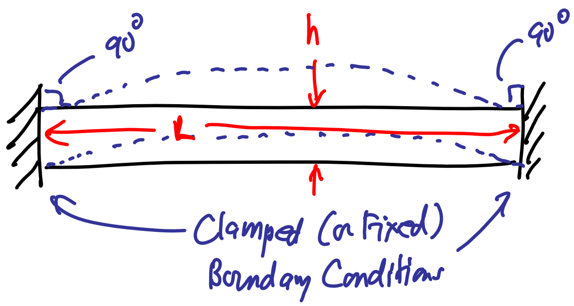
- Last Time:
- Going through module 2

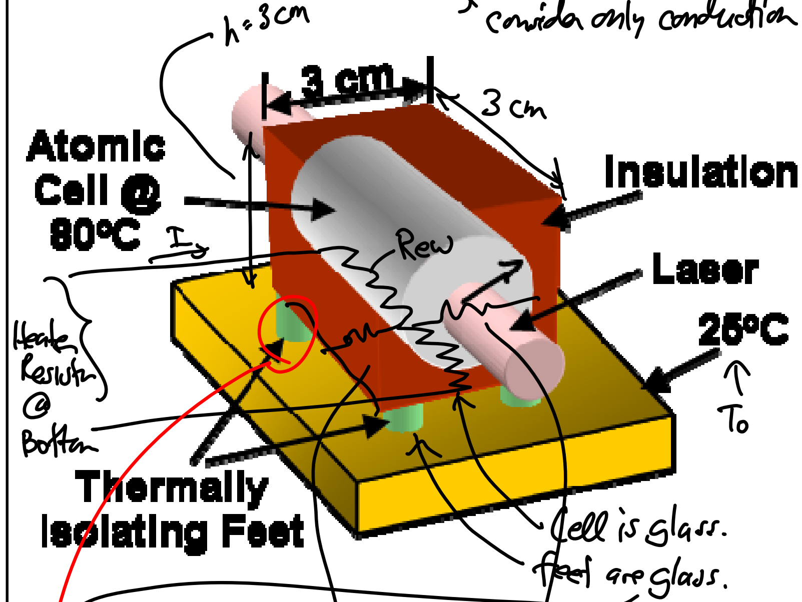


⇒ Eq. for resonance:

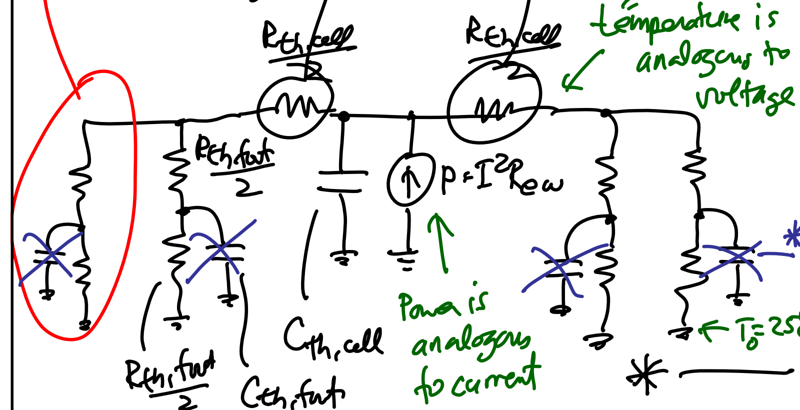
$$f_0 = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = 1.03\sqrt{\frac{E}{\rho}}\frac{h}{L^2} \quad L\downarrow \rightarrow f_0\uparrow\uparrow \qquad (1)$$

where $E \triangleq$ Young's modulus [GPa] $h \triangleq$ thickness [m]
$\rho \triangleq$ density [kg/m³] $L \triangleq$ length [m]

Simplifying Assumption: Everything is in vacuum
→ neglect convection
→ consider only conduction

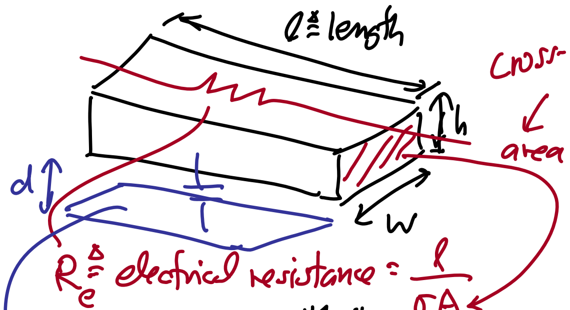


Glass: $\rho_{,glass} = 2500\,kg/m^3$ $K_{glass} = 1.05\,\frac{W}{m\cdot K}$
$C_{p,glass} = 0.5\,J/(g\cdot K)$



Review Electrical Resistance First

(then, attack the thermal R analogy)



$R_e \triangleq$ electrical resistance $= \dfrac{\ell}{\sigma A}$

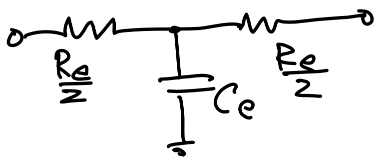
permittivity

electrical conductivity

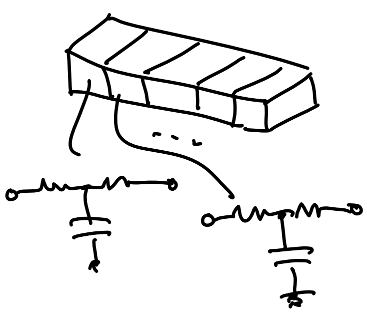
$C_e \triangleq$ capacitance $= \dfrac{\epsilon_o W L}{d}$

Stores Energy (charge energy) $= \frac{1}{2} C V^2 = E$

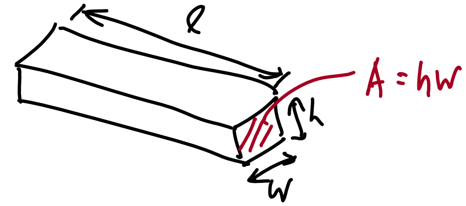
voltage across the capacitor



→ for a distributed σ:



Thermal Ckt



→ thermal capacitance: $C_{th} = \rho V c_p$

specific heat

density

volume

Stores Thermal Energy

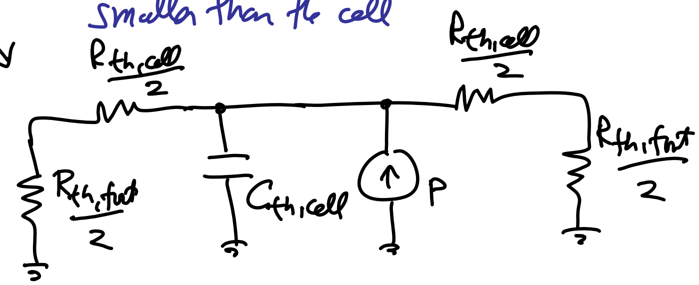
→ thermal resistance: $R_{th} = \dfrac{\ell}{kA}$

length

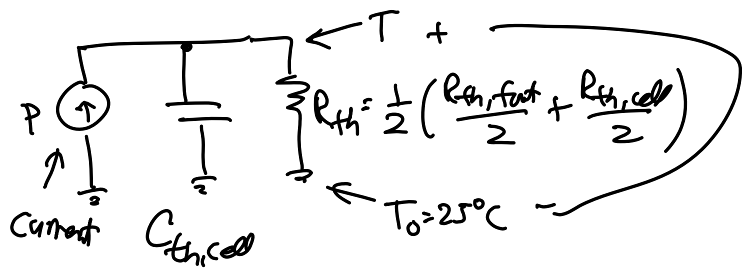
cross-sectional area

thermal conductivity

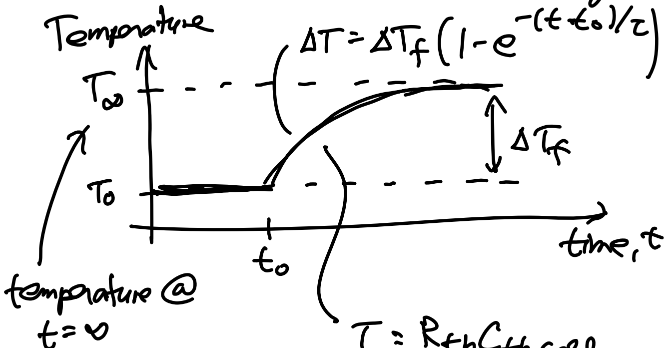
* neglect the $C_{th,foot}$ → feet are much smaller than the cell



reduce



$\Delta T = T - T_0 = P R_{th}$ in steady-steady

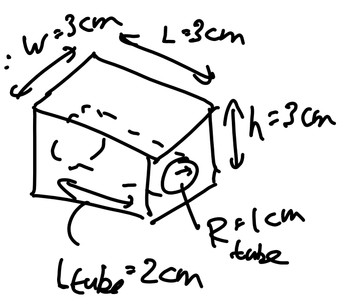


$\tau = R_{th} C_{th,cell}$

time constant determines how fast T_∞ can be achieved

Find $C_{th,cell}$:

⇒ first find the cell volume:



$V_{cell} = hWL - \pi R_{tube}^2 \cdot L_{tube}$

$= 20.7\,cm^3$

specific heat

$C_{th,cell} = \rho_{glass} V_{cell} C_{p,glass}$

$= \left(2500\,\frac{\cancel{kg}}{\cancel{m^3}}\right)\left(1000\,\frac{\cancel{g}}{\cancel{kg}}\right)\left(\frac{1}{100^3}\,\frac{\cancel{m^3}}{\cancel{cm^3}}\right)\left(20.7\,\cancel{cm^3}\right)\left(0.5\,\frac{J}{\cancel{g}\cdot K}\right)$

$\Rightarrow C_{th,cell} = 25.9\,J/K$

